HOTEL CARLTON, WASHINGTON, D. C., SHOWING AIR-CONDITIONING INSTALLATION

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HEATING AND VENTILATING

AIR CONDITIONING

A Home-Study Course and General Reference Work on the Principles, Design, Selection, and Application of Heating and Air-Conditioning Appliances and Systems for Residential, Commercial, Industrial Use.

Illustrated

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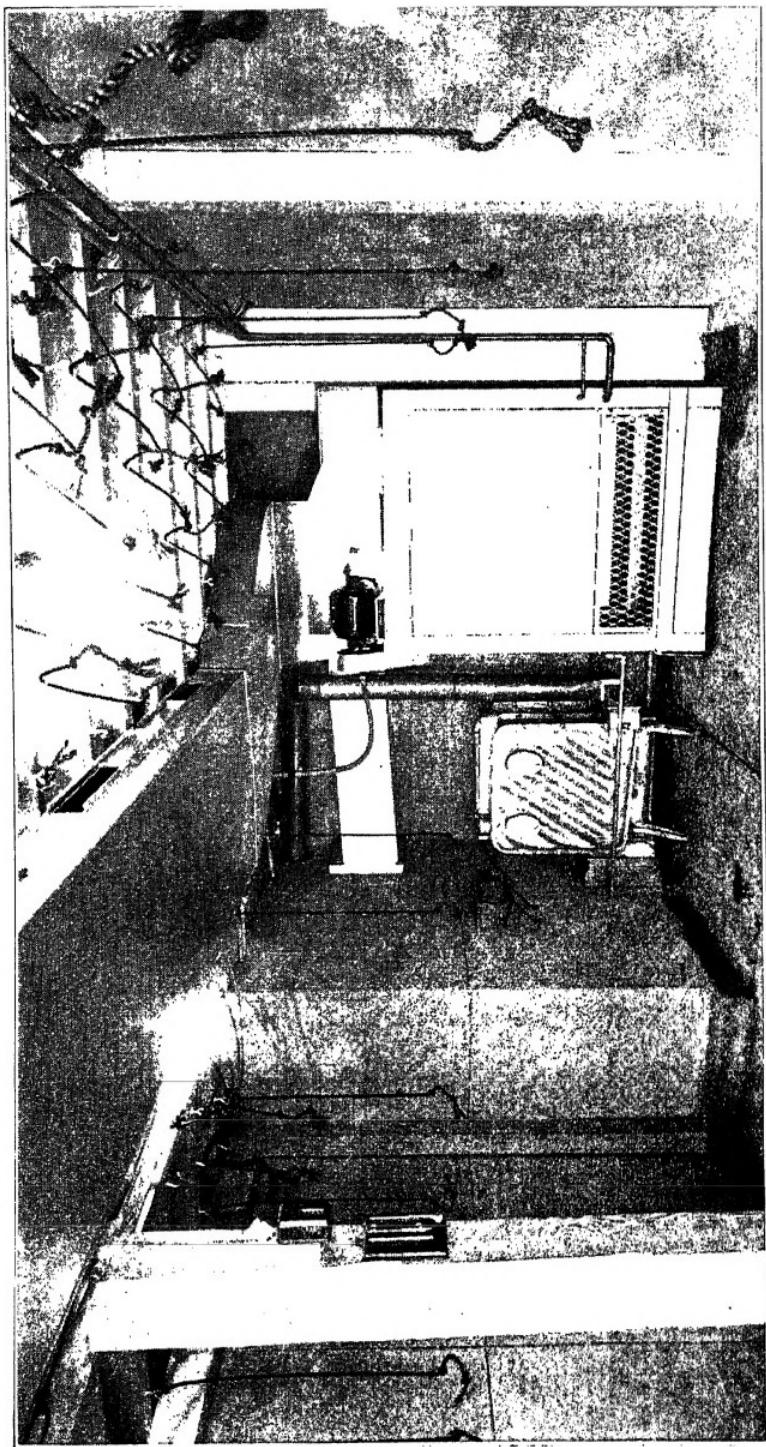
AIR-CONDITIONING OF BANANA STORAGE ROOM

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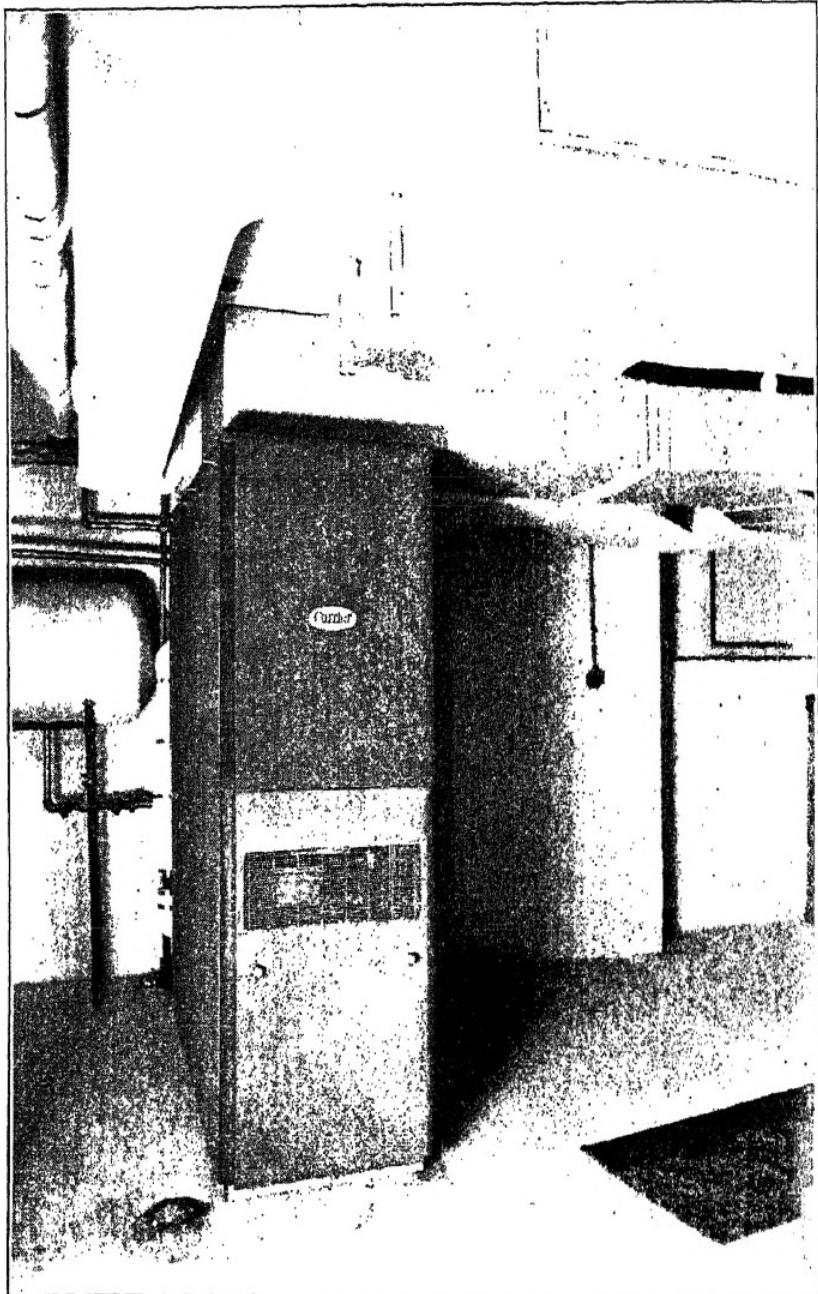
SHOWING AIR-CONDITIONED UNIT IN THE CRARY RESIDENCE,
GREENBURGH, N. Y.

Courtesy of Carrier Corporation, Syracuse, N. Y.

FOREWORD

★Air conditioning is not a new idea. As early as 1911 Willis H. Carrier had formulated some of the principles and laws which are being used in present day air-conditioning engineering. However, he probably did not dream of the possibilities in the field to which he contributed.

★At first, air conditioning was developed only for use in factories where the control of humidity and where summer cooling permitted the continuation of processes previously confined to the cool months of the year. Gradually, the new applications of old principles have brought the possibility of year-round ideal manufacturing conditions in industrial plants.

★The success achieved in industrial air conditioning suggested the possibilities in conditioning primarily for comfort. Theatres, restaurants, and stores—which always had suffered a hot-weather decline in business—offered a fertile field. In theatres, the cooled air increased business during the summer to a point never before known. Restaurants and stores also enjoyed the new summer prosperity. Thus air conditioning for comfort was established. Public demand for greater summer comfort, and the success of air-conditioning engineers in achieving it, gave impetus to the work of residential air conditioning.

★Few industries have enjoyed as rapid a development as air conditioning, and few industries have such potentialities. Air conditioning includes the treatment of air in one or a combination of several of the following ways: heating, cooling, humidifying, dehumidifying, ventilating, and cleaning. The air-conditioning industry is demanding trained men to carry on the work of improving and installing all types of systems, from those used in factories to those used in homes. The training necessary for this work is peculiar to the industry and highly technical.

★These volumes aim to meet the needs of men who, with the demand of the industry for specialized training in mind, look forward to joining those who are building the air-conditioning industry.

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THERMODYNAMICS

CHAPTER I

FUNDAMENTAL PRINCIPLES

Thermodynamics is a very comprehensive science that deals with energy in all its forms and follows it through its transformations from one form to another. Since there are many forms of energy such as mechanical, thermal or heat, chemical, electrical, etc., this science covers a very broad field and is basic to many branches of natural science and engineering. That part of the subject which applies to engineering is generally referred to as Engineering Thermodynamics and, in the main, treats of the relationship between heat energy and mechanical energy commonly referred to as work.

Energy and Work. Energy may be defined as the capacity a body possesses for doing work and is measured in the same units that are used for measuring work. From a mechanical standpoint, it may be potential energy, that is, energy of position, or it may be kinetic energy, which is energy of motion. Water, held behind a dam, has potential energy; if released, the water in its flow has its potential energy changed into kinetic energy. This energy could then be utilized or work could be done equal to the energy possessed by the water by permitting it to flow over a water wheel or through a water turbine.

When a force acts upon a body causing that body to move and to continually overcome a resistance, work is said to be done. This work is equal to the force multiplied by the distance through which it acts. It is measured in a unit which depends upon and indicates the units used for the force and the distance. The time element is of no consideration. The unit of work quite commonly used is the foot-pound, abbreviated ft.-lb. It is the amount of work done by a force of one pound moving through a distance of one foot, or the work done in raising a weight of one pound vertically through a

distance of one foot. It is evident that one foot-pound equals twelve inch-pounds. As a formula for work, we have:

$$W = Fd \qquad (1)$$

where
$$W = \text{work in ft.-lb.}$$
$$F = \text{force in lb.}$$
$$d = \text{distance in ft.}$$

Example. A load of 5 pounds is raised vertically from a floor through a distance of 8 feet. How much work is done?

Solution.

since
$$W = Fd$$
$$W = 5 \times 8 = 40 \text{ ft.-lbs.} \quad Ans.$$

Example. If the load or weight of the preceding example is suddenly released from its suspended position and is allowed to fall freely to the floor, what work is done in bringing it to rest?

Solution.

since
$$W = Fd$$
$$W = 5 \times 8 = 40 \text{ ft.-lbs.} \quad Ans.$$

Example. How much potential energy did the weight of the preceding examples possess in its suspended position relative to the floor?

Solution. Since the weight in falling did an amount of work equal to 40 ft.-lbs., its initial store of energy due to its position, that is, its potential energy with reference to the floor, must have been equal to 40 ft.-lbs. It will be noted that the potential energy of 40 ft.-lbs. was first changed to a kinetic energy of like amount before work was done by the falling body. This principle is employed in many mechanical devices, among which might be mentioned the drop hammer used in die forging.

Motion. A body is said to have motion when it is changing its position. There are many different forms of motion, but the one form generally encountered in machines is plane motion. When a body has plane motion, any point of the body moves continually in a single plane. Its path in this plane may be a straight line or a circle. When the paths of the points of a body are straight lines, the body is said to have a plane motion of translation; when the paths are circles of various diameters, the body is said to have a plane motion of rotation.

The centers of the circular paths are on the same straight line, the axis of rotation, and the body is said to make a given number of revolutions per minute or to have a given r.p.m.

Velocity. Velocity is the rate of motion. There are two kinds, linear velocity, V_L and angular velocity, V_a. A translating body has a linear velocity only, and this is the same for all its points. A rotating body, as such, possesses an angular velocity only, which is also the angular velocity of every one of the points of the body.

In addition to this common angular velocity, each point of a rotating body has its own linear velocity. This is due to the fact that the points have various radial distances from the axis of rotation of the body to which they belong. In general, if

$$V_L = \text{linear velocity in feet per minute}$$
$$d = \text{distance traveled in feet}$$
$$t = \text{time in minutes}$$

We have

$$V_L = \frac{d}{t} \tag{2}$$

Example. A belt travels 5,000 feet in $2\frac{1}{2}$ minutes. Required its linear velocity or speed.

Solution.
From formula (2)

$$V_L = \frac{d}{t} = \frac{5,000}{2\frac{1}{2}} = \frac{5,000}{5} = 5,000 \times \frac{2}{5} = 2,000 \text{ f.p.m. (feet per minute) } Ans.$$

Before the velocities of a point in rotation are considered, it is well to mention a unit of angular measure that will be used to some extent therein. This unit is the radian. It is a much larger unit of angular measure than a degree so when an angle is measured in radians, it will be represented by a much smaller number than that used when it is given in degrees. When an angle of one radian is placed so that its vertex is at the center of a circle (an angle so placed is called a central angle), its sides will naturally become radial lines of the circle. Now these radial sides will cut off or intercept a certain arc on the circumference of the circle. This arc which is intercepted by the radial sides of a central angle of one radian will be exactly equal to

the length of the radius of the circle, no matter what size of circle is used. Hence there can be as many angles of one radian each, or as many radians, at the center of a circle as there are arcs of a length equal to the radius in the circumference of the circle. In other words, if the circumference is divided by R, the radius of the circle, the result will be the number of arcs of this length in the circumference and hence the number of radians there is in the whole central angle of 360 degrees at the center of the circle. Let us perform this division and obtain this relationship between the radian and the degree so that an angle given in one unit can be changed into the other unit. Thus since the circumference of a circle is equal to $2\pi R$, dividing by R, we have

$$\frac{2\pi R}{R} = 2\pi, \text{ the number of radians in 360 degrees}$$

so that

$$2\pi \text{ radians} = 360 \text{ degrees} \qquad (a)$$

Dividing step (a) by 2π,

$$1 \text{ radian} = \frac{360}{2\pi} \text{ degrees} \qquad (b)$$

Dividing step (a) by 360,

$$1 \text{ degree} = \frac{2\pi}{360} \text{ radians} \qquad (c)$$

From step (b) it is evident that an angle given in radians can be changed into degrees by multiplying by $\frac{360}{2\pi}$. From step (c) it is evident that an angle given in degrees can be changed into radians by multiplying by $\frac{2\pi}{360}$. In connection with the radian, it is interesting to note that if a central angle, θ, is given in radians, the arc, S, intercepted by its radial sides can be found by the formula

$$S = R\theta,$$

and S will be in the same unit of linear measure as R, the radius.

In Fig. 1, a is a point on the rim of pulley A, which rotates about its axis, or center O with an r.p.m. N. R_a is the radius of the pulley in feet, and hence is the distance from a to the center of rotation O. In one revolution of A, point a will travel a distance of $2\pi R_a$ feet, in N revolutions it will travel N times as far as in one or a distance equal

to $2\pi R_a N$ feet. Since the pulley makes N revolutions per minute, this distance is traveled by point a in one minute, therefore V_{L_a}, the linear velocity of a, $= 2\pi R_a N$ f.p.m. Evidently the linear velocity of point b located on the hub will be

$$V_{L_b} = 2\pi R_b N \text{ f.p.m.}$$

Likewise, point a, in one revolution of the pulley sweeps through a central angle of 360 degrees or 2π radians. In N revolutions, or in one minute, it will sweep through an angle N times as large, or an angle of $2\pi N$ radians. But angular displacement per unit time is, by
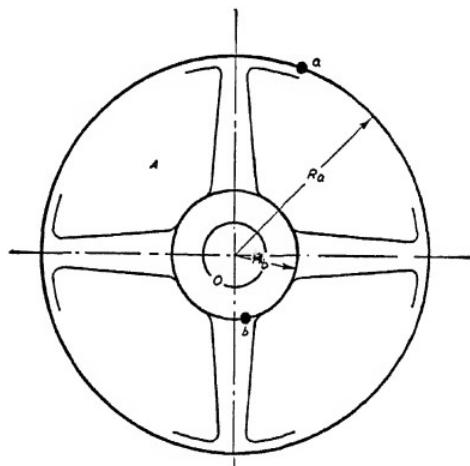
Fig. 1

definition, angular velocity. Therefore the angular velocity of point a is $2\pi N$ radians per minute. Since the angular velocity depends only on the r.p.m. of the rotating body, it is evident that the angular velocities of all points of a rotating body are equal. In general for a rotating body, it follows that

$$V_a = 2\pi N \text{ radians per minute} \tag{3}$$

$$V_L = 2\pi R N \text{ f.p.m.} \tag{4}$$

Example. A pulley with a diameter of 30 inches rotates at 90 r.p.m. Required the linear velocity in f.p.m. and the angular velocity in radians per minute of a point on the rim.

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Solution. From formula (4)

$$V_L = 2\pi R N$$

$$= 2\pi \frac{30}{12} \times 90 = 450\pi = 1,413.72 \text{ f.p.m.} \quad Ans.$$

From formula (3)

$$V_a = 2\pi N$$
$$= 2\pi \times 90 = 180\pi = 565.49- \text{ radians per minute.} \quad Ans.$$

Power. In the previous discussion of work, it was seen and noted that work done is independent of time. Suppose one machine does the same amount of work as another, but in one-third the time. It is evident that the first machine can do three times the work of the second in the same length of time, and hence to compare or rate the machines the time must be considered. This introduces into the rating of machines the term called power. Power is the rate at which work is done. The unit of power is the horsepower, which is equal to 33,000 foot-pounds of work done per minute. Hence, if the power in so-called horsepower units is required, obtain the work in foot-pounds done per minute and divide by 33,000. This can be brought out in a formula in the following manner.

From formula (1)

$$W = Fd \text{ ft.-lbs.}$$

Therefore work done per minute $F\frac{u}{t}$ where t is in minutes. But $\frac{d}{t} = V_L$ (See formula (2)), so that

$$\text{work done per minute } = F \cdot V_L$$

from which statement

$$\text{horsepower, } H = \frac{F V_L}{33,000} \tag{5}$$

Formula (5) is the most fundamental horsepower formula. All other rational formulas which are set up to give horsepowers in specific cases are derived in reality from it.

Example. A load of 2,000 pounds is raised by a cable working around a 24-inch drum of a hoist. If the drum makes 25 revolutions per minute, find the linear velocity of the drum in feet per minute and the horsepower required.

Solution. Applying formula (4); here $R = \frac{12}{12}$ ft., $N = 25$ r.p.m. so that

$$V_L = 2 \times \pi \times \frac{12}{12} \times 25 = 50\pi = 157.08 \text{ f.p.m.} \quad Ans.$$

Applying formula (5), we have $F = 2{,}000$ lbs., $V_L = 157.08$ f.p.m. so that

$$H = \frac{2{,}000 \times 157.08}{33{,}000} = 9.5 \text{ hp.} \quad Ans.$$

Example. In a 12″x18″ double-acting steam engine whose crank-shaft makes 200 r.p.m., the m.e.p. (mean effective pressure, or average unit force) acting upon the piston is 60 pounds per square inch. It is required to find the horsepower developed in the cylinder, called the indicated horsepower, or i.hp.

Note. A 12″x18″ engine is one whose cylinder has an inside diameter of 12″ and whose length of stroke of piston is 18″. Double acting refers to the fact that the steam alternately acts on each side of the piston.

Solution. Since there are two power strokes for each revolution of the crankshaft, the velocity or speed of the piston = $2 \times \frac{18}{12} \times 200 = 600$ f.p.m.

The total force F acting on the piston is equal to the unit pressure, or m.e.p. in lb. per sq. in. multiplied by the area of the piston in sq. in. or

$$F = 60 \times \frac{\pi \times 12^2}{4} = 6{,}785.86 \text{ lbs.}$$

From formula (5)

$$H = \frac{F V_L}{33{,}000}$$

$$= \frac{6{,}785.86 \times 600}{33{,}000} = 123.4 \text{ hp.} \quad Ans.$$

Torque. Fig. 2 illustrates a lever which is keyed to a shaft A. A force F applied to the handle at point B causes the shaft to rotate about its axis O. In doing this, force F, being at a distance R from the axis of the shaft, is said to work through a moment arm R. It should

be noted that the direction of the force is perpendicular to its moment arm, and hence the force acts along a tangent to the circular path of B. In other words F is a tangential force.

The moment of a force in general is the product of the force and the arm through which it works. When a tangential force is applied, as in Fig. 2, its moment is called a turning or twisting moment, or the torque in the rotating shaft. Therefore

$$T_s = FR \tag{6}$$

when
F = tangential force in lbs.
R = radial moment arm in ft.
T_s = torque in ft.-lbs.
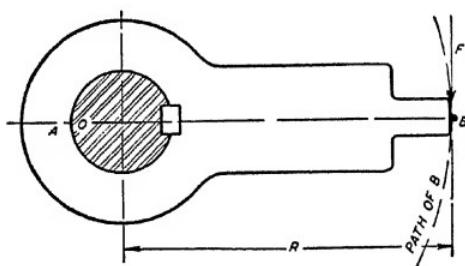
Fig. 2

Many different forms of links could be used in the place of the lever of Fig. 2. For instance, this lever might be a crank on the crankshaft of an engine. In this case, the connecting rod would be attached to the crank at such a point as B, and the connecting rod would deliver the force F to the crank.

When the tangential force F is unknown, but the horsepower and r.p.m. of a shaft are given, the torque can be found from a formula which one can derive as follows:

From formulas (4) and (5)

$$V_L = 2\pi RN$$

$$H = \frac{FV_L}{33,000}$$

Substituting the value of V_L from the first into the second formula, we have

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$$H = \frac{F2\pi RN}{33,000}$$

Rearranging the factors in the second member

$$H = \frac{FR2\pi N}{33,000}$$

But $FR = T_s$, (see formula (6))
therefore

$$H = \frac{T_s 2\pi N}{33,000} \tag{7}$$

Solving for T_s

$$T_s = \frac{33,000H}{2\pi N} \tag{8}$$

where

$T_s =$ torque in ft.-lbs.

$H =$ horsepower

$N =$ r.p.m.

Example. Find the torque in the shaft of a 36-inch pulley to which a tangential force (effective belt pull) of 200 pounds is applied.

Solution.
Using formula (6)

$$T_s = FR$$
$$= 200 \cdot \frac{18}{12} = 300 \text{ ft.-lbs.} \quad Ans.$$

Since 12 inch-pounds = 1 ft.-lb., the above answer could be expressed as 3,600 inch-lbs. and could be obtained directly from formula (6) by substitution for R, its value in inches instead of feet.

Example. If the pulley of the preceding problem makes 250 r.p.m., what horsepower is transmitted?

Solution.
Applying formula (7)

$$H = \frac{300 \cdot 2 \cdot \pi \cdot 250}{33,000} = 14.28 \text{ hp.} \quad Ans.$$

Example. The brake horsepower (that is the horsepower deliv-

ered to the crankshaft of an engine) of an automobile engine is found
to be 80 at 3,400 r.p.m. What is the torque in the crankshaft?

Solution.
Using formula (8)

$$T_s = \frac{33,000 \cdot 80}{2 \cdot \pi \cdot 3,400} = 123.58 \text{ ft.-lbs.} \quad Ans.$$

Temperature. The term temperature is used to define, according
to some arbitrarily selected scale and unit, the hotness of a body. A
body is said to be hot when it has a relatively high temperature; it is
said to be cold when it has a relatively low temperature. It is a known
fact that heat energy has a tendency to flow from a hot body to one
that is cooler and will do so if occasion permits. Thus temperature
determines which way the flow will take place. It is not a measure
of the quantity of heat possessed by the body.

Thermometers. Instruments for the measurement of ordinary
temperatures are called thermometers. Those that record high tem-
peratures are called pyrometers. The design of thermometers is based
on a property certain substances have of expanding when heated and
contracting when cooled.

In the most common forms of thermometers, mercury is placed
in a glass tube of very fine bore. At the lower end of the tube a bulb
is blown. The mercury fills the bulb and part of the tube. The whole
is then heated until the mercury completely fills the tube, after which
the tube is sealed and allowed to cool. In this manner air is eliminated
from the inside of the thermometer. Changes in temperature cause
the mercury to expand or contract, and the liquid will rise or fall
accordingly.

But the thermometer thus made is not yet ready for use. It
must have its divisions properly spaced and in the right places on
the tube so that the height of mercury within the tube can be read
and the temperature thus recorded. All thermometers for accurate
work should have their scales engraved on the tube itself, and not on
a metallic plate to which it may be attached. This is due to the rela-
tive high coefficient of linear expansion of the plate. Before the scale
can be engraved, we must know at least two points on the stem that
correspond to known temperatures. The two points commonly taken
are known as the freezing point and the boiling point.

The freezing point can easily be found by putting the thermometer into finely divided ice which is melting. The boiling point is found by immersing the whole thermometer into steam from boiling water at normal atmospheric pressure. These points are noted carefully on the stem and the distance between them is carefully divided into equal parts called degrees, the number of which divisions depends upon the type of scale to be used. There are two types of scales, the Fahrenheit and the Centigrade.

The Fahrenheit scale divides the distance between the recorded freezing and boiling points into 180 parts or degrees, marking the
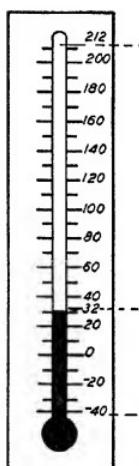
THERMOMETERS

Fig. 3
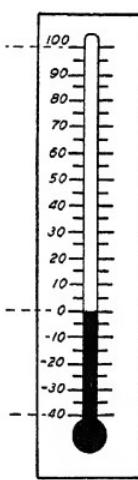
Fig. 4

freezing point as 32° (32 degrees) and the boiling point as 212°. Thirty-two more degrees are laid off below the freezing point, the lowest being the zero point of the scale. This gives 212 degrees between the zero point and the boiling point. The scale may be extended in either direction beyond these points, as is shown in Fig. 3. A point above zero, such as the 40 degree mark, is read, 40° F.; and the point below zero, such as the 20 degree mark, is read −20° F.

The Centigrade scale, Fig. 4, calls the fixed freezing point 0° and the fixed boiling point 100°, hence there are 100 divisions between the two. This scale may also be extended in either direction beyond these points. A point above zero, such as 40 degrees, is read 40° C.; a point below zero, such as 10 degrees, is read −10° C.

A temperature reading on one scale can be converted into a reading on the other scale in the following manner:

Since 100 Centigrade degrees cover the same temperature interval as 180 Fahrenheit degrees, one Centigrade degree is $\frac{180}{100}$ or $\frac{9}{5}$ as long as one Fahrenheit degree. Hence a temperature of m degrees Centigrade ($m°$ C.) is equal to $\frac{9}{5}m$ Fahrenheit degrees above the Centigrade zero. But this point is marked 32° on the Fahrenheit scale, consequently the total reading on the Fahrenheit thermometer will be

$$\frac{9}{5}m + 32$$

The formula for changing C.° Centigrade to its Fahrenheit equivalent, F.° therefore is:

$$F.° = \frac{9}{5} C.° + 32 \qquad (9)$$

and by transposing, we obtain the corresponding formula for changing a Fahrenheit reading to Centigrade:

$$C.° = \frac{5}{9}(F.° - 32) \qquad (10)$$

Example. To what Fahrenheit reading does 100° C. correspond?

Solution. Applying formula **(9)**

$$F.° = \frac{9}{5} \cdot 100 + 32 = 212° \text{ F.} \quad Ans.$$

Example. To what Centigrade reading does 212° F. correspond?

Solution. Applying formula **(10)**

$$C.° = \frac{5}{9}(212 - 32) = 100° \text{ C.} \quad Ans.$$

Absolute Temperature. In taking temperature readings on either the Fahrenheit or the Centigrade thermometer, it is evident that these readings are made with respect to an arbitrarily placed zero of the scale, and that the zero of one scale is not in the same location as the zero of the other. One appreciates that readings of temperatures below zero can be taken on either scale depending upon the range of the particular thermometer being used. And as the readings below zero get larger, the temperatures become lower. It therefore seems

reasonable to suspect that there is some lowest temperature, one below which it is impossible to go. That such a point does exist has been established by research in physics. It is known as Absolute Zero or the Zero of the Absolute Scale. It is located at $-460°$ on the Fahrenheit scale, which is equivalent to $-273°$ on the Centigrade scale.

In Fig. 5 is pictured, diagrammatically, the two scales with the absolute zero placed on each. This design shows that the absolute

FAHRENHEIT	CENTIGRADE
+932	+500
+500	+260
+212 WATER BOILS	+100
+32 ICE MELTS	0
0	-17.78
-40	-40
-460 ABSOLUTE ZERO	-273
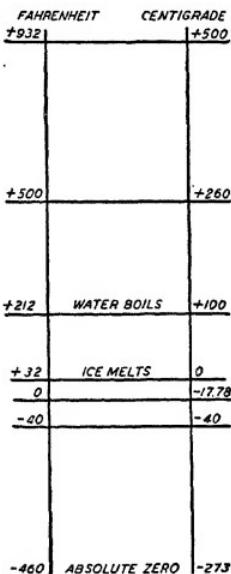
Fig. 5

temperature of a substance in Fahrenheit degrees can be obtained by adding 460° to its Fahrenheit temperature. Likewise, the absolute temperature of a substance in Centigrade degrees can be obtained by adding 273° to its Centigrade temperature. In this text, t will designate a temperature reading on the Fahrenheit scale, and T will designate a temperature on the absolute scale in Fahrenheit degrees; t_c will designate a temperature reading on the Centigrade scale and T_c an absolute temperature in Centigrade degrees. The absolute temperature in Fahrenheit degrees is generally used in Thermodynamics. The relation between these symbols becomes:

$$T = t + 460° \tag{11}$$
$$t = T - 460° \tag{12}$$
$$T_c = t_c + 273° \tag{13}$$
$$t_c = T_c - 273° \tag{14}$$

Example. Find the absolute reading which corresponds to $32°$ F.

Solution. Using formula (11)

$$T = 32° + 460° = 492° \text{ F. absolute.} \quad Ans.$$

Note. This answer is the freezing point reading on the absolute scale in Fahrenheit degrees.

Example. The temperature of a gas is $565°$ F. absolute. What is its Fahrenheit temperature?

Solution. Applying formula (12)

$$t = 565° - 460° = 105° \text{ F.} \quad Ans.$$

Example. Find the absolute reading that corresponds to $-50°$ F.

Solution. From formula (11)

$$T = -50° + 460° = +410° \text{ F. absolute.} \quad Ans.$$

Example. The temperature of a body is $46°$ C. Find its absolute temperature in centigrade degrees.

Solution. From formula (13)

$$T_c = 46° + 273° = 319° \text{ C. absolute.} \quad Ans.$$

Heat. It has previously been stated that heat is a form of energy. This is evident from the fact that heat can be changed into other forms of energy and that other forms of energy can be changed into heat. The first of these statements is demonstrated by all heat engines, such as any steam or gas engine. In these, heat confined in some working substance or medium in the cylinder of the engine is converted into mechanical energy which ultimately becomes the power or torque in the crankshaft. The conversion of mechanical energy into heat or heat energy is noticeable in all machines, where one part rubs on another. Between these parts, which are in contact, there is set up a frictional resistance. Before one part can move with respect to the other, mechanical energy must be supplied to overcome this frictional resistance, and as this is done, heat equal in amount to the mechanical energy supplied for this purpose is generated. This is the reason why bearings get hot, as does also the oil supplied to lubricate them. A basket-ball player receives a floor

burn when he slides violently along the floor because mechanical energy is changed into heat energy. Witness also that the metal chips from a planer or lathe are hot, and that generally speaking when air is compressed its temperature increases, that is, it gets hotter although no heat, as such, is supplied from an external source.

Thus we reason that heat is a form of energy. It is not a substance for when it is added to or absorbed by a body, there is no increase in weight; when it is abstracted from or rejected by a body, there is no decrease in weight.

First Law of Thermodynamics. This law in general states that energy can be neither created nor destroyed, that the transformation of a given quantity of heat energy will yield a definite quantity of mechanical energy and vice versa. It further shows that, if heat energy is added to or absorbed by the working substance or medium of a thermodynamic process, the amount absorbed must be wholly accounted for.

This law is founded on experience and experiment, and like many other general laws and principles of physics, it cannot be proven mathematically. It is known as the Law of the Conservation of Energy. It is now established as an exact law of nature.

British Thermal Unit. Since heat is a form of energy, it may be measured in any unit of energy. This, however, is not customary since units of heat energy have been defined and universally accepted. The unit used in engineering is the British thermal unit, abbreviated B.t.u. It is defined as that amount of heat required to raise the temperature of one pound of water from 63° to 64° F. It is practically the same as the mean B.t.u. which is defined as $\frac{1}{180}$ of the heat required to raise the temperature of one pound of water from 32° F. to 212° F. These statements indicate that the amount of heat required to raise one pound of water one degree Fahrenheit varies with the position of the degree on the thermometric scale, but this variation is very small and need be taken into consideration at relatively high temperatures only.

Mechanical Equivalent of Heat. From the statement of the First Law of Thermodynamics, it is evident that there must be an exact relationship between the unit of heat energy and the unit of mechanical energy. Numerous experiments have been made to de-

termine this relationship. These experiments have proven that

$$1 \text{ B.t.u.} = 778 \text{ ft.-lbs.}$$

The mechanical equivalent of heat is this number, 778. It will be represented in this text by the letter J. Texts universally use this symbol in honor of Joule who in 1843 made the first experiments leading to its determination.

Example. How many foot-pounds of mechanical energy, or work, would result from the complete transformation of 5 B.t.u. of heat energy?

Solution.

Since   1 B.t.u. = 778 ft.-lbs.

           5 B.t.u. = 5×778 ft.-lbs. = 3,890 ft.-lbs.   *Ans.*

Example. How many British thermal units of heat would result from the complete transformation of 1000 foot-pounds of mechanical energy?

Solution.

Since   778 ft.-lbs. = 1 B.t.u.

$$1000 \text{ ft.-lbs.} = \frac{1000}{778} \text{ B.t.u.} = 1.28 \text{ B.t.u.} \quad \textit{Ans.}$$

Example. The heat energy in (or the calorific value of) one gallon of gasoline is about 117,000 B.t.u.

   (a) If this amount of heat energy was completely transformed into mechanical energy, how many foot-pounds of work could be done?

   (b) If the above work was done in lifting vertically a load of one ton, to what distance could the load be lifted?

Solution.

   (a) 117,000×778 ft.-lbs. = 91,026,000 ft.-lbs.   *Ans.*

   (b)   1 ton = 2,000 lbs.

          since $W = Fd$ (see formula (1))

we have by dividing both members of the above equation by F,

$$d = \frac{W}{F}$$

In this example, $W = 91,026,000$ ft.-lbs., and $F = 2,000$ lbs. Sub-

stitution of these values in our formula gives

$$= \frac{91,026,000}{2,000} = 45,513 \text{ ft. } \textit{Ans.}$$

State of a Substance. Any thermodynamic process or change involves the use of a so-called "working substance" or "thermodynamic medium," which has the ability to receive, store, and give out or reject energy as required by the particular process. The medium may be in any one of four physical states of aggregation; namely, solid, liquid, vaporous, and gaseous. The vaporous and gaseous substances are sometimes classified together as gases. The distinction between them will be more clearly defined as this subject is developed. Their use in thermodynamic processes is more general than that of a solid or a liquid.

The mere knowledge of the state of aggregation of a working substance is not enough information to permit one to follow it theoretically through a thermodynamic process. Other characteristics or properties such as pressure, volume, temperature, internal energy, and heat content or enthalpy must be known to determine the "thermodynamic state" of the working medium. A thermodynamic process is made up of a sequence of a great many instantaneous thermodynamic states, some of whose properties may be the same or constant throughout the process, others may be quite different, hence variable. The study of the process in the main consists in finding the exact condition of the final state together with certain energy transformations during the process. This is made possible through the knowledge of the working medium, some of its properties in the initial and final states, and the law governing the change of state throughout the process.

Pressure. Pressure is a force applied over a unit of area. A pressure of 100 pounds per square inch would then be a force of 100 pounds applied over a unit of area, the square inch. It is sometimes spoken of as unit pressure in contrast to total pressure. Evidently, total pressure is equal to unit pressure multiplied by the area upon which the unit pressure acts. In this text, pressure in pounds per square inch will be denoted by p, and pressure in pounds per square foot will be denoted by P. Then $P = 144p$ since there are 144 square inches in 1 square foot.

The pressure, as stated before, is one of the fundamental properties of a working substance. It can be considered as a pressure produced by the working substance, or as a pressure introduced upon the working substance. The former is the point of view generally taken.

Atmospheric pressure is that pressure caused by the weight of the atmosphere. It is measured above the true zero of pressure by a so-called barometer. This is the reason why atmospheric pressure is often called barometric pressure. Atmospheric pressure varies slightly from day to day due to atmospheric conditions. It decreases

GAUGE PRESSURE
60LBS.◻"

ABSOLUTE
PRESSURE
74.7LBS.◻"
(60+14.7)

ATMOSPHERIC PRESSURE

VACUUM GAUGE READING
10LBS. ◻"

ATMOSPHERIC
PRESSURE
14.7LBS.◻"

ABS.PRESSURE   4.7LBS◻"
(14.7−10)

ZERO OF ABSOLUTE SCALE
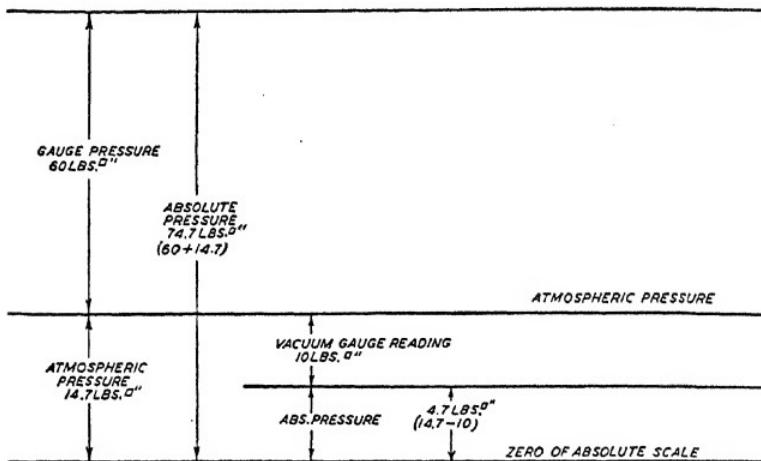
Fig. 6

as the altitude increases. A standard or a normal atmospheric pressure thus becomes convenient. It is based on sea-level conditions and is equal to 29.92 inches of mercury, or its equivalent, 14.7 pounds per square inch.

When a gas, vapor, or fluid in general is confined in a container, the gauge used for recording its pressure measures the difference between the pressure in the container and the atmospheric pressure outside in pounds per square inch. If the pressure within is greater than atmospheric pressure, the gauge is called a pressure gauge, and the pressure obtained by it is referred to as gauge pressure. To obtain a pressure reading relative to the true zero of pressure, the absolute zero, the pressure gauge reading must be added to the

atmospheric pressure. This gives the absolute pressure in pounds per square inch.

Quite often the pressure of a confined fluid is less than that of the surrounding atmosphere. The confined fluid is then said to be under a partial vacuum. In such a case, the instrument used is a vacuum gauge. It records the difference between the atmospheric pressure and that within the vessel in inches of mercury. Hence to obtain the absolute pressure reading, it is necessary to subtract the vacuum gauge reading from the atmospheric pressure. For very careful work, barometer and gauge readings should be made simultaneously. For many calculations, it is accurate enough to assume the atmospheric pressure as normal.

In practically all thermodynamic problems, it is necessary to use the absolute pressure. Fig. 6 graphically shows the relationships existing between gauge, atmospheric, absolute, and partial vacuum pressures.

Remember that for obtaining absolute pressures from a pressure gauge reading:

   absolute pressure = pressure gauge reading+atmospheric pressure;

and for obtaining absolute pressure from a vacuum gauge reading,

   absolute pressure = atmospheric pressure−vacuum gauge reading.

Example. A pressure gauge on a steam boiler reads 101 pounds per square inch. What is the corresponding absolute pressure in pounds per square inch?

Solution. Since no barometer reading is given, the atmospheric pressure is assumed to be normal, or 14.7 lbs. per sq. in. Hence, adding

$$101+14.7 = 115.7 \text{ lbs. per sq. in. absolute.} \quad Ans.$$

Example. A vacuum gauge on a condenser read 27.5 inches of mercury and at the same time the barometer read 29.4 inches of mercury. What was the absolute pressure in the condenser in pounds per square inch?

Solution. Two methods of solution are possible. In one, both pressures are first changed from inches of mercury to pounds per square inch by multiplying each by the conversion factor 0.491. Their difference, obtained by subtracting the vacuum reading from the barometer reading, gives the result in pounds per square inch.

Thus,

$$27.5 \times 0.491 = 13.502 \text{ lbs. per sq. in.}$$
$$29.4 \times 0.491 = 14.435 \text{ lbs. per sq. in.}$$

Subtracting

$$14.435 - 13.502 = 0.933 \text{ lb. per sq. in. absolute. } Ans.$$

In the other method, the difference is first obtained and hence is in inches of mercury. The answer is then changed into the unit, pounds per square inch, by multiplying it by 0.491.

Thus, subtracting,

$$29.4 - 27.5 = 1.9 \text{ in. of Hg. (mercury) absolute}$$
$$1.9 \times 0.491 = 0.933 \text{ lb. per sq. in. absolute. } Ans.$$

Example. Steam is admitted into a steam turbine at a pressure of 300 pounds per square inch, gauge. It expands within the turbine and is exhausted into a condenser which shows a vacuum of 28 inches of mercury. The atmospheric pressure is normal. Find the drop in pressure in pounds per square inch.

Solution. $300 + 14.7 = 314.7$ lbs. per sq. in. absolute.

This is the absolute pressure of the steam as it enters the turbine.

$$29.92 - 28 = 1.92 \text{ in. of Hg.}$$
$$1.92 \times 0.491 = 0.943 \text{ lb. per sq. in. absolute}$$

This is the absolute pressure of the steam as it is exhausted from the turbine.

$314.7 - 0.943 = 313.757$ lbs. per sq. in. drop in pressure. *Ans.*

Specific Volume. The specific volume of a substance is its volume per unit weight or mass. It is generally given in cubic feet per pound. One pound of air at 32° F. and under a pressure of 14.7 pounds per square inch absolute has a volume of 12.39 cubic feet. Therefore the specific volume of air under these conditions is 12.39 cubic feet per pound.

In this text

$$M = \text{weight or mass in pounds}$$
$$V = \text{total volume in cubic feet}$$
$$v = \text{specific volume in cubic feet}$$
$$V = Mv$$

Specific Weight. The specific weight of a substance is its weight per unit volume. This is also known as the density, and is generally stated in pounds per cubic foot. As stated in the preceding article, the specific volume of air is 12.39 cubic feet per pound. Now if 12.39 cubic feet of air weigh 1 pound, 1 cubic foot will weigh $\frac{1}{12.39}$ of 1 pound, or $\frac{1}{12.39}$ pounds. This is the weight in pounds of 1 cubic foot of air. Therefore from our definition it is the specific weight or the density. Comparing this specific volume and the corresponding specific weight, we find the numerical value of the former is 12.39, the numerical value of the latter is $\frac{1}{12.39}$. Hence the density or specific weight of any substance is the reciprocal of the specific volume of the substance. Then it must also be true that the specific volume is the reciprocal of the density.

Heat and the Working Medium. Most of us know the effect of heat upon a body. Experience has taught us that when a body is heated, it gets hot; that is, its temperature rises. Under the conditions met with in daily life, this can not be disputed. We also have noticed that heat may do other things to a body, such as to cause it to increase in size, to melt, to turn to a vapor, and so on. These phenomena, which we have witnessed, have in each case occurred when heat was added to a body. Probably we did not stop to think of what was going on within the body as the heat became resident therein. On the other hand, we have all experienced the flow of heat in the opposite direction, that is, heat leaving a body or being rejected by it. This happens every day to things placed in a refrigerator so that they get cold. Certainly then heat must leave them. And the contents of the refrigerator may get so cold or have so much heat taken from them, that they will actually freeze. We have noted this in the ice cubes produced. Yes, changes such as these, changes in physical state, changes in temperature, are evident to us because of our senses of sight, touch, etc. But changes of state from the standpoint of heat resident in a body, or flowing to and from a body, demand more than sight and touch may give. They demand that we develop some theory as to the internal make-up

of the substance. In other words, one must get some conception of the molecular constitution of matter.

The early Greek philosophers developed the theory that all bodies consist of very small particles called molecules. These molecules are separated by intervening spaces, have an attraction for each other and are further assumed by this theory to be in a state of incessant motion. The proximity of these molecules and the character of their motion are the reasons for the state of aggregation in which the body exists. When the molecules are very closely related and have a rather restricted motion, the body is closely knitted together, or is solid. We say it is a solid. Now it is possible that certain influences at work upon the body may cause its molecules to increase their state of motion. This will permit the molecules to break loose to some extent from their constraint under which they have been held by each other. There is less rigidity to the body due to the greater freedom of its component parts, its molecules. It can not by itself maintain a certain form. In such a state the body is said to be a liquid. We have all noticed this change in physical state from a solid to a liquid. Some have witnessed metals such as iron and lead become molten. We have all experienced ice melting to water. What produces such changes in the state of aggregation? Heat—heat added to the body. It is possible for this influence to be still further exerted. The molecules may find their velocities increased to a point where they can overcome and break away from the constraining influences of the molecules at the surface of the liquid. They project themselves into the space surrounding the liquid. Part of the liquid, if not all, has been changed into a vapor, or using the more general term for this state of aggregation, a gas. As an example, consider the heating of water until it is changed into steam, water vapor. The greater this influence becomes, to change from the liquid to the gaseous state, the less constraint the molecules have upon each other and the more ideal the gaseous state becomes. If the body or substance could be brought to a point where its molecules have no attraction for each other, where constrainment would no longer exist, this ideal state would be reached and the body would be an *ideal gas*. While such a state is never reached, it will be to our advantage to assume theoretically such a state or such an ideal gas. For by doing so, certain laws will

be introduced which are followed closely enough by actual existing gases to permit their use when working with the same.

Internal Kinetic Energy. This theory of the molecular constitution of a substance assumes that the temperature of the substance is due to the incessant motion of the molecules. If heat is added to a body increasing the temperature of the body, the motion of the molecules is assumed to increase even though there is no change in physical state. As mentioned previously, kinetic energy is energy due to motion and external kinetic energy is sometimes called the mechanical kinetic energy of a body. It becomes evident then that due to the motion of the molecules within a body, there must be an accompanying kinetic energy. This is called the internal kinetic energy of the body. Hence, when heat is added to a body, a resulting increase in temperature indicates a corresponding increase in internal kinetic energy. Likewise, a decrease in temperature always indicates a decrease in internal kinetic energy.

Internal Potential Energy. When a body is caused to expand or to change its physical state by the addition of heat, a rearrangement of the molecules must be brought about which increases the distance between them. These molecules have an energy of position or an internal potential energy due to their mutual attraction for each other. If, then, the mean distance between the molecules is increased, internal work must be done in overcoming this attraction, and energy equal in amount to the internal work done must be set up within the body as an increase in internal potential energy. The source of this energy is of course the heat supplied.

Internal or Intrinsic Energy. The sum of the internal kinetic energy and the internal potential energy of a body is known as the internal or intrinsic energy of the body. Therefore the change in internal energy would be the sum of their changes. Interest is not so much in the absolute value of the internal energy as in the change therein.

External Work. If a small rubber balloon is filled with air and then placed in the sun, it will get larger, that is it will expand. The reason for this is to be found in the heat of the sun's rays which has been absorbed or added to the air within the ball. Now this ball is exposed to the pressure of the atmosphere. Hence, when it expands, this atmospheric pressure must be displaced normally (along

a radial line of the spherical ball) through some distance. External work must thus be done by the expanding ball, and energy equal in amount to the external work done must be expended. The source of the energy utilized in this manner is of course the heat from the sun. It is part of the heat added to the ball, but it is not energy set up within the body as an increase in internal or intrinsic energy.

General Energy Equation. The preceding articles have shown what is done with heat energy that is added to a body. The statement of the Law of Conservation of Energy makes it clear that when heat energy is added to a body, it must be wholly accounted for. Therefore the heat energy absorbed by a body must be used in its entirety in any or all of the following ways:

(1) As an increase in internal potential energy

(2) As an increase in internal kinetic energy

(3) In doing external work

In stating the above as a general energy equation let

ΔQ (delta Q) = amount of heat added to a body in B.t.u.

ΔP = that part of ΔQ that is used in increasing the store of internal potential energy of the body in B.t.u.

ΔK = that part of ΔQ that is used in increasing the store of the body's internal kinetic energy in B.t.u.

ΔW = that part of ΔQ that is used in doing external work in B.t.u.

Then we have

$$\Delta Q = \Delta P + \Delta K + \Delta W \tag{15}$$

If ΔU = the increase in internal energy as a whole,

$$\Delta U = \Delta P + \Delta K \tag{16}$$

and

$$\Delta Q = \Delta U + \Delta W \tag{17}$$

The use of the Greek letter, Δ, in front of the above letters is for the distinct purpose of emphasizing that such a symbol represents an increase or, in general, a change. For instance, if U_1 is allowed to represent the internal energy of an initial state of a working medium and U_2 is allowed to represent the internal energy at the final state, $U_2 - U_1$ would be the change in internal energy between the two states, or

$$\Delta U = U_2 - U_1$$

If U_2 happens to be smaller than U_1, ΔU is negative and the resulting increase in internal energy is negative and would be interpreted as a positive decrease in internal energy. To make this clearer, let $U_2 = 500$ B.t.u. and $U_1 = 600$ B.t.u. Then

$$\Delta U = 500 - 600 = -100 \text{ B.t.u.}$$

or the increase in internal energy, ΔU, is negative, but since a negative increase is a positive decrease, we would conclude that there is a decrease in internal energy of $+100$ B.t.u.

In formula (15), any term can be zero, positive, or negative. Should ΔQ be negative, it would be likewise interpreted as negative heat added, which is positive heat rejected.

Thermal Capacity and Specific Heat. The thermal capacity of a body is the number of British thermal units required to raise the temperature of the body one degree Fahrenheit. Nothing is stated in this as to the weight (mass) of the body. So that it is more definite to consider the thermal capacity per unit mass or the thermal capacity of one pound. This is called the Specific Heat and is the number of British thermal units required to raise one pound of the body one degree Fahrenheit.

Let $\quad \Delta Q =$ heat absorbed in B.t.u.

$M =$ mass or weight in lbs.

$t_2 =$ final temperature in degrees F.

$t_1 =$ initial temperature in degrees F.

$c =$ specific heat

Then from the definition of specific heat

$$c = \frac{\Delta Q}{M(t_2 - t_1)} \tag{18}$$

Solving formula (18) for ΔQ

$$\Delta Q = MC(t_2 - t_1) \text{ B.t.u.} \tag{19}$$

Example. Under certain conditions, it required 170.5 B.t.u. to raise 5 pounds of hydrogen from a temperature of 50° F. to a temperature of 60° F. Required the specific heat of hydrogen under these conditions.

Solution. In this example, $\Delta Q = 170.5$ B.t.u., $t_2 = 60°$ F., $t_1 = 50°$ F., $M = 5$ lbs.

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TABLE I

Specific Heats of Gases

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Gas	C_p	C_v	k
Air..	0.2375	0.169	1.406
Hydrogen..................................	3.41	2.42	1.41
Nitrogen...................................	0.244	0.173	1.41
Oxygen....................................	0.218	0.156	1.40
Carbon Monoxide...........................	0.243	0.173	1.403
Carbon Dioxide.............................	0.207	0.162	1.28
Sulphur Dioxide............................	0.154	0.123	1.26

From formula (18)

$$C = \frac{170.5}{5(60-50)} - \frac{170.5}{50} = 3.41 \text{ B.t.u. per lb. } Ans.$$

It follows from the definition of the British thermal unit that the thermal capacity per unit mass, or the specific heat, of water is one B.t.u.

There are two specific heats for all working mediums. They are called the specific heat at constant pressure which is denoted by C_p and the specific heat at constant volume which is denoted by C_v. When the specific heats are variable their mean or average values may be used. The specific heats for an ideal gas and many actual gases, the so-called permanent gases, may be considered as constant, that is, not variable. For solids and liquids the specific heats vary only slightly.

It will be noted in formula (19) that the heat absorbed by a working medium is determined by the continued product of its mass, specific heat, and change in temperature. Since ΔQ is computed by means of a change in temperature, one might be misled into thinking that it is just enough heat to permit this change in temperature. This is not correct. Any ΔQ, heat absorbed, computed on the basis of a specific heat includes not only the heat required to change the temperature, but it also includes all the heat absorbed and stored or utilized in other ways, as, for instance, in doing external work.

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It is at times convenient to use $\dfrac{C_p}{C_v}$ the ratio of the specific heats of a substance. This will be called k in this text. Therefore

$$k = \frac{C_p}{C_v} \tag{20}$$

PROBLEMS

1. A bucket of water weighing 50 pounds is raised by a rope and drum from a well 40 feet in depth.
   (a) How much work in foot-pounds is done?
   (b) What potential energy does the bucket of water possess in its raised position? *Ans.* (a) 2,000 ft.-lbs. (b) 2,000 ft.-lbs.

2. A pressure of 80 pounds per square inch is exerted on the piston of a steam pump throughout its stroke of 12 inches. If the diameter of the piston is 8 inches, what work is done per stroke? *Ans.* 4,021+ ft.-lbs.

3. A train travels 1000 miles in 16½ hours. What is the average speed or linear velocity of the train? *Ans.* 60.6 miles per hour.

4. A flywheel has a diameter of 6 feet. It rotates at 80 revolutions per minute. Find its linear velocity in feet per minute and its angular velocity in radians per minute. *Ans.* 1,508— f.p.m.; 502.7— radians per min.

5. A load of 3,000 pounds is raised by a cable working around a drum with a diameter of 20 inches. The drum makes 18 revolutions per minute. Find the linear velocity, or speed of lift, in feet per minute, and the horsepower required. *Ans.* 94.25 f.p.m.; 8.57 hp.

6. In a 9″x12″ double-acting steam engine whose crankshaft makes 200 revolutions per minute, the mean effective pressure acting upon the piston is 48 pounds per square inch. Find the speed of the piston in feet per minute and the indicated horsepower of the engine. *Ans.* 400 f.p.m.; 37 i.hp.

7. A pulley with an 18-inch diameter is keyed to a shaft. If a tangential force of 240 pounds is applied at the rim of the pulley, what torque is set up in the shaft? *Ans.* 180 ft.-lbs.

8. A shaft transmits 50 horsepower at 100 revolutions per minute. Find the torque in the shaft in foot-pounds. *Ans.* 2,626 ft.-lbs.

9. If the pulley of Problem 7 makes 190 revolutions per minute, what horsepower is transmitted? *Ans.* 6.5 hp.

10. Having given a barometric pressure of 14.5 pounds per square inch and a pressure gauge reading of 125 pounds per square inch, find the absolute pressure in pounds per square inch. *Ans.* 139.5 lbs. per sq. in. absolute.

11. Change a vacuum gauge reading of 10 pounds per square inch to the corresponding absolute reading if the barometric pressure is 29 inches of Hg. (mercury) absolute. *Ans.* 4.24 lbs. per sq. in. absolute.

12. A vacuum gauge on a condenser read 27 inches of Hg. (mercury) absolute and at the same time the barometer read 29.8 inches of Hg. (mercury) absolute. What was the absolute pressure in the condenser in pounds per square inch? *Ans.* 1.37 lbs. per sq. in. absolute.

13. Find the drop in pressure between a pressure gauge reading of 298 pounds per square inch and a vacuum gauge reading of 25 inches of Hg. (mercury) absolute. The atmospheric pressure is normal. *Ans.* 310.28− lbs. per sq. in.

14. How is a pressure reading changed to pounds per square foot?

15. If the temperature reading on the Fahrenheit scale is 77° F., what is the corresponding reading on the Centigrade scale? *Ans.* 25° C.

16. A Centigrade scale shows a temperature of 40° C. What is the absolute temperature in Centigrade degrees? *Ans.* 313° C. absolute.

17. A Fahrenheit scale shows a temperature of 85° F. What is the absolute temperature in Fahrenheit degrees? *Ans.* 545° F. absolute.

18. Change a temperature reading of −20° F. to the corresponding absolute reading in Fahrenheit degrees. *Ans.* 440° F. absolute.

19. A quantity of heat energy is given as 34 British thermal units. Change this to the equivalent amount of mechanical energy in foot-pounds. *Ans.* 26,452 ft.-lbs.

20. Change 5,000 foot-pounds of energy into British thermal units. *Ans.* 6.43− B.t.u.

21. Could an engine do 100,000 foot-pounds of work while being supplied with 100 British thermal units of heat?

22. A certain coal has a calorific value of 13,000 British thermal units.

(a) If this amount of heat energy was completely transformed into mechanical energy, how many foot-pounds of work could be done?

(b) If the above work was done in lifting vertically a load of one ton, to what distance could the load be lifted? *Ans.* (a) 10,114,000 ft.-lbs. (b) 5,057 ft.

23. What is the specific volume of carbon dioxide, if its specific weight, or density, is 0.12323 pound per cubic foot at n.t.p.? *Ans.* 8.115 cu. ft. per lb. at n.t.p.

24. When a body was heated, 2 British thermal units were used in doing external work, 3 British thermal units were used in increasing the internal potential energy, and 12 British thermal units were used in increasing the internal kinetic energy of the body.

(a) How many foot-pounds of external work were done by the body?

(b) What was the total increase in internal energy in B.t.u.?

(c) How much heat was added to the body?

Ans. (a) $\Delta W = 1,556$ ft.-lbs., (b) $\Delta U = 15$ B.t.u., (c) $\Delta Q = 17$ B.t.u.

25. Under certain conditions, it required 30.8 British thermal units to raise the temperature of 10 pounds of sulphur dioxide from 40° F. to 60° F. Find the mean specific heat of sulphur dioxide under these conditions. *Ans.* 0.154 B.t.u. per lb.

CHAPTER II

LAWS OF IDEAL GASES

Ideal Gas. Within every state of aggregation of a substance, the attraction or the molecular force existing between the molecules, as well as the proximity of the molecules, is not the same for a given body throughout that particular physical state. Consider the solid state of a so-called metal. If certain influences are working on the metal, such as heat being gradually added, the solid remains as such for some time but its volume is increasing due to the effect of the heat. Therefore there is a change in the molecular set-up. And as heat is added, this gradual change within the solid state brings it to a point where if more heat is added, the metal becomes slightly molten. It is now entering the liquid state or becoming molten. It will become more liquid, as we say, by the further addition of heat. Changes in other conditions, as in the pressure to which the metal is subjected, might retard or accelerate these changes. And reversal of the heat flow, or cooling the body, reverses the changes. Now in the liquid stage, consider oil. It may be quite viscous, that is, it does not flow readily. But if it is put into the crankcase of an automobile and heated by the running of the engine, it becomes lighter, less viscous, or more fluid. In fact, if the engine gets too hot, the oil is vaporized or gasified and the supply must be renewed.

Such changes within a state of aggregation are probably more evident within the gaseous state than in any other. Also the comparison of a gaseous state of one substance with that of another, both so-called gases, indicates great differences in molecular placement and activity. Certain characteristics become associated with a gas. The fewer of these characteristics possessed by a gas, the nearer it approaches the liquid state. Hence a vapor is a gas that is just removed from the liquid state. The greater the number of these characteristics possessed by a gas, the further it is removed

from this state of being a liquid (state of liquefaction). All the characteristics that a gas should have could be determined by assuming that it is removed from the liquid state so far that internal molecular forces no longer exist. Such a gas is called an *ideal* or *perfect gas*. By assuming such a condition as this ideal gaseous state, certain laws, called the Laws of Ideal Gases can be developed. These are of great value in Thermodynamics because they can be used in problems dealing with many actual gases, such as hydrogen, nitrogen, oxygen, air, carbon monoxide, carbon dioxide, etc., and mixtures thereof without much error. It should then be remembered that actual gases approach the ideal state more closely as their molecules become further and further removed from each other and for this reason exert less and less influence upon each other. The states of aggregation are listed below together with the relative positions of certain substances or subdivisions of the gaseous state. In the latter, as we progress upward toward the ideal state, the more closely the laws of ideal gases will be followed.

States of Aggregation

- Gas
  - The Ideal or Perfect Gas
  - Actual gases
    - Hydrogen, oxygen, nitrogen, air
    - Carbon monoxide
    - Carbon dioxide
    - Sulphur dioxide
  - Vapor
    - Superheated
    - Saturated
- Liquid
- Solid
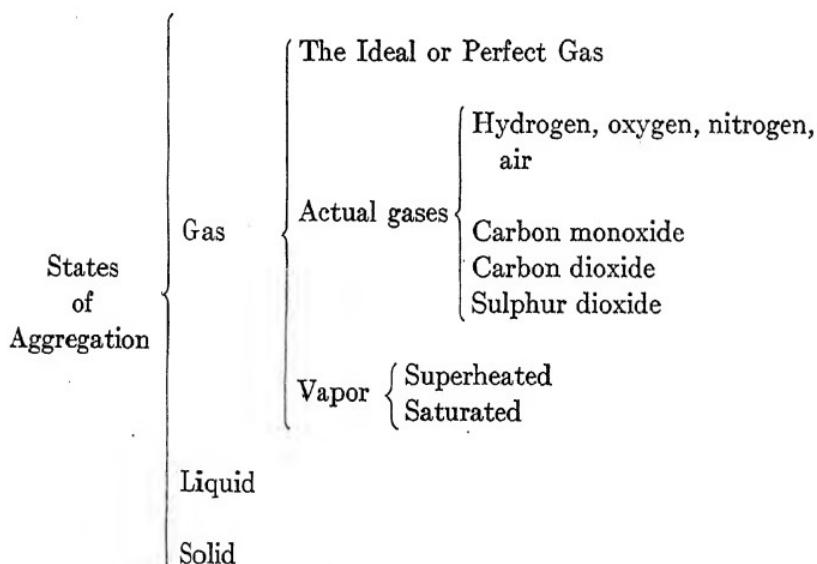
Joule's Law. It has been stated in Chapter I under Internal Potential Energy that in certain cases some of the heat energy added to a body is used to overcome the attraction existing between the molecules, and for this reason an increase in internal potential

energy, ΔP, of the body results. Since it is assumed that there is no attraction between the molecules of an ideal gas, there can be no energy consumed in overcoming that which does not exist, hence none of the heat energy added to an ideal gas can be used in increasing its internal potential energy

$$\therefore \quad \Delta P, \text{ for an ideal gas} = 0 \qquad (21)$$

This was experimentally determined by Joule, and for that reason is called Joule's Law.

Since the general energy equation for any substance is:

$$\Delta Q = \Delta P + \Delta K + \Delta W$$

for an ideal gas, since $\quad \Delta P = 0$

$$\Delta Q = \Delta K + \Delta W \qquad (22)$$

and since

$$\Delta U = \Delta P + \Delta K$$

for an ideal gas

$$\Delta U = \Delta K$$

or the increase in internal energy for an ideal gas must be a change in its internal kinetic energy only. Remember that a change in internal kinetic energy is always accompanied by a change in temperature.

Boyle's Law. A thermodynamic state of a gas is generally given by its absolute pressure in pounds per square foot P, its volume in cubic feet V, and its absolute temperature in degrees Fahrenheit, T. For an initial state of a gas, the subscript 1 will be used so that the initial state will be represented by P_1, V_1, and T_1. Now suppose a thermodynamic change of state (hereafter called change of state) is taking place, the final or second state is represented by P_2, V_2, T_2. It might be that one of the final properties was the same or constant throughout the change of state. If it was the temperature that behaved in this manner, it would be said that the gas is changing its state under a condition of constant temperature. The final state could be thus represented by P_2, V_2, and T_1 for $T_2 = T_1$. Graphically it could be represented as shown below, in Fig. 7.

In the former figure the arrow indicates the direction of the change of state.

When a gas changes its state in this manner, it is said to follow Boyle's Law, which states that when a gas is compressed or

expanded, according to a condition that its temperature remains constant, the absolute pressures vary inversely as the volumes. Hence the absolute pressure increases as the volume decreases and vice versa. Stated mathematically, when T is constant:

$$\frac{P_1}{P_2} = \frac{V_2}{V_1} \tag{23}$$

multiplying both members of the above by P_2

$$\frac{P_1}{P_2} \cdot P_2 = P_2 \cdot \frac{V_2}{V_1} \text{ or } P_1 = P_2 \cdot \frac{V_2}{V_1}$$

multiplying by V_1

$$P_1 V_1 = P_2 \cdot \frac{V_2}{V_1} \cdot V_1 \text{ or } P_1 V_1 = P_2 V_2 \tag{24}$$

$$\therefore PV = \text{a constant} \tag{25}$$
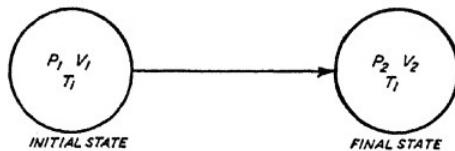
Fig. 7

where P and V are the corresponding pressure and volume for any instantaneous condition, state, or phase of a process in which temperature is constant. In other words, when we have an isothermal process, the product of the pressure and volume at any phase of the process is always equal to the same number as at any other phase of the same process.

Example. If 15 cubic feet of air at 1,600 pounds per square foot absolute pressure expand to 35 cubic feet at the same temperature, what will be the absolute pressure of the final state in pounds per square foot? What will be the corresponding absolute pressure in pounds per square inch?

Solution. Here $P_1 = 1,600$ lbs. per sq. ft., $V_1 = 15$ cu. ft., $V_2 = 35$ cu. ft.

Since $P_1 V_1 = P_2 V_2$ formula (24)

$$1,600 \times 15 = P_2 \times 35$$
$$\text{or} \quad 35 P_2 = 24,000$$

dividing each member of the equation by 35

$$P_2 = \frac{24,000}{35} = 685.71 \text{ lbs. per sq. ft., absolute. } Ans.$$

To change pressure in lbs. per sq. ft. to lbs. per sq. in., divide by 144, since there are 144 sq. in. in 1 sq. ft.
Hence

$$685.71 \div 144 = 4.76 \text{ lbs. per sq. in., absolute. } Ans.$$

Example. 200 cubic feet of air are compressed isothermally (that is at constant temperature) to a final volume of 25 cubic feet. If the initial pressure is 5 pounds per square foot absolute, what is the final pressure in pounds per square foot?

Solution. Here $P_1 = 5$ lbs. per sq. ft., $V_1 = 200$ cu. ft., $V_2 = 25$ cu. ft.
Using formula (24)

$$5 \times 200 = P_2 \times 25$$
$$25P_2 = 1000$$
$$P_2 = 40 \text{ lbs. per sq. ft., absolute. } Ans.$$

Example. 10 cubic feet of air are expanded isothermally to a volume of 30 cubic feet. If the initial pressure is 40 pounds per square inch gauge, what will be the final pressure as recorded by the gauge in pounds per square inch?

Solution. In this example, the initial pressure must be first changed to lbs. per sq. ft. absolute. This is done as follows:
since absolute pressure = gauge pressure + atmospheric pressure
absolute pressure in lbs. per sq. in. $= 40 + 14.7 = 54.7$
$\therefore P_1$, absolute pressure in lbs. per sq. ft. $= 54.7 \times 144 = 7,876.8$
It is given that $V_1 = 10$ cu. ft., and $V_2 = 30$ cu. ft.
From formula (24)

$$7876.8 \times 10 = P_2 \times 30$$
$$30P_2 = 78,768$$

Dividing by 30

$$P_2 = \frac{78,768}{30} = 2,625.6 \text{ lbs. per sq. ft. absolute}$$

$$\frac{2,625.6}{144} = 18.23 \text{ lbs. per sq. in. absolute}$$

Therefore to obtain the gauge pressure:

$$18.23 - 14.7 = 3.53 \text{ lbs. per sq. in., gauge.} \quad Ans.$$

Example. Air is compressed isothermally to a final volume of 2.5 cubic feet and a final pressure of 12 pounds per square foot absolute. If the initial pressure is 3 pounds per square foot absolute, what is the initial volume?

Solution. Here $P_1 = 3$ lbs. per sq. ft. absolute, $P_2 = 12$ lbs. per sq. ft. absolute, $V_2 = 2.5$ cu. ft.
From formula (24)

$$3 \times V_1 = 12 \times 2.5$$
$$3V_1 = 30$$
$$V_1 = 10 \text{ cu. ft.} \quad Ans.$$

Charles' Law Dealing with Constant Pressure. A gas may change its state and have the pressure remain unchanged or constant. Such
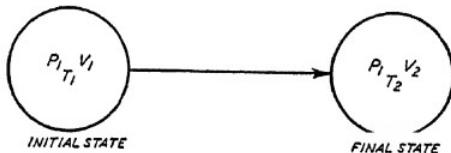
Fig. 8

a change is indicated by Fig. 8, in which the pressure in the final state is indicated by the same symbol as it is indicated by in the first state. This shows it to be the same; otherwise it would necessarily have to take a different symbol.

One of the Laws of Charles deals with this condition. It states that when an amount of gas is changing its state so that the pressure is always the same or constant, the volumes will vary directly as the absolute temperatures. It should be noted that this is a direct variation, so that if the absolute temperature becomes doubled during the change of state, the volume will likewise become twice its original value. In applying this law, always use the absolute temperature. If it is given as Fahrenheit temperature, change it before it is used, so that during the problem nothing but an absolute temperature is involved. Should the answer require a Fahrenheit temperature, obtain first the absolute and then change it to the corresponding Fahrenheit value. The following is the statement of this

law as a formula. When P is constant,

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \qquad (26)$$

To change this to another algebraic form, multiply each member of the above by V_2 which gives

$$\frac{V_1}{V_2} \cdot V_2 = \frac{T_1}{T_2} \cdot V_2 \text{ or } V_1 = \frac{T_1 V_2}{T_2} \qquad (27)$$

Next, multiplying both members by T_2,

$$V_1 T_2 = \frac{T_1 V_2}{T_2} \cdot T_2$$

Hence

$$V_1 T_2 = V_2 T_1 \qquad (28)$$

Example. 5 cubic feet of air are heated from 30° F. to 300° F. at a constant pressure of 20 pounds per square inch absolute. According to the Law of Charles, the final volume must be greater than the initial volume of 5 cubic feet, because the final temperature is greater than the initial. Hence the heat added caused the air to expand to a larger volume. Required this larger or final volume.

Solution. In this example, $V_1 = 5$ cu. ft., $t_1 = 30°$ F., $t_2 = 300°$ F.

It will be necessary to change t_1 to T_1 and t_2 to T_2 before they can be used.

$$T_1 = t_1 + 460° = 30° + 460° = 490° \text{ F. absolute } (11)$$
$$T_2 = t_2 + 460° = 300° + 460° = 760° \text{ F. absolute } (11)$$

Substituting in formula (26)

$$\frac{5}{V_2} = \frac{490}{760}$$

Multiplying both members of the above equation by V_2

$$\frac{5}{V_2} \cdot V_2 = \frac{490}{760} \cdot V_2 \text{ or } 5 = \frac{490 V_2}{760}$$

Multiplying by 760

$$490 V_2 = 5 \times 760$$

Dividing by 490

$$V_2 \quad \frac{5 \times 760}{490} = 7.76 \text{ cu. ft.} \quad Ans.$$

Example. 10 cubic feet of gas are compressed at constant pressure until the final volume is one-half the initial volume. If the initial temperature is 450° F., what is the final temperature?

Solution. Here $V_1 = 10$ cu. ft., $V_2 = 5$ cu. ft., $t_1 = 450°$ F., $T_1 = 450° + 460° = 910°$ F. absolute (11)
Substituting these values in formula (28)

$$10T_2 = 5 \times 910$$
$$10T_2 = 4,550$$
$$\therefore T_2 = 455° \text{ F. absolute. } Ans.$$

To change, if wanted from absolute to Fahrenheit, use formula (12). Thus, $t_2 = 455° - 460° = -5°$ F., or 5° below zero on the Fahrenheit scale. *Ans.*

Example. A gas is cooled at constant pressure from a temperature of 1000° F. absolute, to a temperature of 400° F. absolute. If the initial volume is 40 cubic feet, what is the final volume?

Solution. $V_1 = 40$ cu. ft., $T_1 = 1000°$, $T_2 = 400°$
Using formula (26)

$$\frac{40}{V_2} = \frac{1000}{400}$$

multiplying both members of the equation by V_2,

$$\frac{40}{V_2} \cdot V_2 = \frac{1000}{400} \cdot V_2 \text{ or } 40 = \frac{1000 V_2}{400}$$

multiplying now by 400

$$1000 V_2 = 40 \times 400$$
$$V_2 = \frac{40 \times 400}{1000} = 16 \text{ cu. ft. } Ans.$$

Charles' Law Dealing with Constant Volume. A second law of Charles deals with that change of state in which the volume remains constant, and the temperature and pressure vary. This is shown graphically in Fig. 9.

This law states that, when a given mass of gas is changing its state with the volume remaining constant, the absolute pressures vary directly as the absolute temperatures. Again in this law, as in the other Law of Charles, a direct proportion is involved. Care must be exercised in seeing that the absolute pressures, as well as the absolute temperatures are used in the application of this law.

A conception of the condition of change of state at constant volume can be gained by considering a closed vessel filled with a gas under pressure. Now no matter how many pounds of a certain gas are put into a container, the gas will fill the container and the capacity of the latter becomes the volume of the gas. The gas in the vessel will then have a certain pressure, temperature, and volume. Certainly as long as the gas stays in the container its volume remains constant. Of course there may be no change in the thermodynamic state, then all three characteristics of the gas would remain the same. But suppose heat is applied; maybe the container is placed over a fire. As the gas absorbs heat, the temperature will rise, and according to this law along with it the pressure will likewise increase.

Stated as a formula, this law becomes:

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}, \text{ when } V \text{ is constant} \tag{29}$$
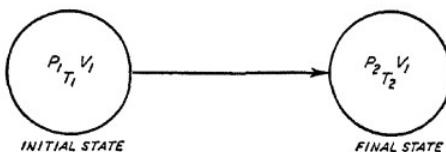
Fig. 9

or multiplying each member of the equation by the product $P_2 T_2$, we obtain

$$P_1 T_2 = P_2 T_1 \tag{30}$$

Example. The volume occupied by a gas remains constant while its temperature is changing from 40° F. to 550° F.

    (a) If the initial gauge pressure is 15.3 pounds per square inch, what is the final absolute pressure in pounds per square foot?

    (b) What are the final absolute and gauge pressures in pounds per square inch?

Solution. (a) It will be necessary to change the temperatures to the absolute readings. Also change the pressure to the absolute reading in lbs. per sq. ft.

$T_1 = 40° + 460° = 500°$ F. absolute

$T_2 = 550° + 460° = 1,010°$ F. absolute

$P_1 = (14.7 + 15.3) \times 144 = 4,320$ lbs. per sq. ft., absolute

Using formula (30)
$$P_2 T_1 = P_1 T_2$$
Evaluating in the above formula
$$P_2 \times 500 = 4{,}320 \times 1{,}010$$

$$P_2 = \frac{4{,}320 \times 1{,}010}{500} = 8{,}726.4 \text{ lbs. per sq. ft. absolute.}\quad Ans.$$

(b) $\dfrac{8{,}726.4}{144} = 60.6$ lbs. per sq. in. absolute.   *Ans.*

$60.6 - 14.7 = 45.9$ lbs. per sq. in. gauge.   *Ans.*

Example. A container is filled with a gas whose temperature is 280° F. and whose gauge pressure is 35 pounds per square inch. Heat is applied to this container and the temperature of its gas within rises to 560° F. What are the final absolute and gauge pressures in pounds per square inch?

Solution. Changing the pressure in lbs. per sq. in. gauge to lbs. per sq. ft. absolute, we obtain
$$P_1 = (35 + 14.7) \times 144 = 7{,}156.8 \text{ lbs. per sq. ft. absolute}$$
Changing the Fahrenheit temperatures to the corresponding absolute temperatures
$$T_1 = 280° + 460° = 740° \text{ F. absolute}$$
$$T_2 = 560° + 460° = 1{,}020° \text{ F. absolute}$$
Applying formula (29)
$$\frac{7{,}156.8}{P_2} = \frac{740}{1{,}020} = \frac{37}{51}$$
multiplying both members of this equation by P_2
$$7{,}156.8 = \frac{37 P_2}{51}$$
Multiplying by 51
$$37 P_2 = 7{,}156.8 \times 51$$
Dividing by 37
$$P_2 = \frac{7{,}156.8 \times 51}{37} = 9{,}864.8 - \text{ lbs. per sq. ft. absolute}$$

$9{,}864.8 \div 144 = 68.5$ lbs. per sq. in. absolute.   *Ans.*

$68.5 - 14.7 = 53.8$ lbs. per sq. in. gauge.   *Ans.*

Example. A quantity of air has an initial absolute pressure of 35 pounds per square inch and an initial temperature of 492° F. absolute. While the volume remains constant, the absolute pressure increases to 100 pounds per square inch. Find the final absolute temperature.

Solution. Here $P_1 = 35 \times 144$ lbs. per sq. ft. absolute, $T_1 = 492°$ F. absolute, $P_2 = 100 \times 144$ lbs. per sq. ft. absolute

Using formula (30), we have

$$35 \times 144 \times T_2 = 100 \times 144 \times 492$$

Dividing by 35×144

$$T_2 = \frac{100 \times 144 \times 492}{35 \times 144} = 1,405.7°\text{ F. absolute. } Ans.$$

Example. A container filled with air is cooled from 200° F. to 0° F. If the initial absolute pressure is 18 pounds per square inch, what is the final gauge pressure in pounds per square inch?

Solution. It is evident that the volume is constant because the air is in the same container throughout the change of state. We have

$$T_1 = 200° + 460° = 660°\text{ F. absolute}$$
$$T_2 = 0° + 460° = 460°\text{ F. absolute}$$
$$P_1 = 18 \times 144\text{ lbs. per sq. ft. absolute}$$

Using formula (30)

$$P_2 \times 660 = 18 \times 144 \times 460$$
$$P_2 = \frac{18 \times 144 \times 460}{660} = 1,806.5\text{ lbs. per sq. ft. absolute}$$

In order to obtain the gauge pressure in lbs. per sq. in. first change the absolute to the sq. in. basis. Thus

$$\frac{1,806.5}{144} = 12.5\text{ lbs. per sq. in. absolute}$$

Since absolute pressure minus atmospheric pressure = gauge pressure,

12.5 lbs. per sq. in. − 14.7 lbs. per sq. in. = − 2.2 lbs. per sq. in. Hence the gauge pressure is recorded below atmospheric pressure, which means that a vacuum gauge is used, for a pressure gauge will not read pressures below atmospheric.

∴ the vacuum gauge reading = +2.2 lbs. per sq. in. *Ans.*

The Characteristic Equation and the Gas Constant. An ideal or perfect gas can change its thermodynamic state without a single one of its characteristics, P, V, and T, remaining constant. In other words during the change of state, the pressure, volume, and temperature may all be variable. This necessitates the derivation of another formula that will apply to such changes.

This formula is known as the Characteristic Equation. It is so fundamental that it is sometimes referred to as the General Law of Ideal Gases. It is derived by combining the Laws of Charles and Boyle. This is illustrated in Fig. 10, where a mass or weight of M pounds of a gas starts with the conditions of pressure, P_1, volume, V_1, and temperature, T_1, at A. This gas is to be ultimately in the state represented by C, where its characteristics have all changed,
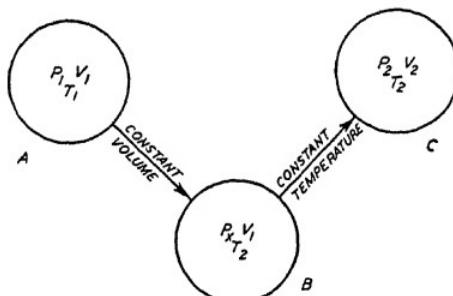
Fig. 10

becoming P_2, V_2, and T_2. It does not go directly from A to C, but passes through an intermediate step, B. From A to B the volume is to remain constant, so that the volume at B is V_1. The temperature at B will be taken as T_2, the final temperature. Hence the pressure to permit this will be some undefined pressure, P_x. The gas changes from state B to state C with its temperature held constant, so that the temperature at C will be T_2. The volume and pressure will change to their final values V_2 and P_2.

Considering the change of state from A to B which follows the Law of Charles since the volume is held constant, formula (30) applies; and

$$P_1 T_2 = P_x T_1$$

$$\text{or } P_x \quad \frac{P_1 T_2}{T_1}$$

The change from B to C being at constant temperature, Boyle's Law is used so that formula (24) gives

$$P_x V_1 = P_2 V_2$$

Substituting the value of P_x from formula (30) for P_x in formula (24)

$$\frac{P_1 T_2}{T_1} \cdot V_1 = P_2 V_2$$

Dividing both members of this equation by T_2,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \tag{31}$$

Let v represent the volume in cubic feet of one pound of the gas, or the specific volume, then

$$V_1 = M v_1 \text{ and } V_2 = M v_2$$

Substituting these values in the preceding equation,

$$\frac{P_1 M v_1}{T_1} = \frac{P_2 M v_2}{T_2}$$

Dividing by M,

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \tag{32}$$

Formula (32) is a statement that for any ideal gas, the product of the absolute pressure P in pounds per square foot and the specific volume, v, in cubic feet divided by the absolute temperature, T, in Fahrenheit degrees at any state, will always be a constant. This constant is generally designated by R, and is called the Gas Constant. So that in general,

$$\frac{Pv}{\ } = R \tag{33}$$

The multiplication of both members of this equation by M produces formula (34) the so-called Characteristic Equation for an ideal or perfect gas:

$$\frac{PMv}{T} = MR$$

or since

$$Mv = V$$
$$PV = MRT \tag{34}$$

where

> P = absolute pressure in lbs. per sq. ft.
> V = volume in cu. ft. of M lbs.
> M = weight or mass in lbs.
> R = the Gas Constant
> T = absolute temperature in Fahrenheit degrees.

The Mol. The mol is a definite weight of a gas in pounds, and is numerically equal to the molecular weight of the gas. If m represents the molecular weight,

$$1 \text{ mol} = m \text{ lbs.} \qquad (35)$$

Since the molecular weight of oxygen, O_2, is 32, one mol of oxygen is equal to 32 pounds. The molecular weight of ammonia, NH_3, is 17; therefore 1 mol of ammonia is equal to 17 pounds.

The so-called normal or standard conditions of temperature and pressure are 14.7 pounds per square inch absolute and 32° Fahrenheit (492° F., absolute). They are represented by the letters, n.t.p. (normal temperature and pressure). It will be shown a little later that the volume of 1 mol of any gas at n.t.p. = 358.7 cu. ft. (36).

Avogadro's Law. This law states that at the same temperature and pressure equal volumes of all gases contain the same number of molecules. Hence under identical conditions of temperature and pressure one cubic foot of hydrogen (H_2) contains the same number of molecules as one cubic foot of oxygen (O_2). Oxygen has a molecular weight of 32, while hydrogen's molecular weight is 2. Therefore each molecule of O_2 has a weight which is $\dfrac{32}{2}$ or 16 times the weight of one molecule of H_2. Since one cubic foot of these two gases contains the same number of molecules, and a molecule of O_2 weighs 16 times as much as a molecule of H_2, it is evident that the density, or specific weight, of oxygen is 16 times the density of hydrogen. So this Law of Avogadro indicates that the specific weights of any two gases are directly proportional to their molecular weights if the gases are at the same temperature and pressure.

The specific weight of oxygen is 0.08922 pounds per cubic foot at n.t.p. Since the specific volume is the reciprocal of the specific weight, the specific volume of oxygen at n.t.p. = $\dfrac{1}{0.08922}$ cubic feet

per pound. If one pound of oxygen has a volume of $\dfrac{1}{0.08922}$ cubic feet, 32 pounds or 1 mol will have a volume of $32 \times \dfrac{1}{0.08922}$ cubic feet $= 358.7$ cubic feet at n.t.p.

Hence from Avogadro's Law,

the volume of 1 mol of any gas at n.t.p. $= 358.7$ cu. ft.　　(36)

Determination of the Gas Constant. In formula (34), let $P = 14.7 \times 144$ lbs. per sq. ft. absolute, $T = 492°$ F. absolute, and $M = m$, the number of pounds in 1 mol. Then V of the formula becomes the volume of 1 mol which under the conditions of pressure and volume equals 358.7 cu. ft. Substitution of these values in the formula gives

$$14.7 \times 144 \times 358.7 = mR \times 492$$

Dividing by 492

$$mR = \frac{14.7 \times 144 \times 358.7}{492} = 1{,}544 \qquad (37)$$

Formula (37) states that the product of the molecular weight, m, and the gas constant of any gas $= 1{,}544$. This product, mR, is known as the Universal Gas Constant.

Solving formula (37) for R,

$$R = \frac{1{,}544}{m} \qquad (38)$$

Formula (38) states that the gas constant of an ideal gas can be found by dividing 1,544 by the molecular weight of the gas. The more closely an actual gas behaves like an ideal gas, the more accurate will be its gas constant when obtained from formula (38).

Since the molecular weights of all gases are available, the gas constant, R, can always be obtained for use in formula (34). For our convenience, the molecular weights and gas constants for a limited number of gases are given in Table II.

Example. The specific weight of air at n.t.p. is 0.0807 pounds per cubic foot. Calculate the gas constant for air from this known data.

Solution. In this example, $P = 14.7 \times 144$ lbs. per sq. ft. absolute, $T = 32° + 460° = 492°$ F. absolute, $V = 1$ cu. ft., $M = 0.0807$ lb.

TABLE II

Molecular Weights and Gas Constants

| Gas | Chemical Symbol | m | | R |
		Approximate	Exact	
Oxygen...	O_2	32	32	48.3
Nitrogen..	N_2	28	28.08	55.1
Hydrogen.	H_2	2	2.016	765.9
Air.............	Gaseous mixture	28.9*		53.3
Carbon Monoxide.	CO	28	28	55.1
Carbon Dioxide...	CO_2	44	44	35.1
Nitric Oxide......	NO	30	30.04	51.4
Sulphur Dioxide..	SO_2	64	64.06	24.1
Acetylene........	C_2H_2	26	26.02	59.4
Ethylene.........	C_2H_4	28	28.03	55.1
Methane.........	CH_4	16	16.03	96.3
Ammonia........	NH_3	17	17.06	90.5

*Apparent molecular weight

Substituting these values in formula (34)

$$PV = MRT$$
$$14.7 \times 144 \times 1 = 0.0807 \times R \times 492$$

Dividing both members of the equation by 0.0807×492

$$R = \frac{14.7 \times 144 \times 1}{0.0807 \times 492} = 53.3 \quad Ans.$$

Example. Having given the gas constant for air, find its apparent molecular weight.

Note. Gaseous mixtures such as air do not have actual molecular weights, so a value determined as such is termed an apparent molecular weight. It acts in the capacity of a molecular weight.

Solution. Using formula (38)

$$R = \frac{1,544}{m}$$

multiplying both members of the equation by m,

$$mR = 1,544$$

Dividing both members by R

$$m = \frac{1,544}{R}$$

Evaluating in the above

$$m = \frac{1,544}{53.3} = 28.9 \quad Ans.$$

Example. A tank having a capacity of 12 cubic feet is filled with 6 pounds of air at a pressure of 200 pounds per square inch, absolute. What is the temperature in degrees Fahrenheit?

Solution. Here, $P = 200 \times 144$ lbs. per sq. ft. absolute, $V = 12$ cu. ft., $M = 6$ lbs., R, for air, $= 53.3$

Substituting these values in formula (34), we have

$$200 \times 144 \times 12 = 6 \times 53.3 \times T$$

Dividing both members of the equation by 6×53.3

$$T = \frac{200 \times 144 \times 12}{6 \times 53.3} = 1,080.7° \text{ F. absolute}$$

$$t = 1,080.7° - 460 = 620.7° \text{ F.} \quad Ans.$$

Example. Compute the gas constant for hydrogen, having given that the molecular weight of hydrogen is 2.016.

Solution. Using formula (38)

$$R = \frac{1,544}{m} = \frac{1,544}{2.016} = 765.9 \quad Ans.$$

Example. A container whose capacity is 3 cubic feet is filled at 1,500 pounds per square inch gauge with oxygen whose temperature is 70° F. Find the weight of oxygen in the tank.

Solution. R for oxygen from Table II $= 48.3$, $V = 3$ cu. ft.

$$T = 70° + 460° = 530° \text{ F. absolute}$$

$$P = (1,500 + 14.7) \times 144 \text{ lbs. per sq. ft. absolute}$$

Evaluating in formula (34), we have

$$1,514.7 \times 144 \times 3 = M \times 48.3 \times 530$$

Dividing by 48.3×530

$$M = \frac{1,514.7 \times 144 \times 3}{48.3 \times 530} = 25.6 \text{ lbs.} \quad Ans.$$

Example. If the container of the previous example were filled with hydrogen instead of oxygen, what would be the weight if the conditions were the same?

Solution. The given data will be the same with the exception of the gas constant which Table II gives as 765.9.

Using formula (34)

$$1,514.7 \times 144 \times 3 = M \times 765.9 \times 530$$

Dividing by the coefficient of M, namely 765.9×530,

$$M = \frac{1,514.7 \times 144 \times 3}{765.9 \times 530} = 1.6 \text{ lbs.} \quad Ans.$$

Example. An automobile tire contains 0.3 pound of air at 30 pounds per square inch gauge pressure and 80° Fahrenheit. What is the volume of the air in cubic inches?

Solution. $M = 0.3$ lb., $R = 53.3$, $P = (30 + 14.7) \times 144$ lbs. per sq. ft. absolute, $T = 80° + 460° = 540°$ F. absolute.

Substituting these values in (34)

$$44.7 \times 144 \times V = 0.3 \times 53.3 \times 540$$

Dividing both members of the equation by 44.7×144

$$V = \frac{0.3 \times 53.3 \times 540}{44.7 \times 144} = 1.3414 \text{ cu. ft.}$$

Change the volume to cubic inches by multiplying by 1,728, since 1 cu. ft. = 1,728 cu. in.

$$1.3414 \times 1,728 = 2,318 - \text{ cu. in.} \quad Ans.$$

Example. It is required to find the specific volume of carbon monoxide at normal temperature and pressure.

Solution.

1st. Method: m for $CO = 28$

since the volume of 1 mol of a gas = 358.7 cu. ft. at n.t.p.

the volume of 28 lbs. of $CO = 358.7$ cu. ft.

∴ the volume of 1 lb. of $CO = \dfrac{358.7}{28} = 12.8$ cu. ft. $Ans.$

2nd. Method: $P = 14.7 \times 144$ lbs. per sq. ft. absolute (normal pressure)

$T = 32° + 460° = 492°$ (normal temperature)

$M = 1$ lb. since the specific volume is the volume of 1 lb

$R = 55.1$ (Table II)

Substituting these values in (34), we have

$$14.7 \times 144 \times V = 1 \times 55.1 \times 492$$

$$V = \frac{55.1 \times 492}{14.7 \times 144} = 12.8 \text{ cu. ft.} \quad Ans.$$

Example. If 10 pounds of air at 60° F. have a volume of 50 cubic feet, what is the pressure in pounds per square inch gauge?

Solution. Here $V = 50$ cu. ft., $M = 10$ lbs., $R = 53.3$
$T = 60° + 460° = 520°$ F. absolute
Using formula (34)

$$P \times 50 = 10 \times 53.3 \times 520$$

$$P = \frac{10 \times 53.3 \times 520}{50} = 5,543.2 \text{ lbs. per sq. ft. absolute}$$

$$\text{absolute pressure in lbs. per sq. in.} = \frac{5,543.2}{144} = 38.5$$

$$\therefore \text{ gauge pressure} = 38.5 - 14.7 = 23.8 \text{ lbs. per sq. in. } Ans.$$

Gas Mixtures. Two or more gases, which under the conditions have no tendency to react chemically with each other, may occupy the same space, and when doing so are called a gas or gaseous mixture. As was stated at the beginning of this chapter, a gas is very loosely made up, or constituted. Its molecules are very far apart. Hence, when one gas is mixed with another of definite volume, the molecules of the one enter the large openings between the molecules of the other and the one assumes the same volume as the other. In other words, diffusion has taken place. A gas can be mixed or thoroughly diffused with another in any proportion. Air is a gaseous mixture, containing 23.6 per cent of oxygen and 76.4 per cent of nitrogen by weight. The laws of ideal gases apply to gaseous mixtures when the individual components of the mixture follow these laws with sufficient accuracy. Besides the three main characteristics of a gas the gaseous mixture has a gas constant and an apparent molecular weight. The latter acts in the same capacity as the actual molecular weight of an individual gas.

Dalton's Law. This law states that the pressure of a gaseous mixture is the sum of the pressures of the individual gases that comprise it. It is not of particular use in obtaining the pressure of a gaseous mixture; a gauge would be used for that purpose. It is, however, of assistance in determining the gas constant of a gaseous mixture and the part of the total pressure of a mixture that is due to each constituent. In applying this law, the volume and temperature of each constituent are the volume (see preceding paragraph on "Gas Mixtures") and temperature respectively of the mixture.

Let us consider a mixture of say three gases, referred to as numbers 1, 2 and 3. The following notation will be used:

	Weight or Mass	Pressure	Volume	Temperature	Gas Constant
Mixture	M	P	V	T	R
Number 1	M_1	P_1	V	T	R_1
Number 2	M_2	P_2	V	T	R_2
Number 3	M_3	P_3	V	T	R_3

Applying formula (34) to the mixture and to each constituent in turn, we have for the mixture

$$PV = MRT \quad (a)$$

for constituent number 1

$$P_1V = M_1R_1T \quad (b)$$

for constituent number 2

$$P_2V = M_2R_2T \quad (c)$$

and for constituent number 3

$$P_3V = M_3R_3T \quad (d)$$

Since it is a mathematical truth or axiom, that if equals are added to equals, their sums are equal, we shall now equate the sum of the first members of equations (b), (c), and (d) to the sum of the second members of these same equations. Thus we obtain

$$P_1V + P_2V + P_3V = M_1R_1T + M_2R_2T + M_3R_3T$$

factoring the two members,

$$(P_1 + P_2 + P_3)V = (M_1R_1 + M_2R_2 + M_3R_3)T \quad (e)$$

But Dalton's Law states that $(P_1 + P_2 + P_3) = P$, the pressure of the mixture. Therefore in (e) we shall substitute for $(P_1 + P_2 + P_3)$ its equal, P, which will give

$$PV = (M_1R_1 + M_2R_2 + M_3R_3)T \quad (f)$$

Since things equal to the same thing are equal to each other, the value of PV in equation (a) must equal the value of PV in equation (f). Therefore

$$MRT = (M_1R_1 + M_2R_2 + M_3R_3)T$$

Dividing both members of the above equation by T,

$$MR = M_1R_1 + M_2R_2 + M_3R_3$$

Dividing by M,

$$R \text{ (the gas constant of the mixture)} = \frac{M_1}{M} R_1 + \frac{M_2}{M} R_2 + \frac{M_3}{M} R_3 \quad (39)$$

It should be noted that in the above formula $\frac{M_1}{M}$, $\frac{M_2}{M}$, etc. are the parts by weight of the individual constituents of the mixture. If parts by volume are given, they must be changed to parts by weight before being used in formula (39). The method of making this change will be brought out later on in this chapter. It should be further noted that there will be the same number of terms in the second member of formula (39) as there are constituent gases in the mixture; hence if there are two constituents, the term $\frac{M_3}{M} R_3$ will disappear, if there are four constituents, one more term $\frac{M_4}{M} R_4$ will appear.

The part of the pressure of a gaseous mixture that is due to any one of its constituent gases can be determined as follows:
Solving formula (a) for V;

$$V = \frac{MRT}{P}$$

Solving formula (b) for V:

$$V = \frac{M_1 R_1 T}{P_1}$$

Equating these two values of V

$$\frac{MRT}{P} = \frac{M_1 R_1 T}{P_1}$$

Multiplying by PP_1 we have,

$$P_1 MRT = PM_1 R_1 T$$

Dividing by T

$$P_1 MR = PM_1 R_1$$

Dividing by MR

$$P_1 = \frac{M_1}{M} \cdot \frac{R_1}{R} \cdot P$$

A similar formula could be derived for any one of the constituents. Hence in general for a constituent, X,

$$P_x = \frac{M_x}{M} \cdot \frac{R_x}{R} \cdot P \quad (40)$$

in which

P = pressure of the mixture in any unit

P_x = part of pressure of mixture due to a constituent, same unit as for P

R_x = gas constant of constituent

R = gas constant of mixture

$\left.\begin{array}{l} M_x = \text{weight of constituent} \\ M = \text{weight of mixture} \end{array}\right\}$ Hence $\dfrac{M_x}{M} = \dfrac{\%}{\text{decimally}}$ by weight, expressed

Example. A gaseous mixture is composed of carbon dioxide, CO_2, and oxygen, O_2. The carbon dioxide represents 20 per cent and the oxygen, 80 per cent by weight.

Find (a) the gas constant and

(b) the apparent molecular weight of the mixture.

Solution. (a) Table II gives R_1 for $CO_2 = 35.1$ and R_2 for $O_2 = 48.3$

$\dfrac{M_1}{M} = 0.20$ (20% stated as a decimal); $\dfrac{M_2}{M} = 0.80$, (80%)

Substituting these values in **(39)**

$$R = 0.20 \times 35.1 + 0.80 \times 48.3$$
$$= 7.02 + 38.64 = 45.7 - \quad Ans.$$

(b) Since formula **(38)** gives

$$R = \frac{1,544}{m}$$

$$m = \frac{1,544}{R}$$

Substituting the value of R, $45.7 -$

$$m = \frac{1,544}{45.7} \quad 33.8 \quad Ans.$$

Example. Assume the absolute pressure of the gaseous mixture of the preceding example to be 40 pounds per square inch. (a) What part of this pressure is due to the carbon dioxide? (b) What part of this pressure is due to the oxygen.

Solution. (a) For evaluating in formula **(40)**, we have

$\dfrac{M_x}{M} = 0.20$, $R_x = 35.1$, $R = 45.7$, $P = 40$ lbs. per sq. in. absolute

$$P_x = 0.20 \times \frac{35.1}{45.7} \times 40 = 6.2 \text{ lbs. per sq. in.} \quad Ans.$$

(b) Here $\dfrac{M_x}{M} = 0.80$, $R_2 = 48.3$, $R = 45.7$, $P = 40$ lbs. per sq. in. abs. Evaluating in formula (40)

$$P_x = 0.80 \times \frac{48.3}{45.7} \times 40 = 33.8 \text{ lbs. per sq. in. } Ans.$$

Volumetric and Weight Analyses. Chemical analyses of gaseous mixtures are generally reported in parts or percentages by volume. Should parts by weight be necessary as, for instance, in formulas (39) and (40), the analysis must be changed from a volumetric to a weight basis, or from parts by volume to parts by weight. The method by which this is accomplished can be brought out clearly by the use of an actual case. Consider the analysis of a flue gas, reported as follows:

carbon dioxide, CO_2... 12 parts or per cent by volume
oxygen, O_2........... 10 parts or per cent by volume
nitrogen, N_2......... 78 parts or per cent by volume
100 parts by volume (the whole)

Step 1. Multiply in turn the number of parts representing each constituent by the molecular weight of that constituent and take the sum of the products thus obtained.

For CO_2, $12 \times 44 = 528$
For O_2, $10 \times 32 = 320$
For N_2, $78 \times 28 = 2,184$
3,032, the sum of the products

Step 2. The fractional part by weight of any constituent is now obtained by dividing the product representing a constituent by the sum of the products. This gives the following values:

Part by weight which is $CO_2 = \dfrac{528}{3,032} = 0.174$ or 17.4 per cent

Part by weight which is $O_2 = \dfrac{320}{3,032} = 0.106$ or 10.6 per cent

Part by weight which is $N_2 = \dfrac{2,184}{3,032} = 0.720$ or 72.0 per cent

Total.....................1.000 or 100 per cent

Example. Air is approximately 21 per cent oxygen and 79 per cent nitrogen by volume.

(a) Determine the corresponding approximate composition of air by weight.

(b) On this basis, what will be the gas constant of air.

Solution:

(a) For O_2, $21 \times 32 = 672$

For N_2, $79 \times 28 = \underline{2,212}$

$2,884$, the sum of the products.

Part by weight which is $O_2 = \dfrac{672}{2,884} = 0.233$ or 23.3 per cent

Part by weight which is $N_2 = \dfrac{2,212}{2,884} = 0.767$ or 76.7 per cent

$\left. \right\}$ *Ans.*

(b) Using formula (39)

$$R = \frac{M_1}{M} R_1 + \frac{M_2}{M} R_2$$

$\dfrac{M_1}{M} = 0.233$; $\dfrac{M_2}{M} = 0.767$; $R_1 = 48.3$ (for O_2); $R_2 = 55.1$ (for N_2)

Evaluating

$$R = 0.233 \times 48.3 + 0.767 \times 55.1$$
$$= 11.25 + 42.26 = 53.51 \quad Ans.$$

(It will be noted that this answer is not far removed from 53.3, the more accurate value of the gas constant of air which has been previously derived and which we shall always use.)

PROBLEMS

1. The temperature remains constant while the pressure of 5 pounds of air is increased from 30 pounds per square inch absolute to 240 pounds per square inch absolute. If the initial volume is 120 cubic feet, what is the final volume? *Ans.* 15 cu. ft.

2. Ten cubic feet of hydrogen are compressed at constant temperature (isothermally) to a final volume of 2 cubic feet. If the initial pressure is 50 pounds per square inch gauge, what are the final absolute and gauge pressures in pounds per square inch? *Ans.* 323.5 lbs. per sq. in. abs.; 308.8 lbs. per sq. in. abs.

3. If a gas is expanded isothermally does its pressure increase or decrease?

If a gas is compressed isothermally does its pressure increase or decrease?

4. A container whose capacity is 25 cubic feet is filled with air at a pressure of 50 pounds per square inch absolute and a temperature of 60° F. The container is heated until the temperature of the air becomes 300°F. What are the final absolute and gauge pressures? *Ans.* 73.1 lbs. per sq. in. abs.; 58.4 lbs. per sq. in. gauge.

5. If the container of Problem 4 is cooled until its final temperature is 20° F. what are the final absolute and gauge pressures in pounds per square inch? *Ans.* 46.1 lbs. per sq. in. abs.; 31.4 lbs. per sq. in. gauge.

6. If a gas is heated at constant volume, does its pressure increase or decrease? If a gas is cooled at constant volume, does its pressure increase or decrease?

7. A gas with an initial pressure of 50 pounds per square inch absolute and an initial temperature of 80° F. is heated at constant volume until the final pressure becomes 150 pounds per square inch absolute. What is the final temperature? State answer in both absolute and Fahrenheit scale readings. *Ans.* 1,620° F. abs.; 1,160° F.

8. A certain mass of air is kept under a constant pressure of 10 pounds per square foot absolute while its absolute temperature is doubled. If the initial volume is 40 cubic feet, what is the final volume? *Ans.* 80 cu. ft.

9. A certain mass of air is kept under a constant pressure of 10 pounds per square foot absolute, while its temperature increases from 50° F. to 100° F. If the initial volume is 40 cubic feet, what is the final volume? (Notice that this problem is identical to Problem 8, except that the Fahrenheit temperature is doubled instead of the absolute. Does it make a difference?) *Ans.* 43.9 cu. ft.

10. A container filled with a gas is under an initial pressure of 2,000 pounds per square foot absolute and an initial temperature of 570° F. absolute. If it is heated until its final temperature is 1,700° F. absolute, what is its final absolute pressure in pounds per square foot? *Ans.* 5,964.9 lbs. per sq. ft. absolute.

11. Find the gas constant of oxygen, whose molecular weight is 32. Check your result with Table II.

12. How many cubic feet are there in one mol of nitrogen at n.t.p.?

13. A tank whose capacity is 200 cubic feet contains 32 pounds of air at a temperature of 60° F. What is the pressure of the air in pounds per square inch gauge? (R for air = 53.3) *Ans.* 16.1 lbs. per sq. in. gauge.

14. For use in the calculation of the test of an air compressor, it is desired to know the density, or weight per cubic foot, of air at 70° F. and normal atmospheric pressure.

(Note. Use $PV = MRT$ with $V = 1$ cubic foot. Solve for M, which then will be the weight of 1 cubic foot, or the density, under these conditions.) *Ans.* 0.075 lb. per cu. ft.

15. A quantity of air has a mass of 0.5 pounds, a pressure of 25 pounds per square inch absolute, and a temperature of 80° F. Find the volume in cubic feet. *Ans.* 4 cu. ft.

16. One pound of air has a pressure of 14.7 pounds per square inch absolute and a volume of 12.4 cubic feet. What must its temperature on the Fahrenheit scale be? *Ans.* 32° F.

17. Two tanks each having a capacity of 4 cubic feet are of the same weight when empty. One tank is filled with oxygen at a pressure of 1,500 pounds per square inch gauge and a temperature of 70° F. The second tank is filled with hydrogen under the same conditions of pressure and temperature. Find the difference in their charged weights. *Ans.* 31.93 lbs.

18. A gaseous mixture has the following composition by volume: Nitrogen, 75 per cent, carbon dioxide, 25 per cent. Required the composition of this mixture on a weight basis. Obtain values of gas constants and molecular weights of nitrogen and carbon dioxide from Table II. *Ans.* $N_2 = 65.6\%$; $CO_2 = 34.4\%$.

19. Find the gas constant and the apparent molecular weight of the gaseous mixture of problem 18. *Ans.* 48.2; 32.03.

20. Assume the absolute pressure of the gaseous mixture of problem 18 to be 80 pounds per square inch.

(a) What part of this pressure is due to the nitrogen?

(b) What part of this pressure is due to the carbon dioxide? *Ans.* (a) 60 lbs. per sq. in.; (b) 20 lbs. per sq. in.

CHAPTER III

THERMODYNAMIC PROCESSES FOR IDEAL GASES

As previously stated, the thermodynamic state of a gas is given or defined by the simultaneous values of its characteristics of pressure, volume, and temperature. If one, or all, of these characteristics becomes changed, a new thermodynamic state is set up and it is said that the gas has undergone a change of state. If some data are known concerning the original and new or changed states of the gas, further information concerning either state can be brought out or calculated, as was shown in Chapter II. Such a change of state of course follows or is according to some law or principle. It is not an instantaneous change in which but two states (generally referred to as the initial and final states) are involved, but on the contrary it is a transition or change consisting of an infinite number of intermediate thermodynamic states between the initial and final conditions. Hence a change of state is in reality a thermodynamic process. A knowledge of what transpires during such processes is necessary in engineering, and is the subject to be discussed in this chapter.

Transfer of Heat. In these thermodynamic processes, various energy transformations occur, which may often involve the transfer of heat between the working substance and an external source or receiver of heat. This heat is known as ΔQ in the general energy equation, formula (15). It may be transferred from one body to another by conduction, convection, or radiation.

Conduction. Gases and liquids are poorer conductors of heat than solids. When a gas is used as a working substance, it is confined in a container of some metallic kind. Hence before heat can get to the gas or be removed from the gas, it must pass through or be conducted through this metal. When the metallic wall is heated, the velocity of the molecules of the wall near the source of heat is increased. The heated molecules communicate their motion to their

immediate neighbors; and the heat energy thus travels through the wall and may be removed by the colder working substance on the other side. This process is called conduction. In this way most of the heat of the products of combustion of a boiler furnace is directly communicated to the water in the boiler, and some of the heat evolved in the cylinder of a gas engine is communicated to the water in the water jacket surrounding the cylinder.

A brass pin held in a gas flame will burn the fingers almost instantly, while a bit of glass may be melted at one end before the other becomes hot, and a match may be burned to the finger-tips without discomfort. It is thus clear that substances differ greatly in thermal conductivity; and the higher the numerical values expressing their thermal conductivities, the more quickly will they conduct heat. See Table III.

If a piece of wire gauze is held over an unlighted gas jet, the gas may be lighted on either side, but the flame will not pass through the meshes. The wire conducts the heat away so rapidly that the gas on the other side does not get hot enough to ignite. This is the principle of the safety lamp, used in coal mines where inflammable gases collect. The lamp flame is surrounded by a wire gauze, and thus kept from igniting the dangerous gases outside. See Fig. 11.

TABLE III
Relative Thermal Conductivities

Silver	100	Lead	7
Copper	66 to 74	Marble	0.0074
Brass	18 to 23	Ice	0.0052
Iron	15	Snow	0.00046

Convection. When a body is heated, say from below, the heated portion expands and rises through the mass, and is replaced by a colder portion, which becomes heated and rises in its turn. In this way what are called convection currents arise, and the heat is distributed throughout the fluid by actual motion within the mass itself. The heated flue gases from the furnace by convection bring their heat to the boiler shell where it is conducted through the shell to the water on the other side. Within the shell convection again takes place as the water is heated. Also convection superheaters are placed in the pathway of the heated flue gases, there to absorb heat to superheat the steam.

Radiation. The theory has been advanced that the molecules of a hot body are in very rapid vibration. Some of the energy of this vibration is communicated in the form of waves to the space surrounding the body. If the motion happens to lie within certain limits, the waves affect our eyes and we call them light-waves. But all such waves, visible or otherwise, represent energy which is sent out by the hot body. When they fall upon any other body, they are reflected or absorbed and transformed into heat energy. Such energy is called radiant energy. Due to the radiant energy of the incandescent fuel
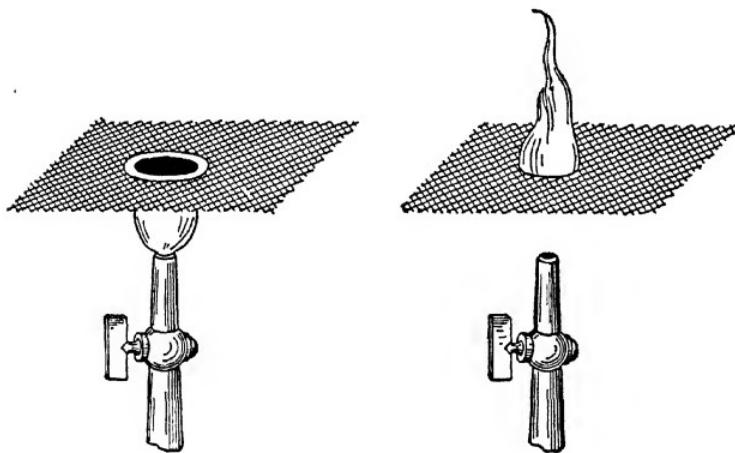
Fig. 11

in a furnace, part of the so-called boiler heating surface is so placed as to receive this energy by radiation.

External Work. Energy transformations in thermodynamic processes often result in external work being done by the working substance when the latter expands, or in external work being done upon the working substance when the latter is compressed. As has been stated in Chapter I, this is due to the fact that in these cases, some force or resistance is moved or displaced through some distance and force multiplied by the distance through which it is displaced always represents work done. Work is expressed directly in foot-pounds, but since 778 foot-pounds equal 1 British thermal unit, work may, as we have seen, be also expressed in the B.t.u. In Chapter I, ΔW was defined as the external work done by the working substance in B.t.u. and W was defined in general as work in foot-pounds. These

symbols will be used to quite an extent in this chapter, so it would be well to notice that

$$\Delta W = \frac{W}{778} \text{ B.t.u.}$$

or

$$W = 778 \times \Delta W \text{ ft.-lbs.}$$

If J represents the mechanical equivalent of heat, 778, these formulas may be written

$$\Delta W = \frac{W}{J} \text{ B.t.u.} \tag{41}$$

or

$$W = J \times \Delta W \text{ ft.-lbs.} \tag{42}$$

It should be further noted that since ΔW is given as the work done
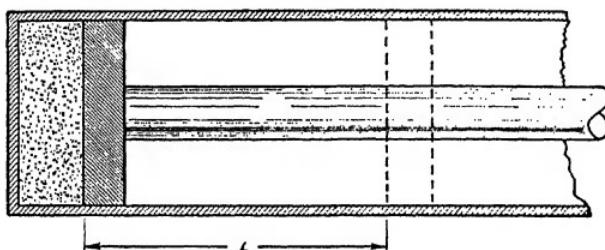
Fig. 12

by the gas, in an expansion ΔW will always receive a positive numerical value, whereas in a compression ΔW will always receive a negative value. Keep in mind that negative work done by the gas is positive work done upon the gas.

Fig. 12 shows a piston of area, A square feet, working in a cylinder. The piston as shown is in its extreme position to the left and there is a certain volume, V_1 cubic feet, between it and the left end of the cylinder. When the piston is moved through the distance, L feet, the volume between it and the left end will become V_2 cubic feet. Therefore the change in volume, or the volume swept through by the piston during its stroke is $(V_2 - V_1)$ cubic feet, which is equal to AL cubic feet since the volume of a cylinder is equal to the area of the base multiplied by the altitude or length.

Now let a force of P pounds per square foot be introduced upon the piston by the working substance until the piston is moved to the

right or dotted position. If P pounds per square foot is the pressure, the total force acting upon the piston throughout its stroke will be $P \times A$, or PA pounds. Now as the piston moves through the distance L feet, PA pounds moves through that distance and an amount of work is done equal to PA pounds multiplied by L feet, or PAL foot-pounds.

Therefore $\qquad W = PAL$ foot-pounds

Since AL, as stated above, equals $V_2 - V_1$, we have

$$W = P(V_2 - V_1) \text{ ft.-lbs.} \qquad (43)$$

or from formula (41)

$$\Delta W = \frac{\iota}{J}(V_2 - V_1) \text{ B.t.u.} \qquad (44)$$

where $\qquad P =$ constant pressure in lbs. per sq. ft.
$\qquad V_2 =$ final volume in cu. ft.
$\qquad V_1 =$ initial volume in cu. ft.
$\qquad J = 778$

It should be carefully noted that formulas (43) and (44) can be used when and only when the pressure is constant.

Pressure=Volume, or PV=Diagram. The co-ordinate axes of mathematics permit one to graphically portray as a point a thermodynamic state of a gas by means of any two characteristics of the gas. The two characteristics often used are the pressure, P and the volume, V. The vertical or Y axis of mathematics is taken as the pressure or P axis and the horizontal, or X axis is taken as the volume, or V axis. Suppose at a given state the pressure of a gas is 500 pounds per square foot and the volume is 1 cubic foot. To show this state on the PV-diagram, Fig. 13, proceed upward on P axis from origin, 0 to the pressure 500 pounds per square foot. Through this point draw a dotted horizontal line as shown. Next proceed from 0 to the right on the V axis until the volume 1 cubic foot, is located. Through this point draw a dotted vertical line. The intersection of the two lines at A graphically represents the given thermodynamic state.

Let it be assumed that this gas undergoes a change of state during which the pressure is always the same, that is, remains constant, and that at the final state B, its volume is 6 cubic feet. Point B can be plotted in exactly the same manner as A. It is evident that

since the pressure is constant in this case, the gradual change in volume from the volume at A, V_a to the volume at B, V_b would create many different states as at C where the volume V_c is 4 cubic feet. Necessarily all these states when graphically shown would lie on a horizontal line from A to B. Therefore the entire thermodynamic process is shown by the line as drawn from A to B, and AB is called the PV-diagram of the process.

From geometry, the area of a rectangle is equal to the product of its two dimensions. In Fig. 13, $A,B,6,1$ is a rectangle whose base dimension is $V_b - V_a$ and whose altitude is P. Therefore its area

VOLUMES IN CU. FT
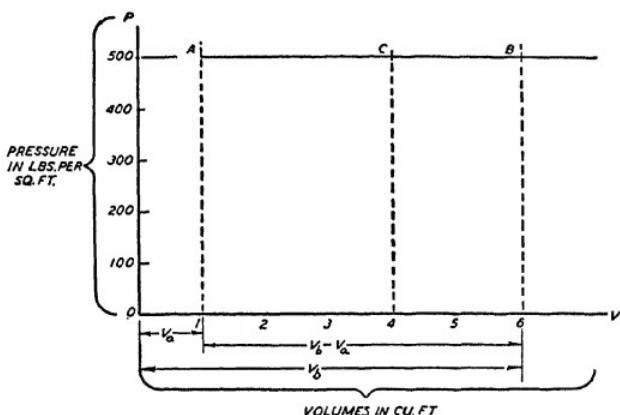
Fig. 13

$= P(V_b - V_a)$. But this is the same statement as formula (43). Hence it is seen that the area between the PV-diagram of the process and the V axis is a measure of the external work done, and with the units as given, the area represents the external work in foot-pounds done by the gas in expanding from A to B.

Since the PV-diagram of a process represented by a horizontal line indicates a constant pressure, it is evident that other processes which do not maintain their pressures constant must have PV-diagrams which are not horizontal lines. Such diagrams will be brought forth in this chapter. However no matter what kind of a line represents a process on the PV-diagram, the area between that line and the V axis always is equal to the external work in foot-pounds involved during the process.

70

Constant Volume Process. For all ideal gases, no matter what process they may undergo the characteristic equation, formula (34) and the general energy equation for an ideal gas, formula (22) apply. Since this process deals with a condition of constant volume, one of Charles' Laws as given by formula (29) also applies here. These are here restated for our convenience in using them and associating them with this process:

Formula (34)

$$PV = MRT$$

Formula (29)

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Formula (22) is here written for the constant volume process noted
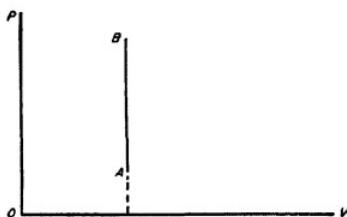
Fig. 14

herein by the use of the subscript, v, attached to each term, thus:

$$\Delta Q_v = \Delta K_v + \Delta W_v, \text{ since for an ideal gas } \Delta P = 0$$

Fig. 14 is the PV-diagram of a constant volume change of state. Consider A as the initial state and B as the final state of the gas. Since $V_a = V_b$, due to the volume being constant, the points A and B must be on the same vertical line. If B is placed above A, the pressure of B is greater than the pressure of A. Since Charles' Law tells us that under this condition of constant volume, the absolute pressures vary directly as the absolute temperatures, the temperature of B is greater than that of A. In another process, these conditions of pressure might be reversed, the initial state of the gas having a higher pressure and hence a higher temperature than the final state.

When a gas changes its state at constant volume, it is needless to say that it is neither being expanded nor compressed. Since

external work, ΔW, is done only during a change in volume, it is evident that for this process

$$\Delta W_v = 0 \qquad (45)$$

This is also shown to be true by examining Fig. 14, which shows the area between the PV-diagram of the process and the V axis to be zero.

Since $\Delta W_v = 0$, formula (22) may be rewritten for this case as

$$\Delta Q_v = \Delta K_v \qquad (46)$$

which states that the heat added to a gas, ΔQ_v, during a constant volume change of state is all used in increasing the internal kinetic energy, ΔK_v. Since an increase in internal kinetic energy is always accompanied by an increase in temperature (Chapter I) and since from Charles' Law an increase in temperature goes with an increase in pressure, evidently in Fig. 14 heat was added to a gas in the initial state A to create the final state B. From this statement, we can draw the conclusion that when the initial state is above the final state on the PV-diagram of this type of process, heat is abstracted or withdrawn from the gas, or in other words ΔQ_v would be negative.

Chapter I called our attention to the fact that C_v is the specific heat used in constant volume processes and that in general $\Delta Q = M_c(t_2 - t_1)$.

Writing this statement for this process, we have

$$\Delta Q_v = MC_v(t_2 - t_1) \qquad (47)$$

Therefore from formula (46)

$$\Delta K_v = MC_v(t_2 - t_1) \qquad (48)$$

It is well to reiterate at this point that the increase in internal kinetic energy is independent of the change in volume and depends only on the change in temperature. Therefore in the future with other processes,

$$\text{any } \Delta K = MC_v(t_2 - t_1) \qquad (49)$$

Example. Ten pounds of air are heated at constant volume from 80° F. to 420° F.

(a) How much heat is supplied?

(b) What is the increase in internal kinetic energy?

(c) How much external work is done?

Solution. (a) Using formula (47) in which $M = 10$ lbs.; C_v, from Table I $= 0.169$; t_2, final temperature, $= 420°$ F.; t_1, initial temperature, $= 80°$ F. we have

$$\Delta Q_v = 10 \times 0.169 (420 - 80) = 574.6 \text{ B.t.u.} \quad Ans.$$

(b) Since $\Delta Q_v = \Delta K_v$, formula (46)

$$\Delta K_v = 574.6 \text{ B.t.u.} \quad Ans.$$

(c) From formula (45)

$$\Delta W_v = 0 \quad Ans.$$

Example. A tank whose capacity is 100 cubic feet is filled with air at a pressure of 35.3 pounds per square inch gauge and a temperature of 100° F. The tank is heated until the temperature of the air becomes 300° F.

(a) How many pounds of air are in the container?

(b) How much heat is supplied?

(c) What is the final gauge pressure in pounds per square inch?

Solution. (a) In formula (34); $P_1 = (35.3 + 14.7)144$ lbs. per sq. ft. $V_1 = 100$ cu. ft.; $R = 53.3$; $T_1 = 100° + 460° = 560°$ F. absolute. Evaluating in the formula (34)

$$(35.3 + 14.7) \cdot 144 \times 100 = M \times 53.3 \times 560$$

Dividing both members of the equation by 53.3×560,

$$M = \frac{(35.3 + 14.7) \cdot 144 \times 100}{53.3 \times 560} = 24.12 \text{ lbs.} \quad Ans.$$

(b) Applying formula (47), in which $M = 24.12$ lbs., $C_v = 0.169$, $t_2 = 300°$ F., $t_1 = 100°$ F. we have

$$\Delta Q_v = 24.12 \times 0.169 \times (300 - 100) = 815.26 \text{ B.t.u.} \quad Ans.$$

(c) Applying Charles' Law, formula (29)

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{P_2}{P_1} = \frac{T_2}{T_1}$$

Multiplying by P_1,

$$P_2 = \frac{P_1 T_2}{T_1}$$

Here
$P_1 = (35.3 + 14.7) \cdot 144$ or 7200 lbs. per sq. ft.
$T_2 = 300° + 460° = 760°$ F. absolute
$T_1 = 100° + 460° = 560°$ F. absolute

Substituting these values in the above equation

$$P_2 = \frac{7,200 \times 760}{560} = 9,771.4 \text{ lbs. per sq. ft. absolute}$$

Changing from lbs. per sq. ft. absolute to lbs. per sq. in. absolute

$$\frac{9,771.4}{144} = 67.9 \text{ lbs. per sq. in. absolute}$$

Changing from lbs. per sq. in. absolute to lbs. per sq. in. gauge

$$67.9 - 14.7 = 53.2 \text{ lbs. per sq. in. gauge.} \quad Ans.$$

Example. Five pounds of air are cooled at constant volume from a temperature of 250° F. to a temperature of 40° F. How much heat energy was abstracted from the air during the cooling?

Solution. Here $M = 5$ lbs., $C_v = 0.169$, t_2, the final temperature, $= 40°$ F., t_1, the initial temperature, $= 250°$ F. Solving for ΔQ_v in formula (47)

$$\Delta Q_v = M C_v (t_2 - t_1)$$
$$\Delta Q_v = 5 \times 0.169 (40 - 250)$$
$$\Delta Q_v = 5 \times 0.169 \times (-210)$$
$$\Delta Q_v = -177.45 \text{ B.t.u.}$$

This result tells us that ΔQ_v, the heat added, is a -177.45 B.t.u.; in other words, a negative quantity of heat has been added during the process. But negative heat added is positive heat abstracted or rejected during the process. Therefore, since -177.45 B.t.u. were added, the heat abstracted $= +177.45$ B.t.u. *Ans.*

Constant Pressure Process. This process follows the Law of Charles which states that as the volumes increase during a constant pressure expansion the absolute temperature will increase and that as the volumes decrease in a constant pressure compression the absolute temperatures will decrease according to formula (26) as follows:

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

The characteristic equation of an ideal gas of course is applicable here for it applies wherever an ideal gas (or one assumed to behave like unto an ideal gas) is involved. And using the subscript, p, to indicate the constant pressure process, the general energy equation for ideal gases, formula (22), will be written as follows:

$$\Delta Q_p = \Delta K_p + \Delta W_p \tag{50}$$

which states that the heat added to a gas or absorbed by a gas from some external source is utilized in increasing the internal kinetic energy and in doing the external work to be done. All terms in this formula are in British thermal units.

For a discussion of the PV-diagram in general, a constant pressure expansion was chosen as given in Fig. 13. Referring to this figure and to formula (44), it is evident that, for this process,

$$\Delta W_p = \frac{P}{J}(V_2 - V_1) \text{ B.t.u.}$$

Applying this formula to the expansion shown in Fig. 13, where we have V_1 or $V_a = 1$ cubic foot, V_2 or $V_b = 6$ cubic feet, $P = 500$ pounds per square foot absolute, $J = 778$

$$\Delta W_p = \frac{500}{778}(6-1) = \frac{500}{778} \times 5 = 3.21 \text{ B.t.u. } \textit{Ans.}$$

Transposing ΔK_p of formula (50), there results;

$$\Delta W_p = \Delta Q_p - \Delta K_p \tag{51}$$

another formula by which ΔW_p may be sometimes obtained.
From formula (49)

$$\Delta K_p = MC_v(t_2 - t_1)\text{B.t.u.} \tag{52}$$

It should be noted that in the above formula although we are obtaining ΔK_p, the increase in internal kinetic energy in British thermal units during a constant pressure process, the specific heat here used is C_v, the specific heat at constant volume.
Transposing ΔW_p of formula (50), there results;

$$\Delta K_p = \Delta Q_p - W_p \tag{53}$$

another formula by which ΔK_p may be sometimes obtained.

Since C_p is the specific heat to be used at constant pressure, formula (19) written for this process becomes

$$\Delta Q_p = MC_p(t_2 - t_1) \text{ B.t.u.} \tag{54}$$

Example. Twenty pounds of air are heated at a constant pressure of 250 pounds per square inch absolute from 40° F. to 60° F.

(a) How much heat is supplied?

(b) What is the increase in internal kinetic energy?

Solution. (a) Here $M = 20$ lbs., $C_p = 0.2375$ (Table I), $t_2 = 60°$ F., $t_1 = 40°$ F.

Substituting these values in formula (54)

$$\Delta Q_p = M C_p (t_2 - t_1)$$

$$\Delta Q_p = 20 \times 0.2375 \times (60 - 40) = 95 \text{ B.t.u., heat added.} \quad Ans.$$

(b) With C_v from Table I = 0.169, we have

$$\Delta K_p = M C_v (t_2 - t_1) \quad \text{formula (52)}$$

$$\Delta K_p = 20 \times 0.169 \times (60 - 40) = 67.6 \text{ B.t.u., increase in internal kinetic energy.} \quad Ans.$$

Example. In the preceding example, how much external work was done by the gas?

Solution. Since formula (51) states that

$$\Delta W_p = \Delta Q_p - \Delta K_p$$

we have, supplying values from the preceding problem

$$\left. \begin{array}{l} \Delta W_p = 95 - 67.6 = 27.4 \text{ B.t.u.} \\ \text{or } W = 27.4 \times 778 \text{ ft.-lbs.} = 21,317.2 \text{ ft.-lbs.} \end{array} \right\} \text{external work}$$

done by the gas during the constant pressure expansion. *Ans.*

Example. One pound of carbon dioxide at 200° F. expands at constant pressure while 30 British thermal units are supplied.

(a) What is the final temperature of the carbon dioxide?

(b) What is the increase in intrinsic energy?

(c) How much external work was done during the expansion?

Solution. Here $M = 1$ lb., $C_p = 0.207$, $C_v = 0.162$, $t_1 = 200°$ F., $\Delta Q_p = 30$ B.t.u.

Substituting in formula (54), we have

$$30 = 1 \times 0.207 \times (t_2 - 200)$$

$$30 = 0.207 \, t_2 - 41.4$$

Transposing $0.207 \, t_2 = 30 + 41.4$

$$t_2 = \frac{30 + 41.4}{0.207} = \frac{71.4}{0.207} = 344.9° \text{ F.} \quad Ans.$$

(b) Since $\Delta K_p = M C_v (t_2 - t_1)$, formula (52),

$$\Delta K_p = 1 \times 0.162 \times (344.9 - 200)$$

$$= 1 \times 0.162 \times 144.9 = 23.47 \text{ B.t.u.} \quad Ans.$$

(c) Since $\Delta W_p = \Delta Q_p - \Delta K_p$, formula (51),

$$\Delta W_p = 30 - 23.47 = 6.53 \text{ B.t.u.} \quad Ans.$$

Example. What is the initial volume of the carbon dioxide in the preceding example if the constant pressure is normal atmospheric?

Solution. Since $PV = MRT$, formula (34) applies, in which $P = 14.7 \times 144$ lbs. per sq. ft. absolute, $V = V_1$, $M = 1$ lb., $R = 35.1$ (Table II), $T = T_1 = 200° + 460° = 660°$ F. absolute.

Substituting these values in our formula, we have

$$14.7 \times 144 \times V_1 = 1 \times 35.1 \times 660$$

$$V_1 = \frac{1 \times 35.1 \times 660}{14.7 \times 144} = 10.94 \text{ cu. ft.} \quad Ans.$$

Example. What is the final volume of the carbon dioxide of the two preceding examples?

Solution. Since Charles' Law holds

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} \text{ or } V_2 = V_1 \times \frac{T_2}{T_1}$$

Here $V_1 = 10.94$ cu. ft., $T_2 = 344.9° + 460° = 804.9°$F. absolute, $T_1 = 200° + 460° = 660°$ F. absolute.

Substituting in the above formula

$$V_2 = 10.94 \times \frac{804.9}{660} = 13.34 \text{ cu. ft.} \quad Ans.$$

Example. Now that the initial and final volumes of the carbon dioxide have been determined, use these in finding the external work done during the expansion of the carbon dioxide. Check this with the value of the external work as previously determined from the data that was at that time available.

Solution. We have $V_2 = 13.34$ cu. ft., $V_1 = 10.94$ cu. ft., $P = 14.7 \times 144$ lbs. per sq. ft. absolute, $J = 778$

Substituting these values in formula (44), we have

$$\Delta W_v = \frac{14.7 \times 144}{778}(13.34 - 10.94)$$

$$= \frac{14.7 \times 144}{778} \times 2.4 = 6.53 \text{ B.t.u. which checks.} \quad Ans.$$

In examining formulas (44), (52), and (54), one finds that the second member of each of these formulas involves a binomial factor. In formula (44), this binomial factor is $(V_2 - V_1)$. In formula (52), it is $(t_2 - t_1)$, and in formula (54), it is also $(t_2 - t_1)$. Since the other factors of the second members of each of these three formulas are always positive algebraically, the algebraic sign of the first member

in each case depends on values assigned to the terms in the binomial factor.

When formula (44) is used in a constant pressure expansion, Fig. 13, the final volume, V_2, must be more than the initial volume, V_1. Therefore the factor, $(V_2 - V_1)$, will result in a positive value and the external work done by the gas, ΔW_p, will be positive. Hence, the gas will actually do external work. But when this same formula is applied to a constant pressure compression, the final volume, V_2, will be less than the initial volume, V_1, and $(V_2 - V_1)$ will result in a negative value, so that the external work done by the gas will be negative. Such a constant pressure compression can be interpreted from Fig. 13 by considering B as the initial state of the gas and A as the final state. This would reverse the direction of the process, making it go from B to A thus causing a decrease in volume which is a compression. Now negative external work done by the gas must be interpreted as positive external work done upon the gas. It is evident that when a gas is compressed, external work is always done upon it, not by it. It should then be remembered that the symbol ΔW_p, or the ΔW of any process, always represents the external work done by the gas; and if external work is done upon the gas (for which there is no symbol directly used), it will be indicated and made evident by the mathematics of the case, and a proper interpretation should be made of the final result.

A similar analysis of formulas (52) and (54) shows that ΔK_p, the increase in internal kinetic energy (or the increase in internal energy in general since for an ideal gas there can be no increase in internal potential energy) of formula (52) and ΔQ_p, the heat added, of formula (54) will both be positive if t_2, the final temperature is more than t_1, the initial temperature, as is the condition during a constant pressure expansion. This relationship of temperatures is evident from the Law of Charles. Hence during such an expansion there will be an actual increase in internal energy and heat will actually be added to the gas from some outside source. But during a constant pressure compression, t_2 will be less than t_1. Hence $(t_2 - t_1)$ will be negative. Therefore both ΔK_p and ΔQ_p will be negative. Now a negative increase in internal energy must be interpreted as a positive decrease and a negative amount of heat added must be interpreted as heat abstracted or rejected by the gas.

A constant pressure compression will always result in negative values of ΔW_p, ΔK_p and ΔQ_p. The interpretations of the latter will be brought out in the following example. One will never go astray in such a problem if initial and final conditions are carefully distinguished from each other for mathematics will do the rest.

Example. Consider as previously mentioned point B of Fig. 13 to be the initial state instead of the final state. Then point A will become the final state and the diagram will indicate a constant pressure compression instead of an expansion. Under these changed conditions, find the external work done. Is it done by the gas or on the gas?

Solution. Use formula (44) in which with the changes made, $V_1 = V_b = 6$ cu. ft., $V_2 = V_a = 1$ cu. ft., $P = 500$ lbs. per sq. ft., abs., $J = 778$

Substituting these values in our formula

$$\Delta W_p = \frac{500}{778}(1-6) = \frac{500}{778} \times (-5) = -3.21 \text{ B.t.u.}$$

Since the external work done by the gas, ΔW_p, is negative, external work $= +3.21$ B.t.u. is done on the gas.

Example. A gas is compressed at a constant pressure of 100 pounds per square inch absolute from a volume of 5 cubic feet to a volume of 2 cubic feet. How much external work is done? Is it done by the gas or on the gas?

Solution. Here $V_1 = 5$ cu. ft., $V_2 = 2$ cu. ft., $P = 100 \times 144$ lbs. per sq. ft. absolute.

Substituting these values in formula (44)

$$\Delta W_p \text{ (work done by the gas)} = \frac{P}{J}(V_2 - V_1) \text{ B.t.u. when } P \text{ is constant}$$

$$\Delta W_p = \frac{100 \times 144}{778}(2-5) = \frac{100 \times 144}{778} \times (-3)$$
$$= -55.527 \text{ B.t.u.}$$

Therefore since the work done by the gas $= -55.527$ B.t.u., the work is actually done on the gas and to an amount $= +55.527$ B.t.u. *Ans.* This answer can be changed to foot-pounds by multiplying by 778, or the original substitution can be made in formula (43), as follows:

W(work done on the gas)$= P(V_2 - V_1)$ ft.-lbs. when P is constant

$\therefore W = 100 \times 144(2-5) = 100 \times 144 \times (-3) = -43,200$ ft.-lbs.

$\therefore$ the work done on the gas $= +43,200$ ft.-lbs.   *Ans.*

Example. Ten pounds of air are compressed at constant pressure. The initial temperature is 600° F.; the final temperature is 100° F.

(a) What amount of heat is added to or rejected by the gas?

(b) What change in internal energy takes place? Is it an increase or decrease in internal energy?

(c) How much external work is done? Is it done by the gas or on the gas?

Solution. (a) Here $M = 10$ lbs., $t_1 = 600°$ F., $t_2 = 100°$ F., $C_p = 0.2375$ (Table I), $C_v = 0.169$ (Table I)

Substituting these values in formula (**54**),

$$\Delta Q_p\text{(heat added)} = MC_p(t_2 - t_1)$$
$$\Delta Q_p = 10 \times 0.2375(100 - 600) = 10 \times 0.2375(-500)$$
$$\Delta Q_p = -1,187.5 \text{ B.t.u., heat added}$$
$$\therefore +1,187.5 \text{ B.t.u. are rejected.}   \textit{Ans.}$$

(b) Substituting in formula (**52**)

$$\Delta K_p\text{(increase in internal energy)} = MC_v(t_2 - t_1)$$
$$\Delta K_p = 10 \times 0.169(100 - 600) = 10 \times 0.169(-500)$$
$$\Delta K_p = -845 \text{ B.t.u., increase in internal (kinetic) energy}$$
$$\therefore +845 \text{ B.t.u. is the decrease in internal (kinetic) energy.}   \textit{Ans.}$$

(c) From formula (**51**)

$$\Delta W_p = \Delta Q_p - \Delta K_p$$

Substituting, in this formula, the values of ΔQ_p and ΔK_p from (a) and (b), we have

$$\Delta W_p = -1,187.5 - (-845) = -1,187.5 + 845 = -342.5 \text{ B.t.u.}$$

$\therefore$ An amount of external work $= +342.5$ B.t.u. or $+266,465$ ft.-lbs. was done on the gas.   *Ans.*

Example. If the constant pressure of the preceding example is 200 pounds per square inch absolute, find

(a) the initial volume

(b) the final volume

(c) Check the external work of the preceding example by using

these volumes in the formula, $\Delta W_p = \dfrac{P}{J}(V_2 - V_1)$

Solution. (a) Here $P = 200 \times 144$ lbs. per sq. ft. absolute, $M = 10$ lbs., $R = 53.3$, $T_1 = 600° + 460° = 1,060°$ F. absolute. Substituting these values in formula (34),

$$PV_1 = MRT_1$$

$$200 \times 144 \times V_1 = 10 \times 53.3 \times 1,060$$

$$V_1 = \frac{10 \times 53.3 \times 1,060}{200 \times 144} = 19.62 \text{ cu. ft. } \textit{Ans.}$$

(b) Since $\dfrac{V_2}{V_1} = \dfrac{T_2}{T_1}$ from formula (26) and $V_1 = 19.62$ cu. ft.,

$T_2 = 100° + 460° = 560°$ F. absolute, and $T_1 = 1,060°$ F. absolute, we have,

$$\frac{V_2}{19.62} = \frac{560}{1,060}$$

$$V_2 = 19.62 \times \frac{560}{1,060} = 10.37 \text{ cu. ft. } \textit{Ans.}$$

(c) $\quad \Delta W_p = \dfrac{\iota}{J}(V_2 - V_1)$

$$= \frac{200 \times 144}{778}(10.37 - 19.62) = \frac{200 \times 144}{778}(-9.25)$$

$$= -342.4 \text{ B.t.u.}$$

$\therefore +342.4$ B.t.u. = the external work done on the gas. *Ans.*

Relation between Specific Heats, C_p and C_v. When a gas changes its state with the volume remaining constant, the heat added, ΔQ_v, must be only that required to permit the change in temperature or in other words to permit the change in internal kinetic energy. This is the statement of formula (46) that $\Delta Q_v = \Delta K_v$. Had this same gas changed its state under a condition of constant pressure, the heat added, ΔQ_p, would have to be sufficient to permit the change in internal kinetic energy and in addition do the external work that must be done. This is evident from formula (50), $\Delta Q_p = \Delta K_p + \Delta W_p$. Hence a given mass of gas would require a larger amount of heat added for a given temperature change under a constant pressure process than it would require for the same temperature change under a constant volume process, that is, its ΔQ_p would be larger than its ΔQ_v. Since under these conditions $\Delta K_p = \Delta K_v$, ΔQ_p would be larger

than ΔQ_v by an amount of heat energy equal to ΔW_p. By formula (54), $\Delta Q_p = MC_p(t_2 - t_1)$ for the constant pressure process, and by formula (47), $\Delta Q_v = MC_v(t_2 - t_1)$ for the constant volume process. Since in this comparison M of one formula or process is equal to M of the other and the temperature change $(t_2 - t_1)$ of one is equal to the temperature change of the other, the only way in which ΔQ_p can be larger than ΔQ_v is for C_p to be larger than C_v for the same gas. By this it becomes evident that for a given gas, C_p is always larger than C_v.

A direct relation between C_p and C_v can be derived by again referring to formula (50)

$$\Delta Q_p = \Delta K_p + \Delta W_p \quad (a)$$

Substituting in (a) for ΔQ_p its equal $MC_p(t_2 - t_1)$, we have

$$MC_p(t_2 - t_1) = \Delta K_p + \Delta W_p \quad (b)$$

Now for ΔK_p, substitute its equal, $MC_v(t_2 - t_1)$, obtaining

$$MC_p(t_2 - t_1) = MC_v(t_2 - t_1) + \Delta W_p \quad (c)$$

But $\Delta W_p = \dfrac{P}{J}(V_2 - V_1)$, which gives

$$MC_p(t_2 - t_1) = MC_v(t_2 - t_1) + \frac{P}{J}(V_2 - V_1) \quad (d)$$

The term, $\dfrac{P}{J}(V_2 - V_1)$ when expanded $= \dfrac{1}{J}(PV_2 - PV_1)$ (e)

in which $PV_2 = MRT_2$ and $PV_1 = MRT_1$ from formula (34) Substituting these values in (e)

$$\frac{P}{J}(V_2 - V_1) = \frac{1}{J}(MRT_2 - MRT_1) = \frac{1}{J}MR(T_2 - T_1) \quad (f)$$

Placing this value for $\dfrac{P}{J}(V_2 - V_1)$ in (d), we have

$$MC_p(t_2 - t_1) = MC_v(t_2 - t_1) + \frac{1}{J}MR(T_2 - T_1)$$

It is evident that $(T_2 - T_1)$ the difference of the absolute temperatures is equal to $(t_2 - t_1)$ the difference of their corresponding Fahrenheit temperatures, therefore

$$MC_p(t_2 - t_1) = MC_v(t_2 - t_1) + \frac{1}{J}MR(t_2 - t_1) \quad (g)$$

Dividing both members of (g) by $M(t_2 - t_1)$,

$$C_p = C_v + \frac{R}{J} \qquad (55)$$

or, transposing C_v to the first member of the equation

$$C_p - C_v = \frac{R}{J} \qquad (56)$$

This last equation in its two forms again clearly shows that for a gas, the specific heat at constant pressure, C_p, is always larger than its specific heat at constant volume, C_v, because R and J are always positive.

Constant Temperature, or Isothermal Process. A change of state of a gas in which the temperature remains constant is called an Isothermal process. The gas may be expanded isothermally in which case it will be shown that heat must be absorbed by the working substance from some external source, or it may be compressed isothermally in which case heat must be abstracted from or rejected by the gas. Since throughout an isothermal process the temperature must always remain the same, the process follows Boyle's Law. This is stated by formula (24) as

$$P_1 V_1 = P_2 V_2$$

and by formula (25) as

$$PV = \text{a constant}$$

These formulas indicate that at any state through which a gas passes during a given isothermal process, the product of the absolute pressure in pounds per square foot and the corresponding volume in cubic feet will always be the same number.

From this it follows that the graph of an isothermal process on the PV-diagram will always be a rectangular hyperbola such as ACB in Fig. 15. It can be constructed by first drawing the horizontal line, pp_1, and the vertical line vv_1, through point, C, which represents a thermodynamic state through which the gas passes. Next, from the origin, O, draw several radial lines as shown. Through the intersections of these radial lines with the line pp_1, draw vertical lines such as bB and through the intersections with the line, vv_1, draw horizontal lines, such as b_1B. The intersection of the vertical line and horizontal line drawn from the same radial line will be a point on the isothermal curve. Hence bB and b_1B, drawn from radial line Ob intersect at B, a point on the PV-diagram.

If the gas starts from an initial state, C, and proceeds to a final state, B, the curve CB will represent its expansion for the volumes are increasing along the curve from C to B since V_b, the volume of B is at a greater distance from the origin, O, than V_c, the volume of C to the same scale. In such a case the area CBV_BV_C will represent the external work in foot-pounds done by the gas during its expansion from C to B and the mathematical solution for ΔW_t in British thermal units or W_t in foot-pounds would result in a positive value.

Likewise if C represents the initial state of the gas which now proceeds along the curve to A, its final state, the gas is being com-
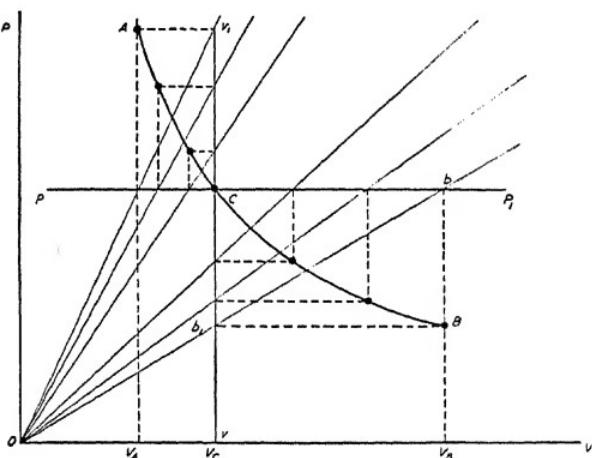
Fig. 15

pressed, for V_A, the volume of A, is less than V_C. The curve from C to A will then represent an isothermal compression and the area CAV_AV_C will be the external work in foot-pounds done on the gas. The mathematical solution for ΔW_t or W_t would in this case of compression result in a negative value, the interpretation of which has been discussed heretofore in this chapter.

Referring again to Fig. 15, it is well to notice that as the volumes get larger, the corresponding pressures become smaller. This is as we would expect it since Boyle's Law states that the absolute pressures vary inversely as the volumes when the temperature is constant.

Since this is a process involving an ideal gas or one assumed to

behave as such, the characteristic equation,

$$PV = MRT$$

will also apply here.

As has been stated, the area between the curve of the isothermal process and the V axis represents the external work done during the process. But in this case the shape of the area is not a geometrical figure whose area can be easily obtained from some formula of geometry as was the case in the constant pressure process. This can however be arrived at and a formula for the external work thus obtained by considering the area below the isothermal curve, divided into a
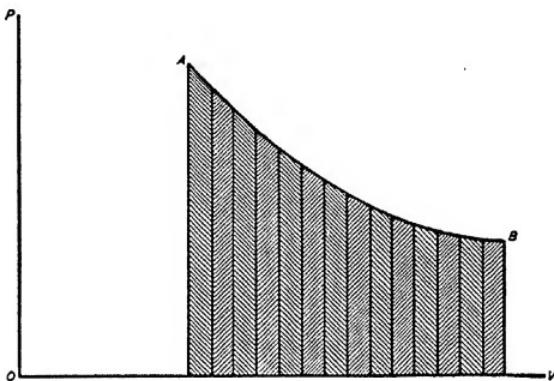
Fig. 16

large number of small areas as in Fig. 16. It is evident that if each sub-division is very narrow, it would very closely approximate a rectangle whose area can be found. The sum of the areas of all the subdivisions would then be very nearly the area below the curve. Mathematics can theoretically produce sub-divisions narrow enough so that when it takes the sum of the small areas, the large area is exactly determined and hence a formula for the external work, W_t, done by the gas is set up. This formula is as follows:

$$W_t = P_1 V_1 \times 2.3 \ \log \frac{V_2}{V_1} \ \text{ft.-lbs.} \tag{57}$$

where P_1 = absolute pressure in lbs. per sq. ft. at the initial state

V_1 = volume in cubic feet at initial state

V_2 = volume in cubic feet at final state

Since $P_1 V_1 = P_2 V_2$ in an isothermal process, it is evident that $P_2 V_2$ can be used in formula (57) in the place of $P_1 V_1$.

In that $P_1 V_1 = MRT$, we may from formula (57) obtain through substitution

$$W_t = MRT \times 2.3 \log \frac{V_2}{V_1} \text{ ft.-lbs.} \tag{58}$$

Since by applying formula (41) to the isothermal process, $\Delta W_t = \frac{W_t}{J}$, formula (57), can be rewritten in terms of British thermal units as follows

$$\Delta W_t = \frac{P_1 V_1}{J} \times 2.3 \log \frac{V_2}{V_1} \text{ B.t.u.} \tag{59}$$

and in a similar manner, we obtain from formula (58),

$$\Delta W_t = \frac{MRT}{J} \times 2.3 \log \frac{V_2}{V_1} \text{ B.t.u.} \tag{60}$$

The general energy equation when written for an isothermal process becomes

$$\Delta Q_t = \Delta K_t + \Delta W_t$$

With no change in temperature, $t_2 = t_1$ and

$$\Delta K_t, \text{ like any } \Delta K, = M C_v (t_2 - t_1) = M C_v (t_1 - t_1) = M C_v \times 0$$

Therefore $\qquad\qquad \Delta K_t = 0 \tag{61}$

Substituting this value of ΔK_t in the general energy equation we have

$$\Delta Q_t = \Delta W_t \tag{62}$$

This formula states that the heat added to the working substance during an isothermal process is, in its entirety, used in doing the external work to be done. It might be well to note that in formula (62) if ΔW_t is negative, ΔQ_t must also be negative. This condition of algebraic signs is met with always in an isothermal compression for, during the latter, external work is actually being done on the gas. Hence it follows from these statements that formula (62) can be further interpreted as stating that the heat rejected during an isothermal compression is equal to the heat equivalent of the external work done on the gas. If this heat were not rejected, it would necessarily go into the gas. This would result in an increase in internal kinetic energy and hence in that which always accompanies the latter,

an increase in temperature. This, of course, is impossible during an isothermal process for the temperature must remain constant.

From formulas (59), (60), and (62), it is evident that

$$\Delta Q_t = \frac{P_1 V_1}{J} \times 2.3 \log \frac{V_2}{V_1} \text{ B.t.u.} \tag{63}$$

and

$$\Delta Q_t = \frac{MRT}{J} \times 2.3 \log \frac{V_2}{V_1} \text{ B.t.u.} \tag{64}$$

In the constant volume and constant pressure processes, the heat absorbed (or rejected) by the working substance was computed by means of a specific heat. The heat absorbed during an isothermal process can not be so determined due to the fact that the change in temperature is zero. It must therefore be determined as shown in formulas (63) and (64). Since Boyle's Law produces formula (23),

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} :: \frac{p_1}{p_2}, \frac{P_1}{P_2} \text{ or } \frac{p_1}{p_2}$$

may be substituted for $\frac{V_2}{V_1}$ in any formula for an isothermal process.

Example. An ideal gas having a pressure of 100 pounds per square inch gauge and a volume of 3 cubic feet is isothermally expanded until its volume is doubled. How much heat energy must be supplied to the gas?

Solution. Here $P_1 = (100+14.7)144$ lbs. per sq. ft. absolute, $V_1 = 3$ cu. ft., $V_2 = 6$ cu. ft.
Evaluating in formula (63)

$$\Delta Q_t = \frac{P_1 V_1}{J} \times 2.3 \times \log \frac{V_2}{V_1}$$

$$\Delta Q_t = \frac{(100+14.7) \times 144 \times 3}{778} \times 2.3 \times \log \frac{6}{3}$$

Since $\log \frac{6}{3} = \log 2 = 0.3010$

$$\Delta Q_t = \frac{114.7 \times 144 \times 3}{778} \times 2.3 \times 0.3010$$

$$\Delta Q_t = 44.09 \text{ B.t.u.} \quad \textit{Ans.}$$

Example. Four pounds of air expand at a constant temperature of 140° F. from a pressure of 240 pounds per square inch absolute to a pressure of 30 pounds per square inch absolute.

(a) What is the change in internal energy?

(b) How much external work in foot-pounds is done by the gas during the expansion?

(c) How much heat must necessarily be added from an external source to permit the expansion in this manner?

Solution.

(a) In any isothermal process since the temperature is constant,

$$\Delta K_t = 0 \quad Ans.$$

(b) p_1 and p_2 are known in this example, therefore we shall substitute their ratio $\dfrac{p_1}{p_2}$ for its equal $\dfrac{V_2}{V_1}$ in formula (58). This gives us

$$W_t = MRT \times 2.3 \log \frac{p_1}{p_2} \text{ ft.-lbs.}$$

Here $M = 4$ lbs., $R = 53.3$, $T = 140° + 460° = 600°$ F. absolute, $p_1 = 240$ lbs. per sq. in. absolute, $p_2 = 30$ lbs. per sq. in. absolute. Evaluating in the above formula,

$$W_t = 4 \times 53.3 \times 600 \times 2.3 \times \log \frac{240}{30}$$

since

$$\log \frac{240}{30} = \log 8 = 0.9031$$

$$W_t = 4 \times 53.3 \times 600 \times 2.3 \times 0.9031$$
$$W_t = 265{,}706 \text{ ft.-lbs.} \quad Ans.$$

(c) Since the external work in ft.-lbs. has just been found, we shall merely convert the answer of part (b) to B.t.u. and this will be the heat added.

$$\Delta Q_t = \frac{W_t}{J} = \frac{265{,}706}{778} = 341.5 \text{ B.t.u.} \quad Ans.$$

Example. Find the initial volume, V_1, and the final volume, V_2, of the air in the preceding example.

Solution. Here $P_1 = 240 \times 144$ lbs. per sq. ft. absolute; $M = 4$ lbs., $R = 53.3$, $T = 600°$ F. absolute

Since $P_1 V_1 = MRT$

$$240 \times 144 \times V_1 = 4 \times 53.3 \times 600$$

Dividing both members of the equation by 240×144

$$V_1 = \frac{4 \times 53.3 \times 600}{240 \times 144} = 3.7 \text{ cu. ft. } Ans.$$

To find V_2, the previous method can be used or formula (23) may be applied. Using the latter, with $P_2 = 30 \times 144$ lbs. per sq. ft. absolute

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

Evaluating

$$\frac{V_2}{3.7} = \frac{240 \times 144}{30 \times 144}$$

(**Note.** The permissible cancellation of 144 in the second member shows why $\dfrac{P_1}{P_2} = \dfrac{p_1}{p_2}$)

Cancel 144 from numerator and denominator of the second member and then multiply both members of the equation by 3.7,

$$V_2 = 3.7 \times \frac{240}{30} = 29.6 \text{ cu. ft. } Ans.$$

Example. An ideal gas is compressed at constant temperature from an initial volume of 4 cubic feet to a final volume of 1 cubic foot. If the initial pressure is atmospheric and the barometer reading is 29.5 inches of mercury,

(*a*) What transfer of heat takes place?

(*b*) Is the heat absorbed or rejected by the gas?

Solution.

(*a*) It will be necessary to change the atmospheric pressure from in. of Hg. to lbs. per sq. in. Thus,

$$29.5 \times 0.491 = 14.5 \text{ lbs. per sq. in. absolute}$$

Then $P_1 = 14.5 \times 144$ lbs. per sq. ft. abs., $V_1 = 4$ cu. ft., $V_2 = 1$ cu. ft. Applying formula (63)

$$\Delta Q_t = \frac{14.5 \times 144 \times 4}{778} \times 2.3 \times \log \frac{1}{4}$$

Now, $\log \dfrac{1}{4} = \log 0.25 = -1.3979$ or in the other logarithmic form, $9.3979 - 10$. Such a logarithm is too unwieldy a form to be used as a factor, for it is in reality an algebraic expression of two terms, in other words a binomial. This is due to the fact that the character-

istic of the logarithm of any number less than unity is negative, while the mantissa is positive in all logarithms. All logarithms like this one can be changed to an expression of one term, that is to a monomial form, by collecting the terms in the original logarithmic form. The resulting monomial will always be negative, and hence will insist that the above product, in which it is a factor, be negative since it is the only negative factor. It must be remembered that the value of the logarithm thus produced is not a logarithmic form itself and must be changed back to such a form if it is to be used in logarithmic calculations. No trouble is occasioned by the logarithms of numbers greater than unity, because their characteristics are positive which permits the logarithm in its logarithmic form to act as a monomial factor. This was evident in the preceding example.

Changing the logarithm of this problem as suggested above,

$$9.3979 - 10 = 9.3979 + (-10)$$

Perform this addition, keeping in mind that when two numbers with unlike signs are added, take the difference of their absolute values and prefix the sign of the greater absolute value. Thus, adding or collecting the terms,

$$-10.0000$$
$$+\ 9.3979$$
$$-\ 0.6021,\ \text{the value of the logarithm or its}$$
monomial form.

Substituting this value for $\log \frac{1}{4}$ in the formula for ΔQ_t,

$$\Delta Q_t = \frac{14.5 \times 144 \times 4}{778} \times 2.3 \times (-0.6021)$$

$$= -14.87 \text{ B.t.u.}\quad Ans.$$

(b) Since ΔQ_t, (heat added or absorbed), is negative, an amount of heat equal to $+14.87$ B.t.u. must be rejected. Keep in mind that negative heat added is positive heat rejected.

Example. The cylinder of an air compressor admits 1.75 cubic feet of air per stroke. The air is taken in at a pressure of 14.6 pounds per square inch absolute and at a temperature of 70° F. It is compressed isothermally to a pressure of 150 pounds per square inch absolute. It is required to find:

(a) the weight of air in the cylinder at the beginning of the compression stroke,

(b) the final volume of the compressed air,

(c) the work done upon the gas during compression per working stroke,

(d) the increase in internal energy,

(e) the heat abstracted from the cylinder per working stroke.

Solution.

(a) Here $P_1 = 14.6 \times 144$ lbs. per sq. ft., absolute, $V_1 = 1.75$ cu. ft., $R = 53.3$, $T = 70° + 460° = 530°$ F., absolute.

Substituting these values in formula (34)

$$P_1 V_1 = MRT$$

$$14.6 \times 144 \times 1.75 = M \times 53.3 \times 530$$

Dividing both members of the equation by 53.3×530

$$M = \frac{14.6 \times 144 \times 1.75}{53.3 \times 530} = 0.130 \text{ lb.} \quad Ans.$$

(b) From formula (23), with $P_2 = 150 \times 144$ lbs. per sq. ft. abs.

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

Substituting in the formula

$$\frac{V_2}{1.75} = \frac{14.6 \times 144}{150 \times 144}$$

Cancelling 144 in the second member and multiplying both members by 1.75

$$V_2 = 1.75 \times \frac{14.6}{150} = 0.170 \text{ cu. ft.} \quad Ans.$$

(c) Substituting values, given or previously determined, in formula (58)

$$W_t = MRT \times 2.3 \times \log \frac{V_2}{V_1} \text{ ft.-lbs.}$$

$$W_t = 0.13 \times 53.3 \times 530 \times 2.3 \times \log \frac{0.17}{1.75}$$

The method of obtaining $\log \frac{0.17}{1.75}$ is as follows:

$$\log 0.17 = 9.2304 - 10$$
$$\log 1.75 = \underline{0.2430}$$

Subtracting, $\log \dfrac{0.17}{1.75} = 8.9874 - 10$

Since $8.9874 - 10 = 8.9874 + (-10)$, we obtain by addition

$$-10.0000$$
Adding $+\ \underline{8.9874}$
$$-\ \overline{1.0126}, \text{ the value of the logarithm,}$$
$$\text{or its monomial form.}$$

Placing this value for $\log \dfrac{0.17}{1.75}$ in the formula for W_t,

$$W_t = 0.13 \times 53.3 \times 530 \times 2.3 \times (-1.0126)$$
$$W_t = -8552.9 \text{ ft.-lbs., work done by the gas}$$

Therefore the work done on the gas $= +8552.9$ ft.-lbs. *Ans.*

(*d*) Since the compression is at constant temperature

$$\Delta K_t = 0, \quad Ans.$$

(*e*) Since $\Delta Q_t = \Delta W_t$, and $\Delta W_t = \dfrac{W_t}{J}$

$$\Delta Q_t = \Delta W_t = \frac{-8552.9}{778} = -10.919 \text{ B.t.u., heat added}$$

$\therefore$ heat abstracted per working stroke $= +10.919$ B.t.u. *Ans.*

Exponential and Logarithmic Computations. In the preceding work on the isothermal process, it was seen that a logarithm is often introduced as a factor in a continued product. It was pointed out that if the logarithm was positive algebraically, the logarithmic form represented the value of the logarithm as a factor in the continued product and could be dealt with in multiplication exactly as any other factor. It was also pointed out that if the logarithm had a negative characteristic, the logarithm was of a binomial form and could not easily be dealt with. In the latter case, the value of the logarithm was obtained by collecting the terms of the logarithm. This simplified it to a single term that was always negative in sign, whence it could be used like unto any other factor in a continued product. In the preceding work, these products were then obtained by actual multiplication and not through logarithmic computation of the multiplication.

Now there are times when a continued product having a logarithm as one of its factors is possibly more easily solved by logarithmic computation than by straight multiplication. As a simple example merely for the purpose of demonstrating this point, let it be required to find the value of $6 \times \log \frac{1}{2}$.

Solution. By straight multiplication;

$$\text{Log } \tfrac{1}{2} = \log 0.5 = 9.6990 - 10$$

Simplifying by collecting or adding the terms of the logarithm

$$
\begin{array}{r}
-10.0000 \\
+\ 9.6990 \\
\hline
-\ 0.3010,
\end{array}
$$
the value of the logarithm

Therefore

$$6 \times \log \tfrac{1}{2} = 6 \times (-0.3010) = -1.8060 \quad Ans.$$

Solution. By logarithmic computation; the work will proceed as before, arriving at the statement,

$$6 \times \log \tfrac{1}{2} = 6 \times (-0.3010)$$

Now in solving by logarithms, keep in mind that the logarithm of a negative number can not be taken. Therefore all factors are considered as positive, and the algebraic sign of the product is determined independently of the logarithmic computation.

Logarithm of 6, or $\qquad$ $\log 6 = 0.7782$
Logarithm of $\log \frac{1}{2}$, or $\log (\log \frac{1}{2})$, or
$\quad \log (9.6990 - 10)$, or $\log (-0.3010) = 9.4786 - 10$
By addition, the log of the absolute value
$\quad$ of product $\qquad = 10.2568 - 10$

Therefore the absolute value of the product $= \log^{-1} 0.2568 = 1.806$
Therefore the product $= -1.806$

It will be noted in the above example, that since one factor was $\log \frac{1}{2}$, its logarithm had to be used the same as the logarithm of any other factor. Hence the log of a log was taken. Since $\log \frac{1}{2} = 9.6990 - 10$, it is evident that in order to secure the log $(\log \frac{1}{2})$, the form, $9.6990 - 10$, had to be changed to the equivalent value, (-0.3010), before its logarithm could be obtained.

In the next process to be discussed in this chapter the formulas are of such a type that logarithmic computation must be resorted to

quite often. The preceding example and those to follow will serve as a review. They should be very carefully studied.

Example. What does $(0.045)^{0.3}$ equal?

Solution. Let our answer be called, N

then
$$N = (0.045)^{0.3}$$

and
$$\log N = (0.3) \times \log(0.045)$$
$$= (0.3) \times (8.6532 - 10)$$

Simplify or collect the terms in $(8.6532 - 10)$ as follows,

$$-10.0000$$
$$+ \ 8.6532$$
$$\overline{- \ 1.3468}$$

Therefore $\log N = (0.3) \times (-1.3468) = -0.4040$

But this is not in its logarithmic form for it is negative and the mantissa of a log is always positive. Before it can be taken to the tables to obtain the number, N, it must be changed to its logarithmic form where the mantissa will be positive. This can always be accomplished by the addition of zero to the value in the form of $(10 - 10)$, as follows

$$10.0000 \quad -10$$
$$- \ 0.4040$$
$$\overline{\ 9.5960} \quad -10, \text{ the logarithmic form in which the character-}$$
istic is (-1) or $(9-10)$, and the mantissa
is $+0.5960$

or $(-0.4040) + 10 - 10 = 10 + (-0.4040) - 10 = 10 - (0.4040) - 10$
$$= 9.5960 - 10$$

Hence $\log N = 9.5960 - 10$

Therefore $N = 0.3945$ *Ans.*

Example. Find the value of P in the equation, $P = 80 \times (\frac{5}{8})^{1.3}$

Solution.

$$\text{Log } 5 = 10.6990 - 10$$
$$\text{Log } 8 = \underline{\ 0.9031}$$

Subtracting, $\overline{9.7959 - 10} = \text{Log } \frac{5}{8}$

$$\text{Log } (\tfrac{5}{8})^{1.3} = 1.3 \times \log \tfrac{5}{8}$$
$$= 1.3 \times (9.7959 - 10)$$
$$= 1.3 \times (-0.2041)$$
$$= -0.2653, \text{ which being entirely negative is}$$
not a logarithmic form

Changing (-0.2653) to its logarithmic form,

$$\log \left(\tfrac{5}{8}\right)^{1.3} = 10 - (0.2653) - 10 = 9.7347 - 10$$
$$\log 80 \qquad\qquad\qquad = \underline{1.9031}$$

By addition of these logs,

$$\log\left[80 \times \left(\tfrac{5}{8}\right)^{1.3}\right] = 11.6378 - 10 \text{ or } 1.6378$$

Hence

$\log P$ which is equal to $\log\left[80 \times \left(\tfrac{5}{8}\right)^{1.3}\right] = 1.6378$

Therefore $\qquad\qquad\qquad\qquad P = 43.43 \quad Ans.$

Example. Solve the equation, $n = \dfrac{\log \tfrac{25}{75}}{\log \cdot}$

Solution.

$$\text{Log } \tfrac{25}{75} = \log 0.333 = 9.5224 - 10$$
$$\text{Log } \tfrac{2}{5} = \log 0.400 = 9.6021 - 10$$

Substituting these logarithms in the equation,

$$n = \frac{9.5224 - 10}{9.6021 - 10} = \frac{-0.4776}{-0.3979} = \frac{0.4776}{0.3979}$$

The above can be solved directly by division or by logarithmic calculation. The latter follows:

$$\log 0.4776 = 9.6790 - 10$$
$$\log 0.3979 = \underline{9.5999 - 10}$$

Subtracting, $\qquad\qquad \log n = \overline{0.0791}$

Therefore $\qquad\qquad n = \log^{-1} 0.0791 = 1.20 \quad Ans.$

Adiabatic Process. An adiabatic change of state, or process, is one in which the gas neither receives nor rejects heat while it expands or is compressed. Hence in this process

$$\Delta Q_a = 0 \qquad\qquad\qquad\qquad (65)$$

so that the general energy equation for an ideal gas, formula (22) becomes

$$0 = \Delta K_a + \Delta W_a \qquad\qquad\qquad (66)$$

or transposing ΔK_a to the other member of the formula,

$$\Delta W_a = -\Delta K_a \qquad\qquad\qquad (67)$$

This formula states that when an ideal gas expands adiabatically, the external work done by the gas is done at the expense of the internal energy of the gas, causing the latter to decrease. Certain it is, that, if a gas expands thus doing external work, some source of energy must be tapped to provide the energy utilized in doing the

external work. Since, in an adiabatic process, no heat energy can be added from an external source, the only supply of available energy is the internal energy of the gas.

If ΔW_a of formula (66) is transposed to the other member of the equation,

$$- \Delta W_a = \Delta K_a \tag{68}$$

This formula states that when a gas is compressed by external work being done upon it, there results an increase in internal energy. So, compressing a gas adiabatically builds up its supply of internal kinetic energy and hence increases the temperature of the gas, while the
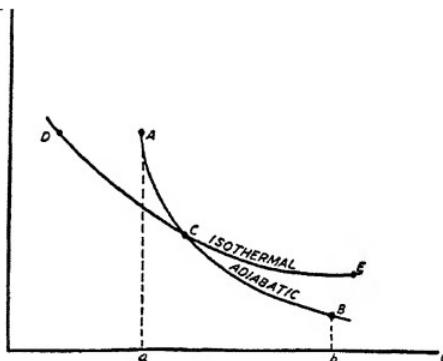
Fig. 17

adiabatic expansion of a gas decreases its supply of internal kinetic energy, cooling the gas, or lowering its temperature.

The so-called adiabatic curve representing the adiabatic process on the PV-diagram is shown in Fig. 17, where it is contrasted with the isothermal curve. It will be noted that when a gas is expanded first isothermally and next adiabatically from the same point such as C, the pressure decreases with the increase in volume more rapidly during the adiabatic than during the isothermal expansion. On the other hand if the gas is compressed from point C, the pressure increases with a decrease in volume more rapidly during the adiabatic than during the isothermal compression.

As in the other processes, the area $ABba$ between the adiabatic curve AB and the V axis represents the external work done by the gas in foot-pounds as it expands adiabatically from A to B. The same area

represents the external work done upon the gas as it is compressed adiabatically from B to A. Of course as in the other processes the value of the latter will be negative in algebraic sign while the former is positive when determined from the formulas for external work done by the gas which are to follow. The algebraic sign for W_a in foot-pounds, or ΔW_a in British thermal units, leads then to the proper interpretation as to whether the work is being done by the gas, in which case the sign is positive, or whether the external work is being done on the gas in which case the sign is negative. Care must always be exercised in distinguishing between the initial and final conditions or thermodynamic states.

The characteristic equation of an ideal gas, $PV = MRT$, (34) is true at all times for an ideal gas. Therefore it is applicable in the adiabatic process. Another relation of P and V, for an adiabatic process only, is

$$PV^k = \text{a constant} \tag{69}$$

where $\qquad k = \dfrac{C_p}{C_v}$ [see formula (20)]

$P = $ absolute pressure in pounds per square foot

$V = $ volume in cubic feet

This formula states that if a gas is changing its state along the adiabatic curve such as AB in Fig. 17,

$$P_a V_a^k = P_c V_c^k = P_b V_b^k$$

or in general

$$P_1 V_1^k = P_2 V_2^k \tag{70}$$

where the subscript, 1, indicates the initial state and the subscript, 2, indicates the final state of the working substance.

In an adiabatic process, P, V, and T all vary or change as the gas goes from one state to another. It is necessary to obtain certain relations that exist between these pressures, volumes, and temperatures at different thermodynamic states in addition to formulas (69) and (70). The general relationship of these characteristics is expressed by formula (31) in the following manner,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Since if equals are divided by equals their quotients are equal, divide the first member of formula (31) by the first member of formula (70) and equate this quotient to the quotient obtained by dividing the second member of (31) by the second member of (70).

$$\frac{\dfrac{P_1 V_1}{T_1}}{P_1 V_1{}^k} = \frac{\dfrac{P_2 V_2}{T_2}}{P_2 V_2{}^k}$$

or

$$\frac{P_1 V_1}{T_1} \times \frac{1}{P_1 V_1{}^k} = \frac{P_2 V_2}{T_2} \times \frac{1}{P_2 V_2{}^k}$$

Now cancel P_1 in the first member and P_2 in the second member

$$\frac{V_1}{T_1 V_1{}^k} = \frac{V_2}{T_2 V_2{}^k}$$

since

$$\frac{V_1}{V_1{}^k} = V_1^1 \times V_1^{-k} = V_1{}^{(1-k)} \quad \text{and} \quad \frac{V_2}{V_2{}^k} = V_2^1 \times V_2^{-k} = V_2{}^{(1-k)},$$

we may substitute these values in the above, obtaining,

$$\frac{V_1{}^{(1-k)}}{T_1} = \frac{V_2{}^{(1-k)}}{T_2}$$

or

$$\frac{T_2}{T_1} = \frac{V_2{}^{(1-k)}}{V_1{}^{(1-k)}}$$

from which

$$T_2 = T_1 \times \frac{V_2{}^{(1-k)}}{V_1{}^{(1-k)}} = T_1 \times \frac{V_1{}^{-(1-k)}}{V_2{}^{-(1-k)}}$$

since

$$-(1-k) = -1 + k = k - 1,$$

we have

$$T_2 = T_1 \times \frac{V_1{}^{(k-1)}}{V_2{}^{(k-1)}} \tag{71}$$

By similar procedure to the above, the following relations can be derived:

$$P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^k \tag{72}$$

$$V_2 = V_1 \times \left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}} \tag{73}$$

$$V_2 = V_1 \times \left(\frac{P_1}{P_2}\right)^{\frac{1}{k}} \tag{74}$$

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \tag{75}$$

$$P_2 = P_1 \times \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} \tag{76}$$

The external work accomplished during this type of process is obtained from the formula,

$$W_a = \frac{P_1 V_1 - P_2 V_2}{k-1} \text{ ft.-lbs.} \tag{77}$$

From the above, since $\Delta W_a = \frac{W_a}{J}$,

$$\Delta W_a = \frac{1}{J} \times \frac{P_1 V_1 - P_2 V_2}{k-1} \text{ B.t.u.} \tag{78}$$

Since $P_1 V_1 = MRT_1$ and $P_2 V_2 = MRT_2$, by substituting in formula (77),

$$W_a = \frac{MRT_1 - MRT_2}{k-1}$$

$$= \frac{MR(T_1 - T_2)}{k-1} \text{ ft.-lbs.} \tag{79}$$

and likewise from (78),

$$\Delta W_a = \frac{1}{J} \times \frac{MR(T_1 - T_2)}{k-1} \text{ B.t.u.} \tag{80}$$

The increase in internal energy, ΔK_a, like any ΔK, is given by the formula

$$\Delta K_a = MC_v(T_2 - T_1) \text{ B.t.u.} \tag{81}$$

From formulas (68) and (78)

$$\Delta K_a = -\Delta W_a = -\left[\frac{1}{J} \times \frac{P_1 V_1 - P_2 V_2}{k-1}\right]$$

$$= \frac{1}{J} \times \frac{-(P_1 V_1 - P_2 V_2)}{k-1} = \frac{1}{J} \times \frac{P_2 V_2 - P_1 V_1}{k-1} \text{ B.t.u.} \tag{82}$$

From formulas (68) and (80),

$$\Delta K_a = -\left[\frac{1}{J} \times \frac{MR(T_1 - T_2)}{k-1}\right] = \frac{1}{J} \times \frac{-MR(T_1 - T_2)}{k-1}$$

$$= \frac{1}{J} \times \frac{MR(T_2 - T_1)}{k-1} \text{ B.t.u.} \qquad (83)$$

Extreme care must again be exercised in all of the formulas of this process to see that the conditions of the initial and final states as indicated by the subscripts are correctly used.

Example. An ideal gas in expanding adiabatically in a cylinder did 28,000 foot-pounds of work upon the piston.

(a) How much heat energy was required to do this work?

(b) What was the source of this heat energy and what effect was there upon this source?

Solution. (a) Since 778 ft.-lbs. = 1 B.t.u., the heat energy required equals $28,000 \div 778 = 36$ B.t.u. *Ans.*

(b) Since $\Delta W_a = -\Delta K_a$ and $\Delta W_a = 36$ B.t.u., $\Delta K_a = -36$ B.t.u., the increase in internal kinetic energy. Therefore the source of the heat energy used in doing the external work was the supply of internal kinetic energy of the gas. The effect upon this source was to decrease the internal kinetic energy by an amount $= +36$ B.t.u. (for a negative increase = a positive decrease).

Example. Three cubic feet of air at a pressure of 150 pounds per square inch absolute expand adiabatically to double their volume. What is the final pressure?

Solution. Here $P_1 = 150 \times 144$ lbs. per sq. ft. absolute, $V_1 = 3$ cu. ft., $V_2 = 6$ cu. ft., $k = 1.406$, from Table I. Using formula (72)

$$P_2 = 150 \times 144 \times \left(\frac{3}{6}\right)^{1.406}$$

$$= 150 \times 144 \times (0.5)^{1.406}$$

To find the value of $(0.5)^{1.406}$

$$\log 0.5 = 9.6990 - 10$$

$$\log (0.5)^{1.406} = 1.406 \times (9.6990 - 10) = 1.406 \times (-0.3010)$$

$$\log (0.5)^{1.406} = -0.4232 \text{ (not a logarithmic form)}$$

To change to the logarithmic form by adding and subtracting 10,

$$\log (0.5)^{1.406} = 10 - 0.4232 - 10 = 9.5768 - 10$$

Therefore $\qquad (0.5)^{1.406} = \log^{-1} 9.5768 - 10$

$$= 0.3774$$

Substituting this value in the formula

$$P_2 = 150 \times 144 \times 0.3774$$

$$P_2 = 8151.84 \text{ lbs. per sq. ft. absolute. } \textit{Ans.}$$

or $\quad p_2 = \dfrac{P_2}{144} = \dfrac{8,151.84}{144} = 56.61 \text{ lbs. per sq. in. absolute. } \textit{Ans.}$

Example. One pound of air at 200° F. expands adiabatically from 250 pounds per square inch absolute to a pressure of 20 pounds per square inch absolute.

(a) What is its final temperature?

(b) What is its initial volume?

(c) What is its final volume?

(d) How much external work is done by the air in expanding?

(e) What is the decrease in internal energy?

(f) How much heat is added from an external source?

Solution. (a) Here $T_1 = 200° + 460° = 660°$ F. absolute, $P_1 = 250 \times 144$ lbs. per sq. ft. absolute, $P_2 = 20 \times 144$ lbs. per sq. ft. abs., $k = 1.406$

Substituting these values in formula (**75**)

$$T_2 = T_1 \times \left(\frac{P_2}{P}\right)^{\frac{k-1}{k}}$$

$$T_2 = 660 \times \left(\frac{20 \times 144}{250 \times 144}\right)^{\frac{1.406-1}{1.406}}$$

$$= 660 \times \left(\frac{20}{250}\right)^{\frac{0.406}{1.406}} = 660 \times (0.08)^{0.289}$$

To obtain $(0.08)^{0.289}$

$$\log 0.08 = 8.9031 - 10$$
$$\log (0.08)^{0.289} = 0.289 \times (8.9031 - 10) = 0.289 \times (-1.0969)$$
$$= -0.3170 \text{ (not a logarithmic form)}$$

Changing to the logarithmic form

$$\log (0.08)^{0.289} = 10 - 0.3170 - 10 = 9.6830 - 10$$

Therefore $\quad (0.08)^{0.289} = \log^{-1} 9.6830 - 10 = 0.482$

Substituting this value in the formula,

$$T_2 = 660 \times 0.482 = 318.12° \text{ F. absolute. } \textit{Ans.}$$

(b) Using formula (34), where $P_1 = 250 \times 144$ lbs. per sq. ft. absolute, $M = 1$ lb., $R = 53.3$, $T_1 = 660°$ F. abs., we have for the initial state

$$P_1 V_1 = MRT_1$$

Evaluating in this formula,

$$250 \times 144 \times V_1 = 1 \times 53.3 \times 660$$

Dividing by 250×144,

$$V_1 = \frac{1 \times 53.3 \times 660}{250 \times 144} = 0.977 \text{ cu. ft. } Ans.$$

(c) Using formula (74), with the data as previously shown,

$$V_2 = 0.977 \left(\frac{250 \times 144}{20 \times 144} \right)^{\frac{1}{1.406}} = 0.977(12.5)^{0.711}$$

To obtain the value of $(12.5)^{0.711}$, we have

$$\log 12.5 = 1.0969$$
$$\log (12.5)^{0.711} = 0.711 \times 1.0969 = 0.7799,$$

which is in its logarithmic form for it is positive, which is as the mantissa of the logarithm must always be.

Therefore $\quad\quad (12.5)^{0.711} = \log^{-1} 0.7799 = 6.024$

Substituting this value in the formula,

$$V_2 = 0.977 \times 6.024 = 5,885 \text{ cu. ft. } Ans.$$

(d) From formula (79)

$$W_a = \frac{MR(T_1 - T_2)}{k - 1} \text{ ft.-lbs.}$$

Evaluating,

$$W_a = \frac{1 \times 53.3 \times (660 - 318.12)}{1.406 - 1}$$

$$= 44,882 \text{ ft.-lbs. } Ans.$$

or

$$\Delta W_a = \frac{W_a}{J} = 57.69 \text{ B.t.u. } Ans.$$

(e) Since $\quad\quad \Delta K_a = -\Delta W_a$
$$\Delta K_a = -57.69 \text{ B.t.u.}$$

Therefore the decrease in internal kinetic energy $= +57.69$ B.t.u. $Ans.$

(*f*) None, since for an adiabatic change of state

$$\Delta Q_a = 0, \text{ always.}$$

Example. Ten cubic feet of air are at a temperature of 70° F. and a pressure of 15 pounds per square inch absolute. They are compressed adiabatically to a final pressure of 120 pounds per square inch absolute.

(*a*) What is the final volume?

(*b*) What is the final temperature?

(*c*) How much external work is done on the gas?

(*d*) What is the increase in internal kinetic energy?

Solution. (*a*) Here $V_1 = 10$ cu. ft., $P_1 = 15 \times 144$ lbs. per sq. ft. absolute, $P_2 = 120 \times 144$ lbs. per sq. ft. absolute, $k = 1.406$

Using formula (74),

$$V_2 = 10 \times \left(\frac{15 \times 144}{120 \times 144}\right)^{\frac{1}{1.406}} = 10 \times (0.125)^{0.711}$$

To obtain the value of $(0.125)^{0.711}$,

Log $0.125 = 9.0969 - 10$

Log $(0.125)^{0.711} = 0.711 \times (9.0969 - 10) = 0.711 \times (-0.9031)$
$= -0.6421$ (not a logarithmic form)

Changing to a logarithmic form

Log $(0.125)^{0.711} = 10 - 0.6421 - 10 = 9.3579 - 10$

Therefore $(0.125)^{0.711} = \log^{-1}(9.3579 - 10) = 0.228$

Substituting this value in the formula

$$V_2 = 10 \times 0.228 = 2.28 \text{ cu. ft.} \quad Ans.$$

(*b*) 1st Method. Solve for M in the formula

$$P_1 V_1 = M R T_1$$

Here P_1 and V_1 are as given in (*a*), $R = 53.3$, $T_1 = 70° + 460° = 530°$ F. absolute

Substituting

$$15 \times 144 \times 10 = M \times 53.3 \times 530$$

Dividing by 53.3×530

$$M = \frac{15 \times 144 \times 10}{53.3 \times 530} = 0.765 \text{ lb.}$$

Substitute known values in

$$P_2V_2 = MRT_2$$

and solve for T_2

$$120 \times 144 \times 2.28 = 0.765 \times 53.3 \times T_2$$

Dividing by 0.765×53.3,

$$T_2 = \frac{120 \times 144 \times 2.28}{0.765 \times 53.3} = 966.4° \text{ F. absolute.} \quad Ans.$$

Therefore $\quad t_2 = 966.4° - 460° = 506.4°$ F. $\quad Ans.$

2nd Method. Using formula (75)

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$= 530 \times \left(\frac{120 \times 144}{15 \times 144}\right)^{\frac{1.406-1}{1.406}} = 530 \times 8^{0.289}$$

To obtain the value of $8^{0.289}$,

$$\log 8 = 0.9031$$

$$\log 8^{0.289} = 0.289 \times 0.9031 = 0.2610, \text{ a logarithmic form}$$

Therefore $8^{0.289} = \log^{-1} 0.2610 = 1.8237$

Substituting this value in the formula,

$$T_2 = 530 \times 1.8237 = 966.6° \text{ F. absolute, which checks closely.}$$

(c) Evaluating in formula (77),

$$W_a = \frac{P_1V_1 - P_2V_2}{k-1} \text{ ft.-lbs.}$$

$$= \frac{15 \times 144 \times 10 - 120 \times 144 \times 2.28}{1.406 - 1}$$

$$= \frac{-17798.4}{0.406}$$

$$= -43838.4 \text{ ft.-lbs., work done by the gas}$$

Therefore the work done on the gas $= +43838.4$ ft.-lbs. $\quad Ans.$

(d) Since $\quad \Delta K_a = -\Delta W_a = -\dfrac{W_a}{J}$,

$$\Delta K_a = \frac{-(-43838.4)}{778} = 56.4 \text{ B.t.u., increase in internal energy } Ans.$$

To check, use formula (83),

$$\Delta K_a = \frac{1}{778} \quad \frac{0.765 \times 53.3 \times (966.6 - 530)}{1.406 - 1}$$

$$= 56.4 \text{ B.t.u., increase in internal energy}$$
checking the above

or to check, use formula (81)

$$\Delta K_a = 0.765 \times 0.169 \times (966.6 - 530)$$

$$= 56.4 \text{ B.t.u., increase in internal energy,}$$
checking again the above.

Polytropic Processes. The thermodynamic processes, which have been heretofore discussed, together with many more processes, can be represented by the equation,

$$PV^n = a \text{ constant} \tag{84}$$

where P = pressure in pounds per square foot
                V = volume in cubic feet
and n = any numerical value, positive or negative

Any such process, no matter by what other name it may be called, is known as a polytropic process. Thus the definition of this process makes it cover a very broad field of thermodynamic changes of state, but its usage in engineering is restricted to a relatively narrow field, as will be shown later on. When the formula (84) of this process is applied to the polytropic change of state of a gas, it expresses a relationship between any two states of the gas such as the initial state, 1, and the final state, 2, as follows:

$$P_1 V_1^n = P_2 V_2^n \tag{85}$$

The point, C, in Fig. 18, represents a given condition as to pressure and volume of an ideal gas. One horizontal line, 1 1_1, one vertical line 2 2_1, and a very large number, in fact an infinite number, of curved lines, such as 3 3_1, 4 4_1, and 5 5_1, could be drawn through point, C. Each would represent a polytropic process by its PV-diagram, and would follow the law, given by formula (84), PV^n = a constant. Although these processes follow the same law, their PV-diagrams are different lines due to the fact that each process has its own numerical value for n. Therefore in the general polytropic field there is an infinite number of processes for, as previously stated, n can take any positive or negative numerical value. Of all of these possible

polytropics, the student of Engineering Thermodynamics is interested in only the four processes previously considered in this chapter namely the constant volume, constant pressure, isothermal, and adiabatic, and one other process which becomes known as The Polytropic. We shall now proceed to connect those four previously considered with formula (84), and to establish our new process. This is accomplished by assuming certain specific numerical values for n of the equation
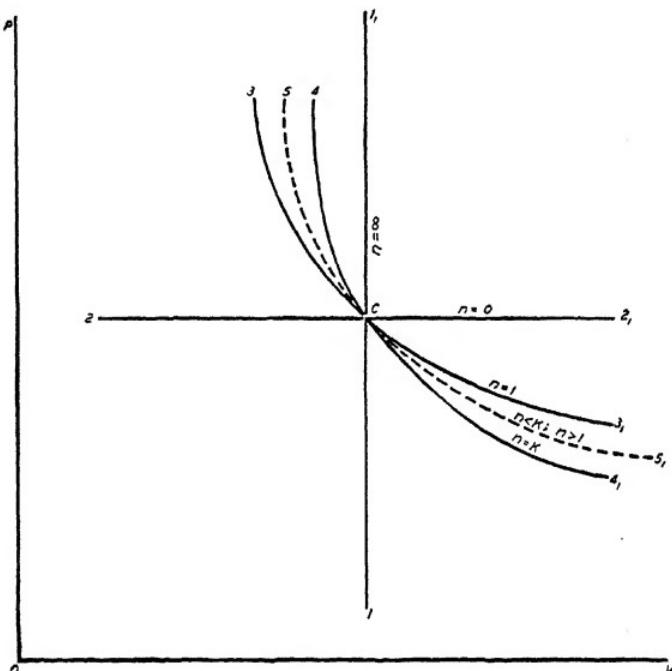
Fig. 18

$PV^n =$ a constant. The assumption of such a number for n brings about a special case of our general polytropic process, which special case follows $PV^n =$ a constant, when and only when n has this value assigned to it.

Let $n =$ infinity, ∞. Substitution of this value in formula (84) gives

$$PV^\infty = \text{a constant}$$

It can be proven mathematically that if such is the case, by raising each member of this equation to a power indicated by the exponent,

$\frac{1}{\infty}$, there is obtained

$$P^{\frac{1}{\infty}}V = \text{a constant}$$

or $$V = \text{a constant}$$

This is the condition of a constant volume process. Hence a polytropic, where $n = \infty$, is the constant volume process with which we have dealt in this chapter. It is indicated in Fig. 18 by the vertical line, 1 1_1, along which $n = \infty$ has been written.

Next, consider the case where $n = 0$. Then formula (84) becomes

$$PV^0 = \text{a constant}$$

But $V^0 = 1$, for any mumber raised to a power indicated by the exponent 0, is equal to unity, for instance, $10{,}000{,}000^0 = 1$.

Substituting for V^0 its equal, 1, in $PV^0 = \text{a constant}$, we have

$$P \cdot 1 = \text{a constant}$$

or $$P = \text{a constant}$$

In other words a polytropic process in which $n = 0$, is a process during which the pressure is constant, or a constant pressure process. This is shown in Fig. 18 by the line, 2 2_1, along which is written $n = 0$.

Now if n is given the numerical value, 1, formula (84) becomes

$$PV^1 = \text{a constant}$$

or $$PV = \text{a constant}$$

But this is the statement of Boyle's Law and can be true only under the condition of a constant temperature; so that when $n = 1$ during a polytropic, the latter is in reality an isothermal or constant temperature process. Fig. 18 presents it as the curve, 3 3_1.

For the next special value to be assigned to n, take k or $\dfrac{C_p}{C_v}$. This when placed for n in formula (84) gives

$$PV^k = \text{a constant}$$

which will be immediately recognized as the fundamental formula of the adiabatic process. Therefore a polytropic change of state during which $n = k$ is an adiabatic process, represented in Fig. 18 by the curve, 4 4_1.

Thus it is seen that the four special values assigned to n of the polytropic result in the four major processes which have previously been considered. These processes have their own individual names, the use of which identifies them. This reserves the name, polytropic, so that it may be used in engineering to apply to those processes in

which n takes values between 1 and k. These are processes which represent the actual compression of air in air compressors and the actual expansion of the working substance in internal-combustion engines. If we were to plot the curve of $PV^n =$ a constant where n is given some value between unity and k, it would fall somewhere between the isothermal curve, 3 3_1, and the adiabatic curve, 4 4_1, of Fig. 18. Such a curve is shown in Fig. 18 by the dotted line. The so-called polytropics of engineering practically always fall within this rather restricted area.

Certain formulas of the polytropic process are the same as those of the adiabatic if n replaces k therein. These are formulas (69) to (80), inclusive. While the heat absorbed during an adiabatic change of state, ΔQ_a, is zero (and hence the specific heat during the adiabatic process is also equal to zero) the heat absorbed during a polytropic process, ΔQ_n, and the specific heat, C_n, are not equal to zero. The general energy equation for the polytropic can be written as;

$$\Delta Q_n = \Delta K_n + \Delta W_n \tag{86}$$

in which

$$\Delta Q_n = MC_n(T_2 - T_1) \text{ B.t.u.} \tag{87}$$

where

$C_n =$ the constant (or mean value of) specific heat of the polytropic. This specific heat, C_n, is given by the formula,

$$C_n = C_v \frac{n-k}{n-1}$$

so that

$$\Delta Q_n = MC_v \frac{n-k}{n-1}(T_2 - T_1) \text{ B.t.u.} \tag{88}$$

The formula for ΔK_n, the increase in internal energy during a polytropic process, like any ΔK, is as follows,

$$\Delta K_n = MC_v(T_2 - T_1) \text{ B.t.u.} \tag{89}$$

and ΔK_n being equal to ΔK_a,

$$\Delta K_n = \frac{1}{J} \times \frac{P_2V_2 - P_1V_1}{k-1} \text{ B.t.u.} \tag{90}$$

or

$$\Delta K_n = \frac{1}{J} \times \frac{MR(T_2 - T_1)}{k-1} \text{ B.t.u.} \tag{91}$$

In formulas (90) and (**91**), it should be noted that k is used as such and is not replaced by n.

Example. Five cubic feet of air are at a pressure of 80 pounds per square inch absolute. This air expands polytropically until the final volume is 8 cubic feet. If $n = 1.3$, what is the final pressure?

Solution. Here $P_1 = 80 \times 144$ lbs. per sq. ft. absolute, $V_1 = 5$ cu. ft., $V_2 = 8$ cu. ft. Using formula (**72**), with n replacing k, we have

$$P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^n$$

Evaluating

$$P_2 = 80 \times 144 \times \left(\frac{5}{8}\right)^{1.3}$$

To obtain the value of $\left(\frac{5}{8}\right)^{1.3}$;

$$\log \frac{5}{8} = \log 0.625 = 9.7959 - 10$$

$$\log \left(\frac{5}{8}\right)^{1.3} = 1.3 \times (9.7959 - 10) = 1.3 \times (-0.2041)$$

$$= -0.2653, \text{ not a logarithmic form}$$
$$= 10 - 0.2653 - 10$$
$$= 9.7347 - 10, \text{ the logarithmic form}$$

Therefore $\left(\dfrac{5}{8}\right)^{1.3} = \log^{-1} 9.7347 - 10 = 0.5429$

Substituting this value in the formula for P_2,

$$P_2 = 80 \times 144 \times 0.5429$$
$$= 6254.2 \text{ lbs. per sq. ft. absolute. } Ans.$$

or $p_2 = \dfrac{6254.2}{144} = 43.43$ lbs. per sq. in. absolute. *Ans.*

Example. In the preceding example, how much external work is done during the expansion?

Solution. Substitute the given values, as shown in the preceding example, in formula (**77**), with n replacing k.

$$W_n = \frac{P_1 V_1 - P_2 V_2}{n - 1} \text{ ft.-lbs.}$$

$$W_n = \frac{80 \times 144 \times 5 - 6254.2 \times 8}{1.3 - 1}$$

$$= 25221 \text{ ft.-lbs. } Ans.$$

$$\Delta W_n = \frac{W_n}{778} = \frac{25221}{778} = 32.4 \text{ B.t.u. } Ans.$$

TABLE IV

Summary of the Processes

	Pressure Change	Volume Change	Temperature Change	Heat Added or Rejected	Internal (Kinetic) Energy Change	Work Done BY or ON the Gas
Constant volume change with $P_2>P_1$	increase	no change	increase	heat added ΔQ_v is $+$	increase ΔK_v is $+$	No work done $\Delta W_v=0$
Constant volume change with $P_2<P_1$	decrease	no change	decrease	heat rejected ΔQ_v is $-$	decrease ΔK_v is $-$	No work done $\Delta W_v=0$
Constant pressure expansion	no change	increase	increase	heat added ΔQ_p is $+$	increase ΔK_p is $+$	BY the gas ΔW_p is $+$
Constant pressure compression	no change	decrease	decrease	heat rejected ΔQ_p is $-$	decrease ΔK_p is $-$	ON the gas ΔW_p is $-$
Isothermal expansion	decrease	increase	no change	heat added ΔQ_t is $+$	no change $\Delta K_t=0$	BY the gas ΔW_t is $+$
Isothermal compression	increase	decrease	no change	heat rejected ΔQ_t is $-$	no change $\Delta K_t=0$	ON the gas ΔW_t is $-$
Adiabatic expansion	decrease	increase	decrease	no heat from external source $\Delta Q_a=0$	decrease ΔK_a is $-$	BY the gas ΔW_a is $+$
Adiabatic compression	increase	decrease	increase	no heat from external source $\Delta Q_a=0$	increase ΔK_a is $+$	ON the gas ΔW_a is $-$
Polytropic expansion, $n=1$ to k	decrease	increase	decrease	heat added ΔQ_n is $+$	decrease ΔK_n is $-$	BY the gas ΔW_n is $+$
Polytropic compression, $n=1$ to k	increase	decrease	increase	heat rejected ΔQ_n is $-$	increase ΔK_n is $+$	ON the gas ΔW_n is $-$

Summary. The summary presented in Table IV indicates the changes in pressure, volume, and temperature, and the heat or energy changes for all the processes. The student should study this summary very carefully and refer to it during the solution of problems.

PROBLEMS

1. Draw the PV-diagram of a constant volume process, in which the initial pressure, p_1, is equal to 15 pounds per square inch absolute, the initial volume, V_1, is 4 cubic feet, and the final pressure, p_2, is 150 pounds per square inch absolute.

Is the final temperature higher or lower than the initial temperature? Why?

2. Five pounds of air are heated at constant volume from 100° F. to 500° F.

(a) How much heat is supplied?

(b) What is the increase in internal kinetic energy? Is this the only increase in internal energy? Why?

(c) How much external work is done?

Ans. (a) 338 B.t.u. (b) 338 B.t.u. (c) None.

3. A tank, whose capacity is 20 cubic feet, is filled with carbon dioxide, CO_2, at a pressure of 25.3 pounds per square inch gauge and a temperature of 60° F. After heat is applied to the tank, the pressure gauge shows a pressure of 100 pounds per square inch.

(a) How many pounds of CO_2 are there in the tank?

(b) What is the final temperature on both the Fahrenheit and absolute scales?

(c) How much heat is supplied?

(d) What is the increase in intrinsic or internal energy?

Ans. (a) 6.31 lbs. (b) 1,491° F., absolute, 1,031° F. (c) 992.6 B.t.u. (d) 992.6 B.t.u.

4. A tank containing 15 pounds of air at 240° F. is cooled until the temperature of the air is 30° F.

(a) How much heat is added?

(b) How much heat is rejected?

(c) What is the increase in internal energy?

(d) What is the decrease in internal energy?

Ans. (a) −532.35 B.t.u. (b) +532.35 B.t.u. (c) −532.35 B.t.u. (d) +532.35 B.t.u.

5. Ten pounds of oxygen, kept under a constant pressure of 100 pounds per square inch absolute is caused to expand by the addition of heat. If the initial temperature of the oxygen is 70° F. and the final temperature is 200° F.

(a) How much heat is supplied?

(b) What is the increase in internal kinetic energy?

(c) How much external work is done?

Ans. (a) 283.4 B.t.u. (b) 202.8 B.t.u. (c) 80.6 B.t.u., or 62,707 ft.-lbs.

6. What are the initial and final volumes of the oxygen in the preceding problem?

Ans. $V_1 = 17.77$ cu. ft., $V_2 = 22.13$ cu. ft.

7. Plot the process of Problem 5 on the PV co-ordinate plane.

8. Three pounds of air are compressed at a constant pressure of 150 pounds per square inch absolute from an initial volume of 5 cubic feet to a final volume of 1 cubic foot. How much external work is done upon the gas?

Ans. 86,400 ft.-lbs.

9. In the preceding problem,

(a) What is the initial temperature?

(b) What is the final temperature?

(c) What is the decrease in internal kinetic energy?

(d) Does the internal kinetic energy of a gas decrease during every thermodynamic process when the temperature decreases? Why?

(e) Does the internal kinetic energy of a gas increase during every process when the temperature increases? Why?

(f) Does the temperature increase or decrease during constant pressure compression?

Ans. (a) 675.42° F., absolute, (b) 135.08° F., absolute, (c) 273.95 B.t.u. (d) Yes (e) Yes (f) Decreases.

10. Plot an isothermal curve as in Fig. 15 going through a point, C, whose pressure (P_c) is 100 pounds per square inch absolute and volume (V_c) is 3 cu. ft.

11. Two cubic feet of air at an initial pressure of 90 pounds per square inch gauge are expanded isothermally to a volume of 5 cubic feet.

(a) How much heat energy is supplied?

(b) What is the increase in internal energy during the expansion?

(c) How much external work is done by the air in expanding?

Ans. (a) 35.47 B.t.u. (b) No change (c) 35.47 B.t.u. or 35.47×778 ft.-lbs.

12. The cylinder of an air compressor admits 1.5 cubic feet of air per stroke. The air is taken in at a pressure of 14.6 pounds per square inch absolute and at a temperature of 65° F. It is compressed isothermally to a pressure of 130 pounds per square inch absolute. It is required to find:

(a) The weight of air in the cylinder at the beginning of the compression stroke,

(b) the final volume of the compressed air,

(c) the work done upon the gas during compression per working stroke,

(d) the increase in internal energy,

(e) the heat abstracted from the cylinder per working stroke.

Ans. (a) 0.113 lb. (b) 0.168 cu. ft. (c) 6,900 ft.-lbs. approx. (d) Zero (e) 8.9 B.t.u.

(Note. In solving this problem, follow step by step the solution of a similar example of this chapter.)

13. What does $(0.039)^{0.4}$ equal? *Ans.* 0.2731

14. An ideal gas while expanding adiabatically in a cylinder did 35,000 foot-pounds of work upon the piston.

(a) How much heat energy was required to do this work?

(b) What was the source of this heat energy and what effect was there upon this source?

(c) Was any heat supplied from an external source with which to do this work?

Ans. (a) 45— B.t.u. (b) Internal energy. Decrease. (c) No.

15. One pound of air at 250° F. expands adiabatically from 200 pounds per square inch absolute to a pressure of 25 pounds per square inch absolute.

(a) What is its final temperature?

(b) What is its initial volume?

(c) What is its final volume?

(d) How much external work is done by the air in expanding?

(e) What is the decrease in internal energy?

(f) How much heat is added from an external source?

Ans. (a) 389.3° F., absolute   (b) 1.314 cu. ft.   (c) 5.763 cu. ft.   (d) 42,100 ft.-lbs. approx.   (e) 54.1 B.t.u.   (f) None.

16. Five cubic feet of air at a temperature of 70° F. and a pressure of 14.7 pounds per square inch absolute are compressed adiabatically to a volume of ½ cubic foot.

(a) What is the final pressure?

(b) How much external work was done on the gas?

(c) What is the increase in internal energy?

(d) How much heat is rejected during the adiabatic compression?

Ans. (a) 374.41 lbs. per sq. in. absolute   (b) 40,329 ft.-lbs.   (c) 51.8 B.t.u (d) None.

17. One pound of air at 250° F. expands polytropically from 200 pounds per square inch absolute to a pressure of 25 pounds per square inch absolute. If $n = 1.2$,

(a) What is its final temperature?

(b) What is its initial volume?

(c) What is its final volume?

(d) How much external work is done by the air in expanding?

(e) What is the decrease in internal energy?

(f) How much heat is added from an external source?

(Note. Compare results obtained in this problem with those of problem 15.)

Ans. (a) 501.8° F., absolute   (b) 1.314 cu. ft.   (c) 7.43 cu. ft.   (d) 55,476 ft.-lbs.   (e) 35.13 B.t.u.   (f) 36.2 B.t.u.

18. What is the source of energy with which external work is done by the gas during a polytropic expansion where n lies between *1* and *k*? Contrast this with the source of energy in an adiabatic expansion and in an isothermal expansion.
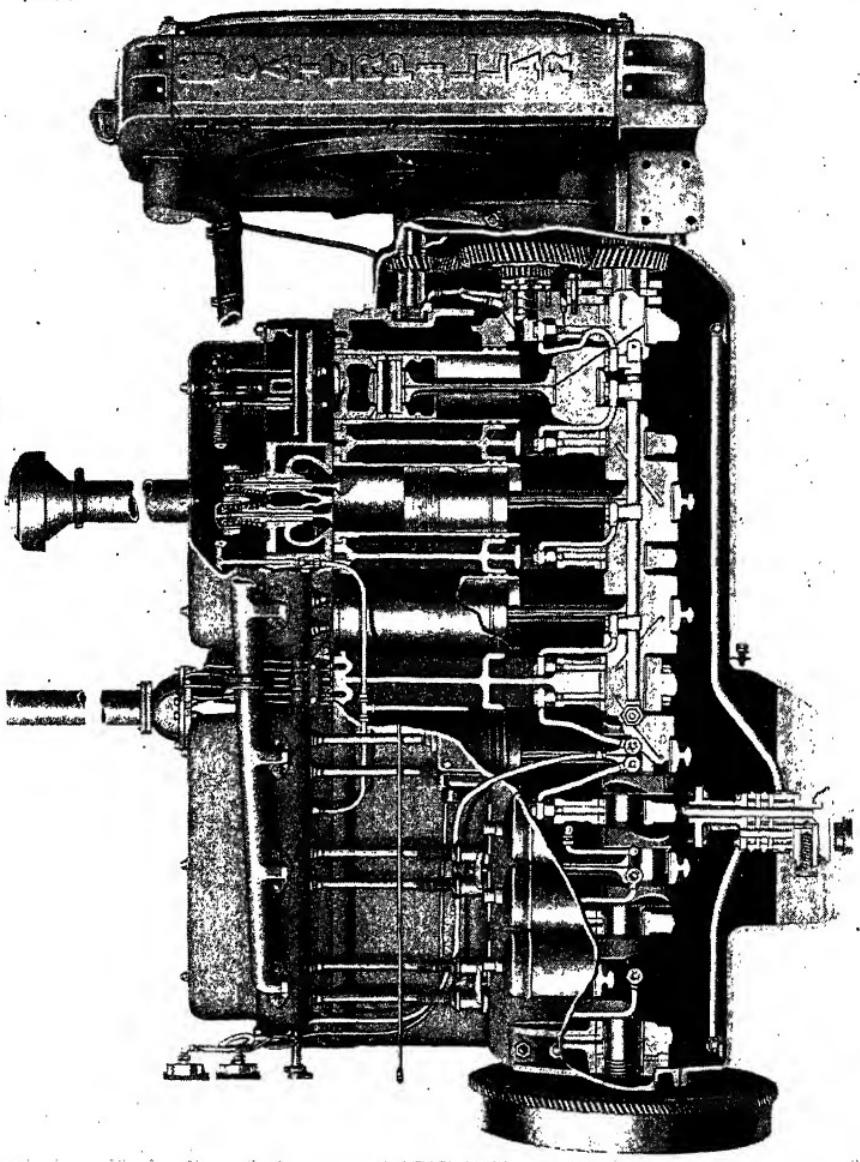
CATERPILLAR DIESEL ENGINE
Longitudinal Section View Showing Construction and Lubrication System
The Science of Thermodynamics is basic to this

CHAPTER IV

HEAT-ENGINE CYCLES

Introduction. A heat-engine or thermodynamic cycle is a series of thermodynamic processes through which a working substance or heat medium passes in a certain designated sequence, the completion of which returns the working substance to its initial state. Hence at the end of the cycle, the working substance has the same pressure, volume, temperature, and, of course, internal energy that it had at the beginning. Somewhere during every cycle, heat is received by the working substance. It is then the object of the cycle to convert as much of this heat energy as possible into mechanical energy or work. The heat energy which is not thus converted is rejected by the working substance during some process of the cycle.

Heat Engine. Any machine designed to carry out a thermodynamic cycle and thus convert heat energy supplied to it into mechanical energy is called a heat engine. Hence the cycle it operates on is known as a heat-engine cycle. It is generally made up of a piston and cylinder, together with the following elements,

a *Hot Body*, a name applied to the source of the heat which is received during the cycle,

a *Cold Body* or *Refrigerator*, whose function is to receive the heat rejected during the cycle,

and the *Working Substance* or *Heat Medium*, which receives the heat directly from the hot body, rejects heat to the cold body, does external work upon the piston during expansion, and has external work done upon it by the piston during compression.

The heat engine may be of the following types:

Steam Engine, in which the working substance is steam,

Hot-Air Engine, in which the working substance is heated air,

Internal Combustion Engine, the so-called Gas Engine, which uses as a working substance a mixture of air and gas, or a mixture of air and petroleum vapor.

The cycles which will be presented in this chapter are ideal cycles which apply to the last two types of heat engines and are generally spoken of as gas cycles and the working substance is generally referred to as a gas.

Available Energy, Net Work, and Efficiency of a Cycle. It was stated in a preceding paragraph that it was the function of any heat-engine cycle to receive heat from some external source, the hot body, and transform as much of this heat as possible into mechanical energy, or work done on the piston of the engine. This amount of heat which is transformed into mechanical energy is known as the Available Energy of the cycle. It is equal to the difference between the heat received or supplied to the cycle from the hot body and the heat rejected to the cold body. This statement is of course a direct consequence of the law of conservation of energy.

Let Q = available energy of the cycle in British thermal units

Q_1 = heat received during the cycle from the hot body, in B.t.u.

Q_2 = heat rejected during the cycle to the cold body, in B.t.u.

Then from our definition,

$$Q = Q_1 - Q_2 \qquad (92)$$

Every cycle contains thermodynamic processes involving both expansions and compressions. During the former, work upon the piston of course is done by the gas, while during the latter, work is done on the gas by the piston. The difference between the work done by the gas and the work done on the gas during the complete cycle is called the Net Work of the cycle. It is necessarily equal to the available energy of the cycle.

If $\qquad W_k$ = the net work of the cycle in B.t.u.

then $\qquad W_k = Q$ B.t.u. $\qquad (93)$

and therefore from formula (92)

$$W_k = Q_1 - Q_2, \text{ B.t.u.} \qquad (94)$$

The efficiency of a cycle, e, is defined in the following manner:

$$e = \frac{\text{Heat equivalent of the net work of the cycle}}{\text{Heat received during the cycle from the hot body}}$$

or $\qquad e = \dfrac{\text{the available energy of the cycle}}{\text{Heat received during the cycle from the hot body}}$

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Hence

$$e = \frac{Q}{Q_1} = \frac{W_k}{Q_1} \qquad (95)$$

or from formula (92)

$$e = \frac{Q_1 - Q_2}{Q_1} \qquad (96)$$

The statements which have so far been made are applicable to any cycle. Hence formula (96) is the most general statement of the efficiency of any heat-engine cycle.

Example. A heat engine receives 100 British thermal units of heat energy from the hot body per cycle. Of these heat units 25 are transformed into mechanical energy.

(a) What is the available energy of the cycle?

(b) How much heat is rejected to the cold body?

(c) What is the net work of the cycle?

(d) What is the efficiency of the cycle?

Solution.

(a) $\qquad\qquad Q = 25$ B.t.u. *Ans.*

(b) Here $\qquad Q_1 = 100$ B.t.u., $Q = 25$ B.t.u.

since from formula (92)

$$Q = Q_1 - Q_2$$

we have by transposing Q_1 to the other member of the equation,

$$Q_2 = Q_1 - Q$$

Substituting in the above

$$Q_2 = 100 - 25 = 75 \text{ B.t.u., rejected.} \quad Ans.$$

(c) $\qquad\qquad W_k = Q$

$$W_k = 25 \text{ B.t.u. or } 25 \times 778 \text{ ft.-lbs.} \quad Ans.$$

(d) From formula (95)

$$e = \frac{100 - 75}{100} = \frac{25}{100} = 0.25 \quad Ans.$$

To express an efficiency in per cent, multiply the value of the efficiency in the decimal form by 100 per cent, thus

$$e = 0.25 = 0.25 \times 100\% = 25\% \quad Ans.$$

Net Work and the PV=Diagram. Let a gas be expanded during some process from A to B, Fig. 19, then compressed during a second

process from B to C, and finally compressed during a third process from C back to its initial condition at A. It is evident then that the gas has passed through some sort of a cycle and that ABC is the PV-diagram of the cycle. In expanding from A to B, external work is done by the gas equal in amount to the area, $ABba$. External work is done on the gas during the compression from B to C equal in amount to the area $CBbc$, and external work is done on the gas again during the compression from C to A of an amount equal to area $ACca$. Thus the total amount of work done on the gas is equal to

Area $ACca$ + area $CBbc$ = area $ACBba$
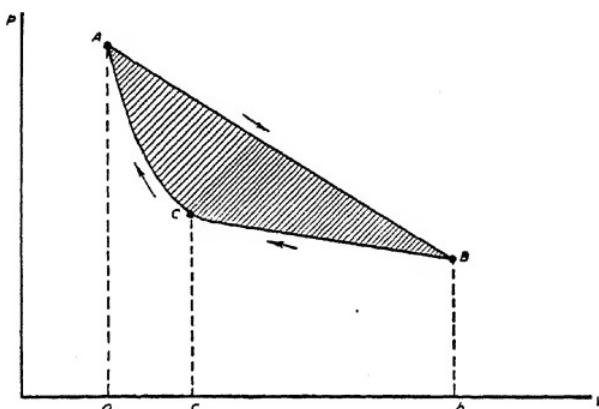
Fig. 19

The work done by the gas is more than the work done on the gas, since area $ABba$ is greater than area $ACBba$. This is as it must be in any heat-engine cycle. Thus Fig. 19 verifies previous statements of this chapter. Since the net work of the cycle was defined as being equal to the difference between the work done by the gas and the work done on the gas during the cycle, it follows that the

Net work of the cycle in ft.-lbs. = area $ABba$ − area $ACBba$

= area ABC (shown cross-hatched in Fig. 19)

Therefore the enclosed area within the PV-diagram of any heat-engine cycle is a measure of the net work of the cycle in foot-pounds.

Carnot Cycle. This cycle was brought out in 1824 by a French engineer named Sadi Carnot. Although its limitations are such that no heat engine has ever been constructed to use it, this cycle theoreti-

cally permits the conversion of the maximum quantity of a given amount of heat energy into mechanical energy. In other words it gives the maximum efficiency that it is possible to obtain in a heat-engine cycle. Hence its usefulness lies in the comparisons which it affords with other heat engines, giving as it does under the conditions the maximum result that they would like to approach.
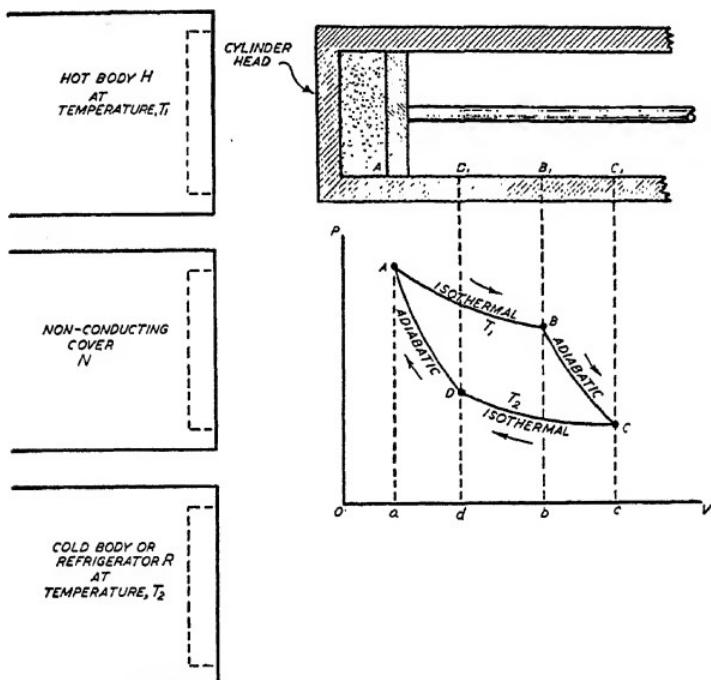
Fig. 20

An engine operating on this ideal cycle, Fig. 20, would require a cylinder and piston as shown, made of perfectly non-conducting material, a cylinder head that will conduct heat perfectly, and three other elements that can be brought into contact with the conducting cylinder head as occasion demands. These three elements are,

the *Hot Body*, *H*, always at a temperature, T_1, the source of heat energy supplied to the working substance of the heat-engine,

the *Non-conducting Cover*, *N*,

and the *Cold Body* or *Refrigerator*, maintained at a temperature of T_2, the minimum temperature of the cycle. This element receives

the heat that is rejected from the working substance of the heat-engine during the cycle.

In Fig. 20, the piston is shown at the left end of its stroke. Between it and the cylinder head is the working substance which is assumed to be a perfect gas. The position of the piston, point A_1, in the cylinder corresponds with point A, of the PV-diagram.

Starting with this point, A, the hot body, H, is brought over the end of the cylinder, the cylinder head. H supplies heat at temperature T_1 to the working substance causing it to expand isothermally until the point, B, is reached. This point is the end of the isothermal expansion throughout which the temperature has been maintained constant at T_1 so that

the temperature at A, T_a = the temperature of B, $T_b = T_1$

This isothermal expansion of the working substance has driven the piston from A_1 to B_1.

At B, the hot body is removed from contact with the cylinder head and is replaced by the non-conducting cover, N. Since all elements of the engine that are now in contact with the working substance are non-conducting, no heat can be added or abstracted from the gas. Hence the continued expansion of the gas as it drives the piston from B_1 to C_1 is along the adiabatic curve, BC, at the end of which the working substance reaches the lowest temperature of the cycle, T_2. Since C is the end of this adiabatic process, $T_c = T_2$.

Point, C_1, is also the right end of the stroke of the piston. Hence the piston must now be brought back to its initial position, A_1, while the working substance is brought back to its initial state at A of the PV-diagram.

Hence at C, the non-conducting cover, N, is removed and the cold body, R, takes its place in contact with the conducting cylinder head. Work is now done upon the piston compressing the gas in front of the piston along the isothermal curve, CD, the piston being moved to D_1. During this compression, the heat necessarily rejected by the gas goes into the cold body R. This makes the isothermal compression at a temperature T_2, possible and permits $T_d = T_c = T_2$.

At D, the cold body is removed and the non-conducting cover, N, again takes the position at the end of the cylinder. The gas is now adiabatically compressed along the curve, DA, until it reaches the starting point, A, of the cycle, where it resumes its initial conditions

of temperature, pressure, and volume, and the piston is returned to the left end of its stroke, A_1. The cycle may now be repeated. The energy with which all the work of compression is done on the gas is derived from the machine itself. It may be from such a source as the energy of motion of a fly-wheel.

From the description of the Carnot cycle as just given, it is evident that certain relations must exist between the pressures and volumes of points A, B, C, and D, so that the working substance can be returned exactly to its condition at A or, in other words, so that the PV-diagram of the cycle will close at A. To do this, the isothermal compression must be stopped at just the right point so that the adiabatic compression which follows may end at point A. To bring about this relationship, we have from the adiabatic compression along DA where D is the initial state and A the final state,

$$\frac{T_d}{T_a} = \left(\frac{V_d}{V_a}\right)^{1-k} \text{ [obtained from (71)]}$$

From the adiabatic expansion from B to C, where B is the initial state and C the final state,

$$\frac{T_c}{T_b} = \left(\frac{V_c}{V_b}\right)^{1-k} \text{ [obtained from (71)]}$$

Since $T_d = T_c = T_2$, of the cycle and $T_a = T_b = T_1$, of the cycle,

$$\frac{T_d}{T_a} = \frac{T_2}{T_1} \text{ and } \frac{T_c}{T_b} = \frac{T_2}{T_1}$$

from which

$$\frac{T_d}{T_a} = \frac{T_c}{T_b}$$

therefore

$$\left(\frac{V_d}{V_a}\right)^{1-k} = \left(\frac{V_c}{V_b}\right)^{1-k}$$

Extracting the root, indicated by the index $(1-k)$, of each member of the equation,

$$\sqrt[1-k]{\frac{V_d^{1-k}}{V_a}} = \sqrt[1-k]{\frac{V_c^{1-k}}{V_b}}$$

or

$$\left(\frac{V_d}{V_a}\right)^{\frac{1-n}{1-k}} = \left(\frac{V_c}{V_b}\right)^{\frac{1-n}{1-k}}$$

which gives

$$\frac{V_d}{V_a} = \frac{V_c}{V_b} \text{ or } \frac{V_b}{V_a} = \frac{V_c}{V_d} \tag{97}$$

This relationship of volumes must exist, if the gas is to be returned to the initial or starting point, A, and thus complete a cycle.

In every cycle, investigation must be made of the heat transfer during each individual process, the heat received, Q_1, during the cycle from the hot body, the heat rejected, Q_2, during the cycle to the cold body, the available energy, Q, which becomes the net work, W_k, of the cycle, and the efficiency of the cycle, together with the relationships existing between the pressures, volumes, and temperatures at various points of the cycle as needed to procure the above. It must be kept in mind at this time that heat received during a cycle is received (or added) during some process of the cycle, so that Q_1 is a ΔQ (heat added) of some process. It follows from this statement that a formula for Q_1 will be identical to a formula for ΔQ of some process. Likewise the heat rejected, Q_2, during a cycle is heat rejected during some process of the cycle and hence is a ΔQ (heat added) of that process which takes for itself a negative value and thus becomes positive heat rejected. This is due to the fact that negative heat added is in reality positive heat rejected. It follows from these statements that a formula if written specifically for Q_2, heat rejected during the cycle, will be identical to a formula for ΔQ, heat added, for some process with the single exception that some slight change has been made in the set-up of the formula that changes the algebraic sign of the value procured from $-$ to $+$ without changing its absolute value.

This can be made clearer by noting a specific case such as a constant pressure process in which, from formula (54), we have

$$\Delta Q_p = MC_p(t_2 - t_1) \text{ B.t.u.}$$

When this formula is used for an expansion, all factors in the second member are positive, hence the resulting value of ΔQ_p is positive and heat is actually added. But if this same formula is used for a compression, while M and C_p are still positive, the other factor $(t_2 - t_1)$ is negative because t_2 is less than t_1, and hence the resulting value is

negative. Therefore negative heat is added (for ΔQ_p is always heat added). Consequently, as has been stated previously, the result is interpreted as positive heat abstracted or rejected. Now suppose we wish to write a formula specifically for the heat rejected during such a compression. By just changing the position of t_2 and t_1 in the original factor (t_2-t_1) it would become (t_1-t_2) and its algebraic sign would become positive due to the change. Hence we could write

heat rejected during a constant pressure compression (or Q_2)

$$= MC_p(t_1-t_2)$$

It should be noted that we refrain from using ΔQ_p in the first member of the above.

Investigation of the Carnot cycle reveals that relationships of P, V, and T between points A and B and between points C and D can be found by using Boyle's Law and that similar relationships for points B and C, and points D and A can be obtained from formulas (69) to (76) inclusive, the formulas for an adiabatic process. In addition to these there is formula (97) which has been derived for this cycle.

Since no heat is added or rejected during the adiabatic processes of this cycle, the only heat received during the cycle is the heat added along the isothermal expansion from A to B, and the only heat rejected during the cycle is along the isothermal compression from C to D.

Hence, applying formula (64), heat received from A to B equals

$$Q_1 = \Delta Q_t = \frac{MRT_1}{J} \times 2.3 \log \frac{V_2}{V_1} \text{ B.t.u.}$$

$$= \frac{MRT_1}{J} \times 2.3 \log \frac{V_b}{V_a} \text{ B.t.u.} \tag{98}$$

and, applying the same formula (64), written however for Q_2, heat rejected from the cycle from C to D

$$Q_2 = -\Delta Q_t = -\left(\frac{MRT_2}{J} \times 2.3 \log \frac{V_2}{V_1}\right)$$

$$= -\left(\frac{MRT_2}{J} \times 2.3 \log \frac{V_d}{V_c}\right)$$

Since

$$-\log \frac{V_d}{V_c} = +\log \frac{V_c}{V_d},$$

we have

$$Q_2 = \frac{MRT_2}{J} \times 2.3 \log \frac{V_c}{V_d} \text{ B.t.u.} \qquad (99)$$

From formula (92), the available energy of the cycle,

$$Q = \frac{MRT_1}{J} \times 2.3 \log \frac{V_b}{V_a} - \frac{MRT_2}{J} \times 2.3 \log \frac{V_c}{V_d}$$

Since

$$\frac{V_b}{V_a} = \frac{V_c}{V_d} \text{ [formula (97)]}$$

$$Q = \frac{MRT_1}{J} \times 2.3 \log \frac{V_b}{V_a} - \frac{MRT_2}{J} \times 2.3 \log \frac{V_b}{V_a}$$

Factoring

$$Q = \frac{MR(T_1 - T_2)}{J} \times 2.3 \log \frac{V_b}{V_a} \text{ B.t.u.} \qquad (100)$$

Since formula (93) states that W_k, the net work of the cycle in B.t.u., is equal to Q, the available energy of the cycle, W_k can be found by using formula (100). If the net work of the cycle is wanted in foot-pounds, W_k (in B.t.u.) can be multiplied by 778, or formula (100) with J removed from the denominator may be used. Why? As stated earlier in this chapter, the enclosed area of the PV-diagram also represents the net work of the cycle in foot-pounds.

Substituting both the value of Q from formula (100) and the value of Q_1 from formula (98) for Q and Q_1 respectively in formula (95), we have the efficiency of the Carnot cycle,

$$e = \frac{Q}{Q_1} = \frac{\dfrac{MR(T_1 - T_2) \times 2.3 \log \dfrac{V_b}{V_a}}{J}}{MRT_1 \times 2.3 \log \dfrac{V_b}{V_a}}$$

$$e = \frac{MR(T_1 - T_2) \times 2.3 \log \frac{V_b}{V_a}}{J} \times \frac{1}{MRT_1 \times 2.3 \log V_b}$$

$$e = \frac{T_1 - T_2}{T_1} \qquad (101)$$

This formula states that the efficiency of the Carnot cycle is equal to the difference between the absolute temperature of the heat received and the absolute temperature of the heat rejected divided by the absolute temperature of the heat received. It is evident from an inspection of the formula that the greater the temperature difference, the higher the efficiency will be. For a heat-engine operating between two such temperatures, this is the highest possible theoretical efficiency.

Example. During a Carnot cycle, heat is received at a temperature of 600° F. and heat is rejected at a temperature of 70° F. Find the efficiency of the cycle.

Solution. Here $T_1 = 600° + 460° = 1,060°$ F. absolute; $T_2 = 70° + 460° = 530°$ F. absolute. Substituting these values in formula (101),

$$e = \frac{1,060 - 530}{1,060} = \frac{530}{1,060} = 0.50, \text{ or } 50\% \quad Ans.$$

Example. If the engine of the preceding example absorbed 2,000 British thermal units per minute from the source of heat, the hot body, find:

(a) the available energy in B.t.u. per minute

(b) the net work of the cycle in B.t.u. and ft.-lbs. per minute

(c) the theoretical horsepower of the engine.

Solution. (a) Here $Q_1 = 2,000$ B.t.u. per minute, $e = 0.50$; from formula (95)

$$e = \frac{Q}{Q_1}$$

Multiplying both members of this equation by Q_1,

$$Q = e \times Q_1 = 0.50 \times 2,000 = 1000 \text{ B.t.u. per minute. } Ans.$$

(b) Since $W_k = Q$,

$W_k = 1000$ B.t.u., net work of cycle in B.t.u. per minute. *Ans.*

Therefore the net work of the cycle $= 1000 \times 778 = 778,000$ ft.-lbs. per minute. *Ans.*

(c) Since 1 hp. $= 33,000$ ft.-lbs. of work done per minute,

$$H = \frac{778,000}{33,000} = 23.6 \text{ hp. } Ans.$$

Example. Referring to Fig. 20, in a Carnot cycle the pressure, p_a, at the beginning of the isothermal expansion is 100 pounds per square inch absolute while the volume at A, V_a, called the clearance volume, is 3 cubic feet. Heat is received during the cycle at a temperature of 300° F. and is rejected from the cycle at a temperature of 50° F. The volume at the end of the isothermal expansion, V_b, is 6 cubic feet. If the working substance is air, find:

    (a) the weight of air used
    (b) the heat received during the cycle
    (c) the efficiency of the cycle
    (d) the net work of the cycle

Solution. (a) Here $P_a = 100 \times 144$ lbs. per sq. ft. absolute, $V_a = 3$ cubic feet, $T_a = T_1 = 300° + 460° = 760°$ F. absolute, $R = 53.3$ From formula (34),

$$P_a V_a = MRT_a$$

Substituting the above values in this formula,

$$100 \times 144 \times 3 = M \times 53.3 \times 760$$

$$M = \frac{100 \times 144 \times 3}{53.3 \times 760} = 1.066 \text{ lb. } Ans.$$

(b) Using formula (98) with the values as given in (a) and $V_b = 6$ cu. ft.

$$Q_1 = \frac{1.066 \times 53.3 \times 760}{778} \times 2.3 \times \log \frac{6}{3}$$

$$Q_1 = \frac{1.066 \times 53.3 \times 760}{778} \times 2.3 \times 0.3010$$

$$Q_1 = 38.42 \text{ B.t.u. } Ans.$$

[**Note.** Formula (63) could also be used to effect this solution.]

(c) Here $T_1 = 760°$ F. absolute, $T_2 = 50° + 460° = 510°$ F. abs. Substituting these values in formula (**101**),

$$\frac{760 - 510}{760} = 0.329 \text{ or } 32.9\% \text{ Ans.}$$

(d) Using formula (**95**)

$$e = \frac{W_k}{Q_1} \text{ or } W_k = e \times Q_1$$

where $e = 0.329$ and $Q_1 = 38.42$ B.t.u.,
$$W_k = 0.329 \times 38.42 = 12.64 \text{ B.t.u. } Ans.$$
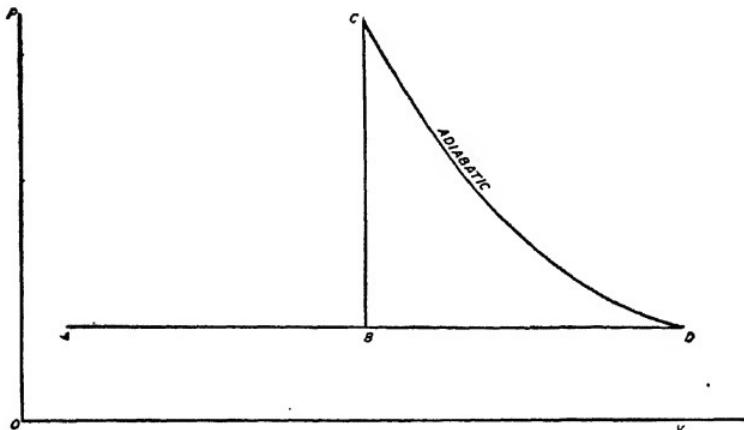
Fig. 21

Therefore the net work of the cycle in ft.-lbs. is

$$12.64 \times 778 = 9,833.9 \text{ ft.-lbs. } Ans.$$

Lenoir Cycle. Even prior to the eighteenth century, attempts had been made to produce power or mechanical work from the heat resulting from the explosion or instantaneous combustion of a mixture of gas and air in a cylinder. However the first successful gas engine to do this was not invented until 1860, when Pierre Lenoir, a Frenchman, produced the so-called Lenoir engine. While it was commercially successful, its poor economy led to its abandonment in a very few years due to gas engines being brought forth that operated on a more efficient cycle.

The theoretical Lenoir cycle is shown in Fig. 21. A mixture of gas and air is drawn into the cylinder at atmospheric pressure while the piston moves from the end of its stroke, A, to nearly mid-stroke, B. This is shown on the PV-diagram by the constant pressure line, AB. At B the mixture is ignited by means of an electric spark. The resultant explosion causes the pressure to rise instantly along the constant volume line, BC. The products of combustion then expand adiabatically along the curve CD, driving the piston before them to the end of its stroke at D. The exhaust port is opened at this time and the products of combustion are expelled from the cylinder along the constant pressure line, DA, as the piston returns to its initial position. This cycle takes place alternately on each side of the piston, and thus two impulses or explosions occur for each revolution of the crankshaft of the engine.

The heat added (or received) during the cycle, Q_1, is added along the constant volume line, BC. Therefore from formula (47),

$$Q_1 = MC_v(t_2 - t_1)$$
$$Q_1 = MC_v(t_c - t_b) \text{ B.t.u.} \tag{102}$$

The heat rejected from the cycle, Q_2, is rejected along the constant pressure line, DB. Since $Q_2 = -\Delta Q_p$, heat added, applying formula (54),

$$
\begin{aligned}
Q_2 &= -\Delta Q_p \\
&= -[MC_p(t_2 - t_1)] \\
&= -[MC_p(t_b - t_d)] \\
&= M \times C_p \times [-(t_b - t_d)] \\
&= MC_p(t_d - t_b) \text{ B.t.u.} \tag{103}
\end{aligned}
$$

for $\qquad -(t_b - t_d) = +(t_d - t_b)$

The available energy, Q, or the net work of the cycle, W_k, which in every cycle is equal to $Q_1 - Q_2$, is given by the formula

$$Q = MC_v(t_c - t_b) - MC_p(t_d - t_b) \text{ B.t.u.} \tag{104}$$

The theoretical cycle efficiency is from formula (95),

$$e = \frac{Q}{Q_1} = \frac{MC_v(t_c - t_b) - MC_p(t_d - t_b)}{MC_v(t_c - t_b)}$$

Dividing each term of the numerator by the denominator,

$$e = \frac{MC_v(t_c - t_b)}{MC_v(t_c - t_b)} - \frac{MC_p(t_d - t_b)}{MC_v(t_c - t_b)}$$

$$= 1 - \frac{C_p}{C_v} \times \frac{t_d - t_b}{t_c - t_b}$$

Since
$$k = \frac{C_p}{C_v},$$

$$e = 1 - k\left(\frac{t_d - t_b}{t_c - t_b}\right) \tag{105}$$

Example. At the time of ignition, in the Lenoir cycle, Fig. 21, the temperature, t_b, equals 85° F., the volume, V_b, equals $1\frac{1}{2}$ cubic feet, and the pressure, p_b, equals 14.6 pounds per square inch absolute. The heat received by the cycle from the combustion of the gas is 20 British thermal units. Assume the specific heats and the value of k the same as for air. Find,

(a) the temperature at point C, the end of the constant volume process

(b) the pressure at point C

(c) the temperature at point D, the end of the adiabatic expansion

(d) the efficiency of the cycle.

Solution. (a) It will be necessary at the beginning to find the weight, M, of the working substance which is considered as air. We have $P_b = 14.6 \times 144$ lbs. per sq. ft., absolute, $V_b = 1.5$ cu. ft., $T_b = 85° + 460° = 545°$ F. absolute, $R = 53.3$
From formula (34),

$$P_b V_b = MRT_b$$

Evaluating,

$$14.6 \times 144 \times 1.5 = M \times 53.3 \times 545$$

Dividing by 53.3×545

$$M = \frac{14.6 \times 144 \times 1.5}{53.3 \times 545}$$

$$= 0.1086 \text{ lb.}$$

Using formula (102) with $Q_1 = 20$ B.t.u., $M = 0.1086$ lb., $C_v = 0.169$, $t_b = 85°$ F. we have

$$Q_1 = MC_v(t_2 - t_1)$$
$$= MC_v(t_c - t_b)$$

or

$$20 = 0.1086 \times 0.169 \times (t_c - 85)$$
$$20 = 0.0184 \, t_c - 1.564$$

Transposing,

$$0.0184 \, t_c = 20 + 1.564$$
$$0.0184 \, t_c = 21.564$$
$$t_c = 1{,}172° \text{ F.} \quad Ans.$$
$$T_c = 1{,}172° + 460° = 1{,}632° \text{ F. absolute.} \quad Ans.$$

(b) Since in a constant volume process, the absolute pressures vary directly as the absolute temperatures

$$\frac{P_c}{P_b} = \frac{T_c}{T_b} \text{ [from formula (29)]}$$

or

$$P_c = P_b \times \frac{T_c}{T_b}$$

Evaluating,

$$P_c = 14.6 \times 144 \times \frac{1{,}632}{545}$$

and

$$p_c = 6{,}294.6 \text{ lbs. per sq. ft. absolute.} \quad Ans.$$

$$P_c = \frac{6{,}294.6}{144} = 43.7 \text{ lbs. per sq. in. absolute.} \quad Ans.$$

(c) Applying formula (75)

$$T_d = T_c \times \left(\frac{P_d}{P_c}\right)^{\frac{k-1}{k}}$$

Evaluating,

$$T_d = 1{,}632 \times \left(\frac{14.6}{43.7}\right)^{\frac{1.406-1}{1.406}}$$

$$= 1{,}632 \times (0.334)^{0.289}$$

To find $(0.334)^{0.289}$

$$\log 0.334 = 9.5237 - 10$$
$$\log (0.334)^{0.289} = 0.289 \times (9.5237 - 10)$$
$$= 0.289 \times (-0.4763)$$
$$= -0.1377, \text{ not a logarithmic form}$$
$$\log (0.334)^{0.289} = 10 - 0.1377 - 10 = 9.8623 - 10, \text{ the logarithmic form}$$

Therefore
$$(0.334)^{0.289} = \log^{-1} (9.8623 - 10)$$
$$= 0.7283$$

Substituting this value in our formula,
$$T_d = 1,632 \times 0.7283$$
$$T_d = 1,188.6° \text{ F. absolute.} \quad Ans.$$

(d) Applying formula (105),

$$e = 1 - 1.406 \times \frac{1,188.6 - 545}{1,632 - 545}$$

$$= 1 - 0.83 = 0.17 \text{ or } 17\% \quad Ans.$$
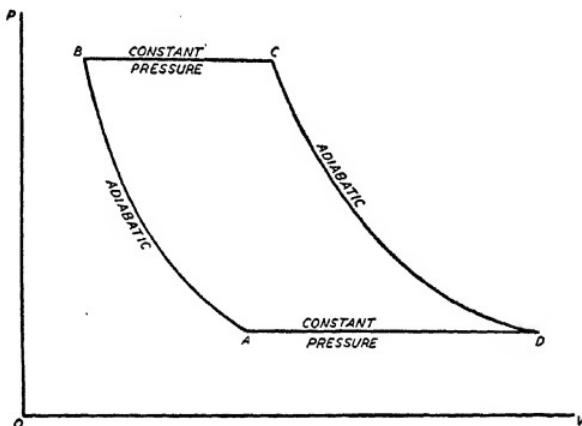
Fig. 22

Joule or Brayton Cycle. This cycle is known under the two names, Joule and Brayton, because it was first proposed by Joule and afterwards in 1872 used by George B. Brayton as the cycle of a gas engine which he designed. This cycle is shown by its PV-diagram in Fig. 22.

The engine of Brayton, in operating on this cycle, uses two cylinders. One of these is of course the main cylinder of the engine while the other is a pump cylinder. A charge of gas and air (the air to support the combustion of the gaseous fuel) is drawn into the pump cylinder at a pressure, P_a, and is then compressed along the adiabatic curve, AB, of the PV-diagram to a pressure, P_b, at point B. The compressed mixture of gas and air now enters a receiver from which it is received by the main cylinder of the engine. The mixture, passing

from the receiver to the main cylinder, is ignited in the cylinder in such a way that expansion occurs within the cylinder at constant pressure along the line, BC. The flow of the mixture from the receiver to the cylinder is cut off at point C, whereupon the resulting products of combustion are expanded adiabatically along the curve, CD, until the pressure at point D is nearly atmospheric. The piston has now been advanced to the end of its stroke. Finally the products of combustion, or burned gases, are expelled to the atmosphere along the constant pressure line, DA, thus completing the cycle.

The maintaining of a constant pressure within the cylinder from B to C is made possible through the relatively slow or gradual burning of the gases within the main cylinder during this Brayton cycle. Such combustion is spoken of as combustion at constant pressure. When the combustible is burned very rapidly, it is said to explode, or it is said an explosion takes place. Such combustion of the fuel is theoretically combustion at constant volume. It will be noted that the latter was the type of combustion involved in the Lenoir cycle, Fig. 21.

During the constant pressure process from B to C,

$$P_c = P_b \qquad (a)$$

During the constant pressure process from D to A,

$$P_d = P_a \qquad (b)$$

Since, if equals are divided by equals, their quotients are equal, division of step (b) by step (a) will produce the following relationship of absolute pressures during the cycle.

$$\frac{P_a}{P_b} = \frac{P_d}{P_c} \qquad (106)$$

Applying formula (76) to the adiabatic expansion from C to D,

$$P_d = P_c \times \left(\frac{T_d}{T_c}\right)^{\frac{k}{k-1}}$$

Dividing both members of the above equation by P_c,

$$\frac{P_d}{P_c} = \left(\frac{T_d}{T_c}\right)^{\frac{k}{k-1}} \qquad (c)$$

Applying formula (76) to the adiabatic compression from A to B,

$$_z = P_b \times \left(\frac{T_a}{T_b}\right)^{\frac{k}{k-1}}$$

Dividing both members of the above equation by P_b,

$$\frac{P_a}{P_o} \quad \frac{T_a}{T_b}\bigg)^{\frac{k}{k-1}} \quad (d)$$

Since by formula (106), $\frac{P_d}{P_c}$ of step $(c) = \frac{P_a}{P_b}$ of step (d), and since things equal to the same thing are equal to each other, from steps (c) and (d) we have

$$\left(\frac{T_d}{T_c}\right)^{\frac{k}{k-1}} = \left(\frac{T_a}{T_b}\right)^{\frac{k}{k-1}}$$

Raising each member of the above equation to the power indicated by the exponent $\frac{k-1}{k}$, the following relationship of absolute temperatures during the cycle is obtained

$$\frac{T_d}{T_c} = \frac{T_a}{T_b} \tag{107}$$

from which

$$\frac{T_d}{T_a} = \frac{T_c}{T_b} \tag{108}$$

and from proportion (108)

$$\frac{T_d - T_a}{T_c - T_b} = \frac{T_a}{T_b} \tag{109}$$

Since no heat can be added or rejected during an adiabatic process, we have only to refer to the constant pressure processes for Q_1 and Q_2. Heat is added to or received by the cycle from B to C, so that

$$Q_1 = \Delta Q_p = MC_p(t_2 - t_1)$$
$$Q_1 = MC_p(t_c - t_b) \text{ B.t.u.} \tag{110}$$

Heat is rejected from the cycle from D to A, so that

$$Q_2 = -\Delta Q_p$$
$$Q_2 = -\left[MC_p(t_a - t_d)\right]$$
$$Q_2 = +MC_p(t_d - t_a) \text{ B.t.u.,} \tag{111}$$

since $-(t_a - t_d) = +(t_d - t_a)$

The formula for the available energy or the net work of this

cycle becomes

$$Q = W_k = Q_1 - Q_2$$
$$= MC_p(t_c - t_b) - MC_p(t_d - t_a) \text{ B.t.u.} \qquad (112)$$

The efficiency of this cycle will be

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{Q}{Q_1}$$

$$e = \frac{MC_p(t_c - t_b) - MC_p(t_d - t_a)}{MC_p(t_c - t_b)}$$

Performing the indicated division of the second member

$$e = 1 - \frac{MC_p(t_d - t_a)}{MC_p(t_c - t_b)}$$

$$= 1 - \frac{t_d - t_a}{t_c - t_b}$$

$$= 1 - \frac{T_d - T_a}{T_c - T_b}$$

From formula (109),

$$e = 1 - \vdots \qquad (113)$$

Example. In a Brayton cycle, p_a is equal to 14.6 pounds per square inch absolute and p_b, the pressure after the compression of the mixture of gas and air, is equal to 140 pounds per square inch absolute. It is required to find the efficiency of the cycle when k is assumed as 1.4.

Solution. From step (d) above,

$$\left(\frac{T_a}{T_b}\right)^{\frac{k}{k-1}} = \frac{P_a}{P_b}$$

Raising both members of this equation to the power indicated by the exponent $\frac{k-1}{k}$,

$$\left[\left(\frac{T_a}{T_b}\right)^{\frac{k}{k-1}}\right]^{\frac{k-1}{k}} = \left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}}$$

from which is obtained

$$\left(\frac{T_a}{T_b}\right)^{\frac{k}{k-1}\times\frac{k-1}{k}}=\left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}}$$

since

$$\frac{k}{k-1}\times\frac{k-1}{k}=1,$$

$$\frac{T_a}{T_b}=\left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}}$$

From formula (113), $e=1-\dfrac{T_a}{T_b}$

$$e=1-\left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}} \tag{114}$$

and thus we have in reality derived another formula for the efficiency of this cycle during the solution of our example. Evaluating in formula (114),

$$e=1-\left(\frac{14.6\times144}{140\times144}\right)^{\frac{1.4-1}{1.4}}=1-(0.104)^{0.286}$$

To find $(0.104)^{0.286}$

$$\log 0.104=9.0170-10=-0.9830$$
$$\log (0.104)^{0.286}=0.286\times\log 0.104=0.286\times(-0.9830)$$
$$\log (0.104)^{0.286}=-0.2811, \text{ not a logarithmic form}$$
$$\log (0.104)^{0.286}=10-0.2811-10=9.7189-10, \text{ the logarithmic form}$$
$$(0.104)^{0.286}=\log^{-1}(9.7189-10)=0.5235$$

Substituting this value in our formula,
$$e=1-0.5235=0.4765 \text{ or } 47.65\% \quad Ans.$$

Otto Cycle. This ideal heat engine cycle was proposed in 1862 by Beau de Rochas. In 1876, Dr. Otto designed an engine to operate on this cycle. The Otto engine immediately became so successful from a commercial standpoint that its name was affixed to the cycle used by it. This cycle has endured until today it is in use in all gas and gasoline engines, together with some oil engines. Along with the Diesel cycle, it completely dominates the internal-combustion

engine field. The Otto cycle is generally completed in four strokes of the piston, or two revolutions of the crankshaft. It is then called a four-stroke cycle. Actual engines are also designed to complete this cycle in two strokes. The original theoretical cycle was proposed on a four stroke basis, and as such will be here introduced.

Referring to Fig. 23, the PV-diagram of the Otto cycle, the cycle starts at point A_1 with the piston at the end of its stroke and ready to begin the out stroke, or the movement toward the crankshaft end of the cylinder. The horizontal distance from A_1 to the
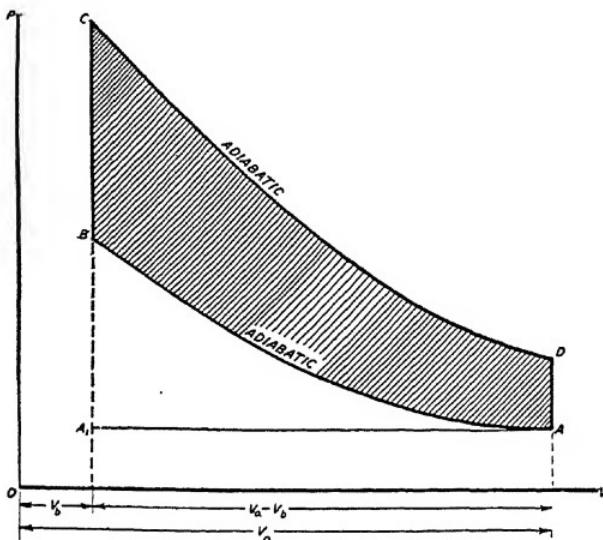
Fig. 23

pressure axis is the clearance volume, $V_{a1} = V_b = V_c$. This is the volume between the piston and the cylinder head when the piston is at this end of its stroke. The clearance volume is generally given as a percentage of the piston displacement volume.

If the latter is $(V_a - V_b)$, the clearance volume, V_b, is given in per cent by the following formula

$$\text{Clearance in per cent} = \frac{V_b \times 100}{V_a - V_b} \tag{115}$$

This is a general formula and is applicable to all heat engines.

At point A_1 the cylinder is in communication with the outside air

through the intake valve, hence the pressure at A_1 represented as it is by the vertical distance from A_1 to the V axis, is atmospheric.

As the piston makes its first stroke from A_1 to A, a mixture of gas and air, in which air predominates, is sucked into the cylinder at about atmospheric pressure. During the second or return stroke this mixture is compressed adiabatically along the adiabatic curve, AB. At point B the mixture in its compressed state is ignited by an electric spark. Combustion occurs so rapidly that it is called an explosion. This causes theoretically a constant volume process to occur from B to C with its accompanying increases in pressure and temperature. The combustion of the mixture liberates the heat energy of the fuel which heat energy becomes the so-called heat added to or received by the cycle. Thus heat is added at constant volume while the piston remains at the end of its second stroke. From point C to point D, as the piston makes its third stroke, the hot products of combustion expand adiabatically along the adiabatic curve, CD. At D the exhaust valve opens, permitting the burnt gases to cool at constant volume from D to A, with heat being rejected from the cycle and with a decrease in pressure to atmospheric at A. During the fourth stroke, the exhaust valve is open, the burnt gases are exhausted from the cylinder at atmospheric pressure, and the cycle is completed.

The closed part of the PV-diagram is cross-hatched in Fig. 23 and represents the net work of the cycle in foot-pounds. The actual heat cycle in reality starts at point A, and going through points B, C, and D returns to A.

It is evident from the description of the ideal Otto cycle, that

$$V_c = V_b \text{ and } V_a = V_d \qquad (a)$$

From formula (71), during the adiabatic compression, AB,

$$T_b = T_a \times \frac{V_a^{k-1}}{V_b^{k-1}} \qquad (b)$$

Multiplying both members of this equation by V_b^{k-1}, we have

$$T_b \times V_b^{k-1} = T_a \times V_a^{k-1} \qquad (c)$$

In a similar manner, applying formula (71) to the adiabatic expansion, CD,

$$T_d = T_c \times \frac{V_c^{k-1}}{V_d^{k-1}} \text{ or } T_d \times V_d^{k-1} = T_c \times V_c^{k-1} \qquad (d)$$

Since if equals are divided by equals, their quotients are equal, we may obtain from steps, (c) and (d)

$$\frac{T_a \times V_a^{k-1}}{T_d \times V_d^{k-1}} = \frac{T_b \times V_b^{k-1}}{T_c \times V_c^{k-1}} \qquad (e)$$

If from step (a), $V_a = V_d$, then any power of V_a is equal to the same power of V_d. Hence

$$V_a^{k-1} = V_d^{k-1}$$

In a similar manner, since $V_c = V_b$

$$V_b^{k-1} = V_c^{k-1}$$

Cancelling the above equal factors in step (e)

$$\frac{T_a}{T_d} = \frac{T_b}{T_c} \qquad (116)$$

From the above proportion

$$\begin{array}{cc} T_d - T_a & T_a \\ T_c - T_b & T_b \end{array} \qquad (117)$$

Dividing both members of the equation of step (c) by $T_b \times V_a^{k-1}$,

$$\frac{T_a \times V_a^{k-1}}{T_b \times V_a^{k-1}} = \frac{T_b \times V_b^{k-1}}{T_b \times V_a^{k-1}} \text{ or } \frac{T_a}{T_b} = \frac{V_b^{k-1}}{V_a^{k-1}} = \left(\frac{V_b}{V_a}\right)^{k-1} \quad (f) \text{ or } [\text{formula } (71)]$$

Applying formula (75) to the adiabatic compression, AB, of our cycle

$$T_b = T_a \times \left(\frac{P_b}{P_a}\right)^{\frac{k-1}{k}}$$

Dividing by T_a,

$$\frac{T_b}{T_a} = \left(\frac{P_b}{P_a}\right)^{\frac{k-1}{k}} \text{ or } \frac{T_a}{T_b} = \left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}} \quad (g)$$

In the Otto cycle, the only heat added is received by the cycle along the constant volume process, BC. Hence from formula (47)

$$Q_1 = \Delta Q_v = MC_v(t_c - t_b) = MC_v(T_c - T_b) \text{ B.t.u.} \qquad (118)$$

and the heat is rejected from the cycle along the constant volume

process, DA. Hence from formula (47)

$$Q_2 = -\Delta Q_v = -[MC_v(T_a - T_d)] = MC_v(T_d - T_a) \text{ B.t.u.} \quad (119)$$

Therefore the available energy, Q, or net work, W_k, of the cycle is given by

$$Q = W_k = Q_1 - Q_2$$
$$Q = MC_v(T_c - T_b) - MC_v(T_d - T_a) \text{ B.t.u.} \quad (120)$$

Hence for the efficiency of the Otto cycle, we have,

$$e = \frac{Q}{Q_1} = \frac{MC_v(T_c - T_b) - MC_v(T_d - T_a)}{MC_v(T_c - T_b)}$$

Expanding the second member of the above equation,

$$e = \frac{MC_v(T_c - T_b)}{MC_v(T_c - T_b)} - \frac{MC_v(T_d - T_a)}{MC_v(T_c - T_b)}$$

or

$$e = 1 - \frac{T_d - T_a}{T_c - T_b}$$

Therefore from formula (117)

$$e = 1 - \frac{T_a}{T_b} \quad (121)$$

and substituting for $\dfrac{T_a}{T_b}$ in the above, its equals from steps (f) and (g), we have

$$: 1 - \left(\frac{V_b}{V_a}\right)^{k-1} = 1 - \left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}} \quad (122)$$

The Otto cycle gives combustion at constant volume, while the Brayton cycle gives combustion at constant pressure. They are both so-called compression cycles, in that ignition of the combustible mixtures does not occur until after the charge has been compressed. The formulas representing their efficiencies are identical as will be noted and show that their efficiencies depend upon the extent to which the charge is compressed.

Example. In an ideal Otto cycle, the charge taken in is assumed to be air at a temperature of 70° F. and a pressure of 14.7 pounds per square inch absolute. If the clearance volume is 25 per cent, find,

(a) the pressure at the end of the adiabatic compression in pounds per square inch absolute,

(b) the temperature at the end of the adiabatic compression in degrees Fahrenheit.

Solution. (a) It will be necessary to obtain a relationship between V_a and V_b (See Fig. 23), which can be done by evaluating in formula (115)

$$25 \text{ (per cent)} = \frac{V_b \times 100}{V_a - V_b}$$

Multiplying each member of the equation by $(V_a - V_b)$,

$$25 (V_a - V_b) = V_b \times 100$$
$$25 V_a - 25 V_b = 100 V_b$$

Transposing

$$25 V_a = 100 V_b + 25 V_b$$
$$25 V_a = 125 V_b$$
$$V_a = \frac{125 V_b}{25 V_b} = 5 V_b$$

Applying formula (70), we have

$$P_a V_a^k = P_b V_b^k$$

in which $P_a = 14.7 \times 144$ lbs. per sq. ft. absolute, $V_a = 5 V_b$, $k = 1.406$, $P_b = 144 p_b$

Evaluating

$$14.7 \times 144 \times (5 V_b)^{1.406} = 144 \times p_b \times V_b^{1.406}$$
$$14.7 \times 144 \times 5^{1.406} \times V_b^{1.406} = 144 \times p_b \times V_b^{1.406}$$

Dividing both members of the equation by $144 \times V_b^{1.406}$,

$$p_b = \frac{14.7 \times 144 \times 5^{1.406} \times V_b^{1.406}}{144 \times V_b^{1.406}}$$

Cancelling like factors in the second member of our equation,

$$p_b = 14.7 \times 5^{1.406}$$

To obtain $5^{1.406}$

$$\log 5 = 0.6990$$
$$\log 5^{1.406} = 1.406 \times 0.6990 = 0.9828, \text{ a logarithmic form}$$

Therefore

$$5^{1.406} = \log^{-1} 0.9828 = 9.612$$

Substituting this value in our formula for p_b,

$p_b = 14.7 \times 9.612 = 141.3$ lbs. per sq. in. absolute. *Ans.*

(b) Applying formula (71), we have

$$T_b = T_a \times \left(\frac{V_a}{V_b}\right)^{k-1}$$

in which $T_a = 70° + 460° = 530°$ F. absolute, and $\dfrac{V_a}{V_b} = 5$

Evaluating in our formula for T_b,

$$T_b = 530 \times (5)^{1.406-1}$$
$$T_b = 530 \times 5^{0.406}$$
$$T_b = 530 \times 1.922 = 1018.7° \text{ F. absolute}$$

or $t_b = 1018.7° - 460° = 558.7°$ F.   *Ans.*

Example. What is the efficiency of the Otto cycle of the previous example?

Solution. Here $\dfrac{V_b}{V_a} = \dfrac{1}{5}$.

Substituting this value in formula (122)

$$e = 1 - \left(\frac{1}{5}\right)^{1.406-1} = 1 - (0.2)^{0.406}$$

To obtain the value of $(0.2)^{0.406}$,

$$\log 0.2 = 9.3010 - 10$$
$$\log (0.2)^{0.406} = 0.406 \times (9.3010 - 10) = 0.406 \times (-0.6990)$$
$$\log (0.2)^{0.406} = -0.2838, \text{ which, being negative, is not a}$$
$$\text{logarithmic form}$$
$$\log (0.2)^{0.406} = 10 - 0.2838 - 10 = 9.7162 - 10, \text{ the log-}$$
$$\text{arithmic form}$$

Therefore

$$(0.2)^{0.406} = \log^{-1} 9.7162 - 10 = 0.5203$$

Substituting this value in the formula for e,

$$e = 1 - 0.5203 = 0.4797 \text{ or } 47.97\% \text{   *Ans.*}$$

Example. In an Otto cycle, the pressure and volume at the beginning of the adiabatic compression are 14.6 pounds per square inch absolute and 14 cubic feet respectively. At the end of the adiabatic compression the volume is 2.25 cubic feet while the temperature is 640° F. At the beginning of the adiabatic expansion the

temperature is 1,140° F. Assuming that the working substance is air, find

(a) the temperature at the beginning of the adiabatic compression,

(b) the weight of air used,

(c) the pressure at the beginning of the adiabatic expansion in pounds per square inch absolute,

(d) the temperature and pressure at the end of the adiabatic expansion,

(e) the heat added to or received by the cycle,

(f) the heat rejected from the cycle,

(g) the available energy in British thermal units,

(h) the net work of the cycle in foot-pounds,

(i) the efficiency of the cycle.

Solution. (a) Referring to Fig. 23, we have $V_a = 14$ cu. ft., $V_b = 2.25$ cu. ft., $T_b = 640° + 460° = 1100°$ F., absolute, $k = 1.406$
Applying formula (71)

$$T_b = T_a \times \left(\frac{V_a}{V_b}\right)^{k-1} \text{ or } T_a = T_b \times \left(\frac{V_b}{V_a}\right)^{k-1}$$

Evaluating in the above

$$T_a = 1,100 \times \left(\frac{2.25}{14}\right)^{1.406-1} = 1,100 \times (0.1607)^{0.406}$$

$$T_a = 1,100 \times 0.476 = 523.6° \text{ F. absolute}$$
$$t_a = 523.6° - 460° = 63.6° \text{ F. } Ans.$$

(b) Since the temperature, pressure, and volume are now known at point A of the cycle, the weight can be found by applying formula (34), which written for point A becomes,

$$P_a V_a = M R T_a$$

in which R, for air = 53.3 (Table II), $P_a = 14.6 \times 144$ lbs. per sq. ft., $V_a = 14$ cu. ft., $T_a = 523.6°$ F. absolute
Evaluating

$$14.6 \times 144 \times 14 = M \times 53.3 \times 523.6$$

Dividing both members of the equation by 53.3×523.6

$$M = \frac{14.6 \times 144 \times 14}{53.3 \times 523.6} = 1.05 \text{ lbs. } Ans.$$

(c) Using formula (34) for the conditions of point C

$$P_c V_c = MRT_c \text{ or } 144\, p_c V_c = MRT_c$$

Here $V_c = V_b = 2.25$ cu. ft., $M = 1.05$ lb., and $T_c = 1,140° + 460° = 1,600°$ F. absolute

Evaluating

$$144\, p_c \times 2.25 = 1.05 \times 53.3 \times 1,600$$

Dividing both members of the equation by 144×2.25;

$$p_c = \frac{1.05 \times 53.3 \times 1,600}{144 \times 2.25} = 276.37 \text{ lbs. per sq. in. abs.} \quad Ans.$$

(d) From formula (70),

$$P_d V_d^k = P_c V_c^k \text{ or } p_d V_d^k = p_c V_c^k$$

Here $V_d = V_a = 14$ cu. ft., $p_c = 276.37$ lbs. per sq. in. absolute, $V_c = 2.25$ cu. ft.

Evaluating

$$p_d \times 14^{1.406} = 276.37 \times 2.25^{1.406}$$

$$p_d = \frac{276.37 \times 2.25^{1.406}}{14^{1.406}} = 21.15 \text{ lbs. per sq. in. abs.} \quad Ans.$$

Logarithmic computation for p_d:

log 276.37 $= 2.4415$	log 2.25 $= 0.3522$
log $2.25^{1.406} = 0.4952$	log $2.25^{1.406} = 1.406 \times 0.3522 = 0.4952$
log of num. $= \overline{2.9367}$	
log $14^{1.406} = 1.6114$	log 14 $= 1.1461$
log p_d $= \overline{1.3253}$	log $14^{1.406} = 1.406 \times 1.1461 = 1.6114$

Therefore

$$p_d = 21.15$$

Now use formula (34) to obtain the temperature at point D.

$$P_d V_d = MRT_d$$

Evaluating

$$144 \times 21.15 \times 14 = 1.05 \times 53.3 \times T_d$$

Dividing both members of the equation by the coefficient of T_d

$$T_d = \frac{144 \times 21.15 \times 14}{1.05 \times 53.3} = 761.9° \text{ F. absolute}$$

$$t_d = 761.9° - 460° = 301.9° \text{ F.} \quad Ans.$$

During the solution of a problem, it is often the case that several different methods can be used in finding the value of some of the unknown quantities. This is particularly true when the solution of a problem of some length is well under way. In the event that another method does present itself, a second solution thus effected should produce the same result as the first and thus serve as a check on the accuracy of the work. A few such checks are highly desirable during a rather long solution. In this example, several have already been made but not included in the text. At this point however, let us note that in arriving at the value of T_d, we might have used formula (116) instead of formula (34). Formula (116) can be used as a check formula in two ways. We may solve for T_d by this formula and compare the result obtained with that obtained by (34), or we may place the value of T_d as determined by (34) along with other known values in (116). If the solution is correct, a numerical identity or nearly such will result. In the latter manner from formula (116) for the Otto cycle

$$\frac{T_a}{T_d} : : \frac{T_b}{T_c}$$

Evaluating all terms of the above

$$\frac{523.6}{761.9} = \text{(should equal)} \ \frac{1,100}{1,600}$$

Simplifying both members,

$$0.687 = \text{(does equal)} \ 0.687$$

This checks not only T_d but also T_a for which we have previously solved. Indirectly this check is of even deeper significance at this time.

(e) Since the heat is added to the cycle from B to C, formula (118) is used. Thus

$$Q_1 = \Delta Q_v = MC_v(t_c - t_b) = MC_v(T_c - T_b) \ \text{B.t.u.}$$

or $\qquad Q_1 = 1.05 \times 0.169 \ (1,600 - 1,100) = 88.725 \ \text{B.t.u.} \quad Ans.$

(f) The heat is rejected from the cycle from D to A. Applying formula (119)

$$Q_2 = -\Delta Q_v = MC_v(T_d - T_a) \ \text{B.t.u.}$$
$$Q_2 = 1.05 \times 0.169 \ (761.9 - 523.6) = 42.286 \ \text{B.t.u.} \quad Ans.$$

(g) Since Q, the available energy, is in any cycle equal to $Q_1 - Q_2$,

$$Q = 88.725 - 42.286 = 46.439 \text{ B.t.u.} \quad Ans.$$

(h) The net work of any cycle is equal to

$$778 \times W_k = 778 \times Q = 778 \times 46.439 = 36,129.5 \text{ ft.-lbs.} \quad Ans.$$

(i) The efficiency of any cycle,

$$e = \frac{Q}{Q_1} = \frac{46.439}{88.725} = 0.523 \text{ or } 52.3\% \quad Ans.$$

or the efficiency of the Otto cycle is given by formula (122) as

$$e = 1 - \left(\frac{V_b}{V_a}\right)^{k-1} = 1 - \left(\frac{2.25}{14}\right)^{1.406-1} = 1 - 0.476$$

$$= 0.524 \text{ or } 52.4\%, \text{ a close check}$$

Diesel Cycle. Internal-combustion engines of today operate on heat cycles which approximate either the ideal Otto cycle or the ideal Diesel cycle. In 1897, Dr. Rudolph Diesel constructed the first successful Diesel engine. This engine was designed to operate theoretically on a new heat cycle devised by him. Since its advent, the Diesel-engine power plant has been used to a considerable extent in stationary, marine, and locomotive practice. It is comparatively heavy and costly and has a high actual thermal efficiency which combined with its ability to use low-grade fuel oils makes it very economical in operation. Diesel engines of both the two-stroke and four-stroke types are constructed.

The ideal Diesel cycle is similar in some respects to both the Brayton and Otto cycles. Like these cycles, it uses both an adiabatic compression and an adiabatic expansion. These are carried on in the same cylinder. It will be remembered that this is like unto the Otto cycle but unlike the Brayton, which uses a separate cylinder for the compression of its mixture. The Brayton and Otto cycles compress their mixtures of fuel and air and then ignite them. The Diesel cycle compresses air only; and the latter is compressed to such a high pressure that the corresponding temperature causes ignition of the fuel as it is injected into the cylinder. Hence no so-called ignition system is necessary with a Diesel engine. The fuel is injected into the cylinder of the Diesel engine at just the right rate from the standpoint of the ideal Diesel cycle to assure combustion at constant pressure. This is similar to the Brayton cycle. Finally

the burnt gases are cooled at constant volume in the Diesel cycle as in the Otto.

The *PV*-diagram of the ideal Diesel cycle is given in Fig. 24. Air only is admitted to the cylinder along A_1A during the entire first stroke of the piston. It is then compressed adiabatically along the adiabatic curve, *AB*, during the return stroke of the piston. At point *B*, the end of the compression stroke, the pressure is about 500 pounds per square inch, and the corresponding temperature of about 1,000° F. is sufficiently high to ignite the fuel when it is injected into the cylinder in the form of a finely divided spray. This
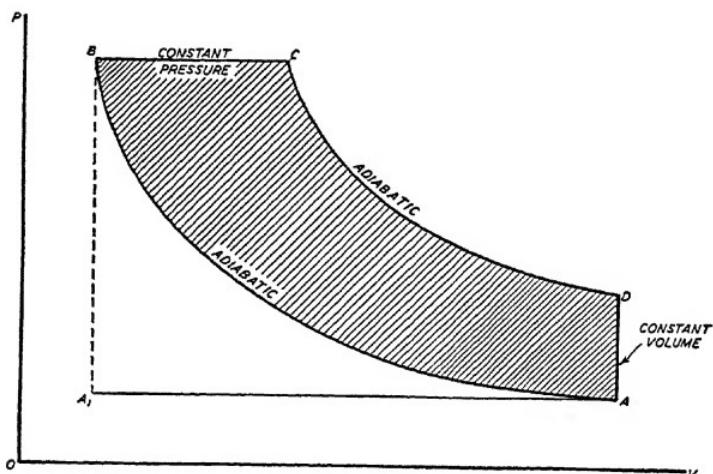
Fig. 24

injection of the fuel now takes place giving combustion at constant pressure and therefore heat is added to the cycle at constant pressure from *B* to *C* as the piston starts its third stroke. No so-called explosion results as the burning of the fuel is too gradual. The fuel supply is cut off at *C* and the resultant products of combustion expand adiabatically along the adiabatic curve, *CD*, driving the piston to the end of its third stroke. At point *D*, the exhaust valve opens permitting the burnt gases to cool at constant volume from *D* to *A*, and therefore along *DA* heat is rejected from the cycle to the cold body. The piston now returns to its initial position along AA_1, expelling the burnt gases from the cylinder as it completes the fourth stroke and also the cycle. The cross-hatched area of the closed part

of the diagram again represents the net work of the cycle in foot-pounds.

The previous cycles have made it clear that, if one is acquainted with the processes that go to make up a specific cycle, the temperatures, pressures, and volumes at the various points of the cycle can be determined from the formulas given for the individual processes in Chapter III. However in applying these formulas, the student should be very careful not only to associate the proper formulas with the process but also to correctly determine or interpret the initial and final states. One can easily interchange the latter and thus introduce a serious error.

Heat is received by the Diesel cycle along the constant pressure line, BC.

By applying formula (54)

$$Q_1 = \Delta Q_p = MC_p(t_2 - t_1)$$

Therefore

$$Q_1 = MC_p(t_c - t_b) = MC_p(T_c - T_b) \tag{123}$$

In the above since C is the final state and B is the initial state, t_2 and t_1 of the general formula are replaced by t_c and t_b respectively. And as stated before the difference in the Fahrenheit temperatures is equal to the difference in their corresponding absolute temperatures, thus $(T_c - T_b)$ can be substituted in the above formula for $(t_c - t_b)$. In a ratio of two temperatures, however, the ratio of the Fahrenheit temperatures can never be substituted for or used in the place of the ratio of the corresponding absolute temperatures. Why not?

Heat is rejected from this cycle during the constant volume process from D to A. With D, the final state, and A, the initial state, formula (47) applies. Thus

$$Q_2 = -\Delta Q_v = -[MC_v(t_2 - t_1)] = M \times C_v \times [-(t_2 - t_1)]$$
$$Q_2 = MC_v(t_1 - t_2) \text{ for } -(t_2 - t_1) = +(t_1 - t_2)$$

Therefore

$$Q_2 = MC_v(T_d - T_a) \text{ B.t.u.} \tag{124}$$

The available energy, Q, or the net work, W_k, of the cycle is given by formulas (93) and (94) as:

$$Q = W_k = Q_1 - Q_2 \text{ (in any cycle)}$$

Substituting in the above, values of Q_1 and Q_2 from formulas (123) and (124) respectively

$$Q = MC_p(T_c - T_b) - MC_v(T_d - T_a) \qquad (125)$$

The efficiency of this cycle is then given by formula (95) as,

$$e = \frac{Q}{Q_1} \text{ (in any cycle)}$$

Substituting in the above the values of Q and Q_1 from formulas (125) and (123) respectively,

$$e = \frac{MC_p(T_c - T_b) - MC_v(T_d - T_a)}{MC_p(T_c - T_b)}$$

Simplifying the second member,

$$e = \frac{MC_p(T_c - T_b)}{MC_p(T_c - T_b)} - \frac{MC_v(T_d - T_a)}{MC_p(T_c - T_b)}$$

or

$$e = 1 - \frac{C_v}{C_p} \times \frac{T_d - T_a}{T_c - T_b}$$

since $\dfrac{C_p}{C_v} = k$, giving $\dfrac{C_v}{C_p} = \dfrac{1}{k}$,

$$e = 1 - \frac{1}{k} \times \frac{T_d - T_a}{T_c - T_b} \qquad (126)$$

Example. An ideal Diesel engine takes in its charge of air at a pressure of 14.5 pounds per square inch absolute and at a temperature of 80° F. At the beginning of the compression stroke, the volume is 8 cubic feet. If 150 British thermal units are added per cycle at a pressure of 550 pounds per square inch absolute and the working substance is assumed to be air, find

    (a) the weight of air used per cycle
    (b) the volume and temperature at point B (See Fig. 24)
    (c) the volume and temperature at point C
    (d) the pressure and temperature at point D
    (e) the heat rejected from the cycle
    (f) the available energy of the cycle
    (g) the net work of the cycle in foot-pounds
    (h) the efficiency of the cycle.

Solution. (a) We have given, $P_a = 14.5 \times 144$ lbs. per sq. ft. abs. $V_a = 8$ cu. ft., $T_a = 80° + 460° = 540°$ F. absolute, $R = 53.3$

Using formula (34)

$$P_a V_a = MRT_a$$

Evaluating in the above

$$14.5 \times 144 \times 8 = M \times 53.3 \times 540$$

$$M = \frac{14.5 \times 144 \times 8}{53.3 \times 540} = 0.58 \text{ lb.} \quad Ans.$$

(b) To obtain the volume V_b, we have P_a and V_a as given above, and $P_b = 550 \times 144$ lbs. per sq. ft. absolute. Applying formula (70),

$$P_b V_b^k = P_a V_a^k$$

Evaluating

$$550 \times 144 \times V_b^{1.406} = 14.5 \times 144 \times 8^{1.406}$$

$$V_b^{1.406} = \frac{14.5 \times 144 \times 8^{1.406}}{550 \times 144}$$

$$V_b^{1.406} = \frac{14.5 \times 144 \times 18.61}{550 \times 144} \text{ (Since } 8^{1.406} = 18.61)$$

$$V_b^{1.406} = 0.49$$

Raising each member of the above to a power indicated by the fractional exponent $\frac{1}{1.406}$

$$(V_b^{1.406})^{\frac{1}{1.406}} = (0.49)^{\frac{1}{1.406}}$$

$$V_b^{1.406 \times \frac{1}{1.406}} = (0.49)^{\frac{1}{1.406}}$$

Therefore

$$V_b = 0.49^{\overline{1.406}} = 0.602 \text{ cu. ft.} \quad Ans.$$

To find the temperature T_b, using formula (71),

$$T_b = T_a \times \left(\frac{V_a}{V_b}\right)^{k-1}$$

where $T_a = 540°$ F. absolute, $V_a = 8$ cu. ft., $V_b = 0.602$ cu. ft. Evaluating in our formula,

$$T_b = 540 \times \left(\frac{8}{0.602}\right)^{1.406-1}$$

$$T_b = 540 \times 2.858 = 1,543.3° \text{ F. absolute.} \quad Ans.$$

$$t_b = 1{,}543.3° - 460° = 1{,}083.3° \text{ F.} \quad Ans.$$

(c) Heat is added to the cycle along the constant pressure line, BC. Hence formula (123) applies here. In this formula, $M = 0.58$ lb., C_p, for air, $= 0.2375$ (Table I), $T_b = 1{,}543.3°$ F. absolute, $Q_1 = 150$ B.t.u.

Evaluating in our formula,

$$Q_1 = MC_p(T_c - T_b)$$
$$150 = 0.58 \times 0.2375(T_c - 1543.3)$$
$$150 = 0.13775\ T_c - 212.5896$$

Transposing,

$$0.13775\ T_c = 150 + 212.5896$$
$$0.13775\ T_c = 362.5896$$
$$T_c = 2{,}632.2° \text{ F. absolute.} \quad Ans.$$
$$t_c = 2{,}632.2° - 460° = 2{,}172.2° \text{ F.} \quad Ans.$$

Applying Charles' Law as given by formula (28)

$$V_c \times T_b = V_b \times T_c$$

Here $T_b = 1{,}543.3°$ F. absolute, $T_c = 2{,}632.2°$ F. absolute, and $V_b = 0.602$ cu. ft.

Evaluating in our formula,

$$V_c \times 1{,}543.3 = 0.602 \times 2{,}632.2$$

$$V_c = 0.602 \times \frac{2{,}632.2}{1{,}543.3} = 1.027 \text{ cu. ft.} \quad Ans.$$

(d) To find the pressure, p_d, use formula (70). Thus

$$P_d \times V_d^k = P_c \times V_c^k \quad \text{or} \quad p_d \times V_d^k = p_c \times V_c^k$$

Here $V_d = V_a = 8$ cu. ft., $V_c = 1.027$ cu. ft., and $p_c = 550$ lbs. per sq. in. absolute.

Evaluating in our formula,

$$p_d \times 8^{1.406} = 550 \times 1.027^{1.406}$$

Dividing both members of the equation by $8^{1.406}$,

$$p_d = \frac{550 \times 1.027^{1.406}}{8^{1.406}}$$

$$p_d = \frac{550 \times 1.035}{18.61} = 30.59 \text{ lbs. per sq. in. absolute.} \quad Ans.$$

To find the temperature, T_d, use formula (30), since the change of state from D to A is at constant volume.

$$P_a \times T_d = P_d \times T_a \text{ or } p_a \times T_d = p_d \times T_a$$

Here $p_a = 14.5$ lbs. per sq. in. absolute, $p_d = 30.59$ lbs. per sq. in. absolute, $T_a = 540°$ F. absolute
Evaluating in the above formula

$$14.5 \times T_d = 30.59 \times 540$$

Dividing by 14.5

$$T_d = \frac{30.59 \times 540}{14.5} = 1,139.02° \text{ F. absolute. } Ans.$$

$$t_d = 1,139.02° - 460° = 679.02° \text{ F. } Ans.$$

(e) Heat is rejected along DA. Applying formula (124)
$$Q_2 = 0.58 \times 0.169 \times (1,139.02 - 540)$$
$$Q_2 = 0.58 \times 0.169 \times 599.02 = 58.716 \text{ B.t.u. } Ans.$$

(f) Since $Q = Q_1 - Q_2$
$$Q = 150 - 58.716 = 91.284 \text{ B.t.u. } Ans.$$

(g) Since the net work of the cycle in ft.-lbs. is equal to $778 \times$ the net work of the cycle in B.t.u.,
the net work of cycle $= 778 \times 91.284 = 71,019$ ft.-lbs. $Ans.$

(h) applying formula (95), which applies to all cycles,

$$\frac{91.284}{150} = .608 \text{ or } 60.8\% \quad Ans.$$

Now let us check the above by using the formula which is specifically applicable to the Diesel cycle. This is formula (126).

$$e = 1 - \frac{1}{1.406} \times \frac{1,139.02 - 540}{2,632.2 - 1,543.3}$$

$$e = 1 - \frac{1}{1.406} \times 0.5501$$

$$e = 1 - 0.392 = .608 \text{ or } 60.8\%, \text{ which checks the above result.}$$

Mean Effective Pressure and Indicated Horsepower. The mean effective pressure, or m.e.p., of a cycle or heat engine is the average net unit pressure in pounds per unit area that operates on the piston throughout its stroke. It is then the average height of the PV- or

indicator diagram of any heat engine. Since the area of the indicator diagram is equal to the net work of the cycle in foot-pounds ($J \times Q$ or $J \times W_k$), it is evident that if P_m is the m.e.p. in pounds per square foot, and p_m is the m.e.p. in pounds per square inch

$$P_m = \frac{J \times Q}{V_a - V_b} \text{ lbs. per sq. ft.} \tag{127}$$

and

$$p_m = \frac{J \times Q}{144(V_a - V_b)} \text{ lbs. per sq. in.} \tag{128}$$

where $J = 778$

Q = available energy or net work per cycle in B.t.u.

$V_a - V_b$ = piston displacement in cu. ft. (See Figs. 23 and 24).

But $V_a - V_b = \frac{A}{144} \times L$ (a)

where A = Area of piston in sq. in.

L = length of stroke in feet

Substituting the value of $V_a - V_b$ from step (a) in (128)

$$p_m = \frac{J \times Q}{144 \times \frac{A}{144} \times L} - \frac{J \times Q}{A \times L}$$

Multiplying by $A \times L$

$J \times Q = p_m \times A \times L$, net work of cycle in ft.-lbs.

Let N_e represent the number of completed cycles per minute or the number of explosions per minute. Then

net work done per minute $= J \times Q \times N_e = p_m \times A \times L \times N_e$ ft.-lbs.

The indicated horsepower of an engine, called the i.hp., is the horsepower which is developed in the cylinder of the engine. Since work done per minute divided by 33,000 gives the horsepower,

$$\text{i. hp.} = \frac{J \times Q \times N_e}{33,000} \tag{129}$$

or

$$\text{i. hp.} = \frac{p_m \times A \times L \times N_e}{33,000} \tag{130}$$

Example. What is the mean effective pressure in pounds per square inch of the Otto engine of the example on page 132?

Solution. Here $J \times Q$, the net work of the cycle, $= 36,129.5$ ft.-lbs., $V_a = 14$ cu. ft., $V_b = 2.25$ cu. ft. Applying formula (128)

$$p_m = \frac{36,129.5}{144(14-2.25)} = 21.35 \text{ lbs. per sq. in. } Ans.$$

Example. In the preceding problem, the engine makes 400 revolutions per minute. What is the theoretical horsepower?

Solution. Since 1 cycle is completed in two revolutions or in four strokes of the piston, there will be $\frac{400}{2}$ cycles, or 200 cycles completed in 1 minute. Hence $N_e = 200$. We also have $Q = 46.439$ B.t.u. (see page 135). Substituting these values in (129)

$$\text{i.hp.} = \frac{778 \times 46.439 \times 200}{33,000} = 219 \text{ hp. } Ans.$$

Example. A six-cylinder single-acting four-cycle Diesel engine has a cylinder diameter of 17 inches and a length of stroke of 25 inches. The engine runs at 200 revolutions per minute. If the mean effective pressure is 100 pounds per square inch, what is the indicated horsepower?

(**Note.** When a cylinder is single-acting power is applied on but one side of the piston.)

Solution. Here $P = 100$ lbs. per sq. in., $A = \frac{\pi \times 17^2}{4} = 227$ sq. in., $L = \frac{25}{12}$ ft.

Since there are four strokes per cycle, there are 2 revolutions of the crankshaft per cycle. Hence the number of cycles, N_e, completed per minute for each cylinder will equal $\frac{200}{2} = 100$.

Then N_e for 6 cylinders $= 6 \times$ the number for 1 cylinder

Therefore N_e for 6 cylinders $= 6 \times 100 = 600$ cycles or explosions per minute.

The horsepower per cylinder can be computed on a basis of $N_e = 100$, and the result obtained multiplied by the number of cylinders to secure the total i. hp.; or the latter can be directly obtained by using N_e, for the six cylinders $= 600$. Using the latter and applying formula (130),

$$\text{total i.hp.} = \frac{100 \times 227 \times \frac{25}{12} \times 600}{33,000}$$

$$= 860 \text{ hp. } Ans.$$

Example. Find the torque in the crankshaft of the engine of the preceding example. Let it be assumed that the horsepower delivered to the crankshaft is 80 per cent of the indicated horsepower.

Solution. Here $N = 200$ r.p.m. and $H = 0.8 \times 860 = 688$ hp. Use formula (8)

$$T_s = \frac{33,000 \, H}{2\pi N}$$

Evaluating

$$T_s = \frac{33,000 \times 688}{2 \times \pi \times 200} = 18,067 \text{ ft.-lbs. } Ans.$$

Actual Versus Ideal Cycle Efficiencies. It has been stated heretofore that internal-combustion engines of today are designed to follow as closely as possible the ideal Otto and Diesel cycles. That they cannot exactly follow and hence operate with the efficiencies of these ideal cycles is evident when one thinks of some of the practical limitations that are involved. For instance, both ideal cycles use an adiabatic compression and an adiabatic expansion. The adiabatic process insists that no heat be added or rejected throughout its duration and hence requires that the working substance be surrounded by a material that is a perfect non-conductor of heat. The cast-iron cylinders of our engines are not non-conductors of heat and hence the process carried on within them can only approach or approximate the adiabatic. For this reason combined with others an expansion or compression as carried on in the actual engine follows a polytropic whose value of n in the equation, $PV^n = a$ constant, is somewhere around 1.35 instead of 1.406 as in an adiabatic. Combustion in the Otto engine cannot be instantaneous as it is in the ideal Otto cycle. This results in a sloping combustion line on the PV-diagram (Indicator diagram) that tends to decrease the area of the diagram and thus is indicative of a tendency toward a lower actual or operating efficiency. Likewise in the actual Diesel engine

cycle, there is a similar tendency in that the combustion line instead of being maintained horizontal, as in the ideal Diesel cycle, tends to slope downward. The efficiency of the ideal cycle is known as the air-standard efficiency, since it is computed, as the examples of this chapter will show, on the basis of the working substance being air throughout the cycle. Therefore the specific heats for air are used. It is evident that this is in variance with the actual cycle.

The total effect of these differences is to produce an efficiency of the actual cycle that is from one-half to two-thirds of the ideal or air-standard efficiency. Well-designed and constructed internal-combustion engines when properly operated should tend toward the upper limit of two-thirds. The efficiency of the ideal cycle can be computed before the engine is constructed, and hence indicates the maximum approachable efficiency of the completed engine. Should the completed engine not closely approach this efficiency, alterations and improvements may be made to bring about the desired result.

The Second Law of Thermodynamics. In all heat-engine cycles, the higher the efficiency, the larger will be the available energy or that part of the heat received that is available for doing external work. For formula (95) states that $e = \dfrac{Q}{Q_1}$ from which statement $Q = e \times Q_1$. Hence for a given value of heat received, Q_1, the value of the available energy, Q, will depend on the value of the efficiency, e. As e approaches 100% (or unity in the formula), Q approaches Q_1 and, since by formula (92), $Q = Q_1 - Q_2$, the heat rejected, Q_2, will become smaller and smaller and approach zero. This heat rejected, Q_2, whatever its value, is not available, as has been noted, for doing work. It is the unavailable part of Q_1 and is said to be the part of Q_1 that is degraded. It might then be stated that the higher the efficiency of a specific cycle, the lower will be the unavailable energy.

It is the objective then of a heat engine to keep this unavailable energy down to a minimum. That it, the unavailable energy, cannot be reduced to zero even in the most perfect cycle has been demonstrated in this chapter and its verification is the purpose of the Second Law of Thermodynamics which may be stated in the following manner:—In any heat engine cycle, the transformation of heat energy into mechanical energy, or work, must always be accompanied by some heat being wasted or degraded by being rejected to the

cold body, or refrigerator. To demonstrate further the truth of this second law, let us investigate the Carnot cycle by assuming that its efficiency is unity. Substitute this value of e into the formula

$$e = \frac{T_1 - T_2}{T_1}$$

and there results
$$1 = \frac{T_1 - T_2}{T_1}$$

Simplifying the equation by multiplying both members by T_1

$$T_1 = T_1 - T_2$$

But T_1 can only equal T_1, therefore T_2 in the above equation must equal zero. Hence the possibility of the ideal Carnot cycle having an efficiency of 100% demands that the temperature of the cold body or refrigerator be equal to zero of the absolute scale. This, however, is impossible, for the temperature of the cold body from a practical standpoint must be at least at the temperature of the surrounding objects whose temperature cannot be at the zero of the absolute scale. It follows that the efficiency of this most perfect cycle cannot be equal to 100% and therefore some heat must be rejected to the cold body as stated by the Second Law of Thermodynamics.

The Heat Pump or Refrigerating Machine. A thermodynamic process is said to be reversible when it can start from some final state it has previously reached and retrace its original path back to the initial state. For instance in Fig. 17 a working substance starting at the initial point A expands adiabatically to the final point B tracing the path ACB. Now let this working substance start at point B and be compressed adiabatically. It will exactly retrace its former path reversing its direction and, if compressed far enough, will return to point B. The heat processes which we have studied have been ideal or reversible processes. A thermodynamic cycle is reversible when it can be performed in the reverse direction. It then becomes evident that a reversible cycle is a closed cycle and is made up entirely of reversible processes. The Carnot cycle is theoretically reversible, and when so operating, will trace the PV-diagram of Fig. 20 in the direction indicated by $ADCBA$. When

an ideal cycle, such as the Carnot cycle, is used in its reversed form as the cycle of a machine, the latter becomes an ideal heat pump or refrigerating machine. Just as actual heat engines cannot exactly carry out or fulfill the requirements of their ideal cycles, or in other words operate exactly as the ideal engines of these cycles, actual refrigerating machines cannot exactly perform according to their ideal cycles.

When a heat cycle is used in its reverse form in a refrigerating machine, everything is reversed in direction. Let us consider the Carnot cycle, which in its reversed form is the most perfect cycle for refrigerating machines just as it is the most perfect cycle for heat engines when it is operated in a direct manner. Starting with point A, Fig. 20, along the reverse cycle, AD becomes an adiabatic expansion instead of an adiabatic compression. Likewise DC is an isothermal expansion instead of an isothermal compression, and the two processes that follow to complete the reversed cycle are now compressions instead of expansions. In the direct cycle, Q_2 is the heat that is rejected from the cycle to the cold body or refrigerator, that is, Q_2 leaves the working substance. In the reversed cycle Q_2 changes its direction of flow and flows into or is received by the working substance from the cold body. In a similar manner, Q_1, the heat received from the hot body by the working substance in a heat engine cycle flows from or leaves the working substance in the refrigerating machine or reversed cycle and enters the hot body. Hence heat is made to flow from a cold body to a relatively warmer one, the hot body. This is not in accordance with another statement of the Second Law of Thermodynamics which states that in a self-acting engine heat cannot be made to flow from a cold body to one at higher temperatures. Therefore in a sense a refrigerating machine is not a self-acting engine, but a machine that requires a real heat engine to operate in conjunction with it and in a certain way as a part of it. The necessity of having the latter becomes evident when we again notice that the direction of everything is changed or reversed during the reversed cycle. Consider the net work of the direct cycle. It is energy flowing out from the cycle. Then in the reversed cycle, energy of like amount must, in changing its direction of flow, flow into or enter the working fluid. But some other machine then must furnish this energy to do the net work done on the cycle. This is the function

of the heat engine. The latter drives a compressor cylinder, within which part of the reversed cycle is carried on.

The cold body in this reversed cycle is the box or room that is to be cooled and kept cool or maintained at a temperature below that of its surroundings. The hot body is an element of the machine through which flows some circulating medium such as air or water which will dissipate the heat received by it. The heat engine may indeed be replaced by an electric motor.

It is interesting to note that all the energy changes occurring in the reversed cycle are obtained mathematically in the same manner as in the direct cycle. Hence the formulas for the direct cycle will apply in refrigeration but, in making these applications, care must be constantly exercised to interpret properly the direction of flow of these energy changes so as to make no mistakes in the algebraic signs of the various results.

PROBLEMS

1. A heat engine receives 150 British thermal units of heat energy from the hot body per cycle. Of these heat units, 50 B.t.u. are transformed into mechanical energy.

    (a) What is the available energy of the cycle?

    (b) How much heat is rejected to the cold body or refrigerator?

    (c) What is the net work of the cycle in foot-pounds?

    (d) What is the efficiency of the cycle?

    Ans. (a) 50 B.t.u. (b) 100 B.t.u. (c) 38,900 ft.-lbs. (d) 33⅓%

2. The efficiency of a heat cycle is 40 per cent. If 80 British thermal units are supplied per cycle, how much heat is rejected per cycle. *Ans.* 48 B.t.u.

3. A single-acting one-cylinder four-cycle (four-stroke cycle) Diesel engine makes 250 revolutions per minute. If 100 British thermal units are received by the engine per (heat) cycle, how much heat is received per minute? *Ans.* 12,500 B.t.u.

4. If a Diesel engine is the same as the one of Problem 3, with the exception that it is double-acting, how much heat is received per minute? *Ans.* 25,000 B.t.u.

5. During a Carnot cycle, heat is received at a temperature of 900° F. and is rejected at 80° F. Find the efficiency of the cycle. *Ans.* 60.3%

6. If the ideal engine of Problem 5 absorbed 3,000 British thermal units per minute from the source of heat, the hot body, find

    (a) the available energy per minute

    (b) the available energy per cycle if 100 cycles are completed per minute

    (c) the net work of the ideal engine in British thermal units and foot-pounds per minute

    (d) the net work per cycle, assuming, as in part *b*, 100 cycles per minute

    (e) the theoretical horsepower of the engine.

Ans. (a) 1,809 B.t.u. (b) 18.09 B.t.u. (c) 1,809 B.t.u., 778×1,809 ft.-lbs. (d) 18.09 B.t.u., 778×18.09 ft.-lbs. (e) 42.6 hp.

7. Is the combustible mixture compressed before ignition in the Lenoir cycle? In the Brayton cycle? In the Otto cycle? In the Diesel cycle?

8. Compare the combustion process in the Brayton cycle with that in the Diesel.

9. State the cycles which give combustion at constant volume, at constant pressure.

10. In what cycle is nothing other than air compressed during the compression process or stroke?

11. In an Otto cycle, the pressure and volume at the beginning of the adiabatic compression are 14.5 pounds per square inch absolute and 10 cubic feet respectively. At the end of the adiabatic compression, the volume is 1.5 cubic feet, while the temperature is 680° F. At the beginning of the adiabatic expansion, the temperature is 1,100° F. Assuming that the working fluid is air, find

    (a) the temperature at the beginning of the adiabatic compression
    (b) the weight of air used
    (c) the pressure at the beginning of the adiabatic expansion in pounds per square inch absolute
    (d) the temperature and pressure at the end of the adiabatic expansion
    (e) the heat added to or received by the cycle
    (f) the heat rejected from the cycle
    (g) the available energy or net work of the cycle in British thermal units
    (h) the efficiency of the cycle.

Ans. (a) 527.7° F. absolute (b) 0.742 lb. (c) 285.6 lbs. per sq. in. abs. (d) 722.1° F. absolute; 19.85 lbs. per sq. in. abs. (e) 52.67 B.t.u. (f) 24.38 B.t.u. (g) 28.29 B.t.u.; 778×28.29 ft.-lbs. (h) 53.7%

12. An ideal Diesel engine takes in its charge of air at a pressure of 14.7 pounds per square inch absolute and at a temperature of 70° F. At the beginning of the compression stroke, the volume is 8.5 cubic feet. If 170 British thermal units are added per cycle at a pressure of 540 pounds per square inch absolute and the working fluid is assumed to be air, find

    (a) the weight of air used per cycle
    (b) the volume and temperature at the end of the adiabatic compression
    (c) the volume and temperature at the beginning of the adiabatic expansion
    (d) the pressure and temperature at the end of the adiabatic expansion
    (e) the heat rejected per cycle
    (f) the available energy or net work per cycle in British thermal units
    (g) the efficiency of the ideal Diesel engine.

Ans. (a) 0.637 lb. (b) 0.655 cu. ft.; 1500.1° F. absolute (c) 1.15 cu. ft.; 2,623.8° F. absolute (d) 32.42 lbs. per sq. in. absolute; 1,168.9° F. absolute (e) 68.8 B.t.u. (f) 101.2 B.t.u. (g) 59.5%

13. Required the mean effective pressure of Problem 12. *Ans.* 69.7 lbs. per sq. in.

14. The engine of Problem 12 has one cylinder and is single-acting. If it makes 100 revolutions per minute, what is its indicated horsepower? *Ans.* 119.3 hp.

15. A six-cylinder, single-acting, four-cycle automobile engine has a cylinder diameter or bore of 3¼ inches and a stroke of 4 inches. If the mean effective

pressure is 120 pounds per square inch, what is the indicated horsepower when the engine is operating at 3,000 revolutions per minute? *Ans.* 90+ hp.

16. Find the torque in the crankshaft of the engine of the preceding problem if the horsepower delivered to the crankshaft is assumed to be 80 per cent of the indicated horsepower. *Ans.* 126 ft.-lbs.

CHAPTER V

VAPORS

Introductory. Since a more or less specific state of aggregation or physical state of a substance is to be dealt with at this time, it would be well for the student to read again the first part of Chapter II that deals with the States of Aggregation. Attention to the states of aggregation there listed will disclose the use of general terms such as solid, liquid, vapor, and gas which define the physical states in which a substance may exist. Along with these general terms are the names of some specific gases given as examples of actual gases and showing by their relative placement in the list their relationship to each other, to the vaporous state below them, and to the ideal gaseous state above them. Now these substances are listed as actual gases because they normally exist as such. It should not be inferred however from this placement in the list that they cannot occur elsewhere or, in other words, take upon themselves physical states other than the gaseous state. For under certain simultaneous conditions of pressure, volume, and temperature, they may occur as even liquids or solids. Let us consider carbon dioxide, CO_2. Its co-ordinates (pressure, volume, and temperature) may be such, while it is worked with, that it exists in a liquid state or even in a solid state. While in the latter, it is known as "dry ice," with which the student is probably familiar. The co-ordinates of carbon dioxide can be easily altered or changed in such a machine as a mechanical refrigerator in which carbon dioxide is the working substance or fluid of the cycle carried on in the refrigerator. Such a machine, by alternately expanding and compressing its working fluid, changes the physical state of the latter from a liquid to a vapor or from a vapor to a liquid. Due to the relative ease with which carbon dioxide (as well as sulphur dioxide) is liquefied, it is often referred to as a vapor. From this statement it may be gathered that a gas which is under such conditions that it may be easily condensed (that is, changed

into a liquid) is then known as a vapor. Hence a vapor is a gas that is near its point of condensation. As such it is said to have approached the state of liquefaction, the state of being a liquid, and it does not now follow the laws of Ideal Gases even approximately. On the other hand, the further it is removed from the state of liquefaction, the more closely will it approximate these laws.

Vaporization and Saturated Vapor. When a liquid is heated at constant pressure, the heat added will cause a rise in the temperature of the liquid and some small increase in its volume. Since the latter is small, very little, if any, of the heat added need be used either in doing external work or in increasing the internal potential energy. Hence practically all of the heat added will be used in increasing the internal kinetic energy which is always accompanied by an increase in temperature. Therefore with ΔP and ΔW assumed equal to zero, the general energy equation, formula (15) becomes

$$\Delta Q = \Delta K$$

As more and more heat is added to the liquid at constant pressure, the temperature continues to rise until a certain point is reached, whereupon an additional supply of heat added to the liquid will cause no change in temperature but will cause a change in the physical state, the liquid being changed to a vapor. This is the process known as Vaporization and the point at which it starts to occur is called the Saturation Point of the liquid. The temperature existing at the saturation point is known as the Saturation Temperature, and the liquid at such a temperature is called a Saturated Liquid. The vapor produced from it is likewise known as a Saturated Vapor since its temperature is the saturation temperature of its liquid. If this additional supply of heat is added very rapidly as is the case when water is heated in a steam boiler, bubbles of vapor form in the liquid and rise to the surface so rapidly that a great deal of turbulence is created. Such vaporization is called Ebullition, or Boiling. Hence saturation temperature is often referred to as Boiling point temperature. There is a certain definite saturation temperature for a given liquid and its saturated vapor to go with each particular pressure. Also the saturation temperatures of one liquid differ from that of another for the same pressures. Thus at pressures of 10 pounds and 50 pounds per square inch absolute, the saturation temperatures of water are 193.21° F. and 281.01° F. respectively; while at the same

pressures, the saturation temperatures of mercury are 637.3° F. and 812.5° F. respectively. Also ammonia at these absolute pressures has saturation temperatures of −41.34° F. and +21.67° F. respectively.

If the action of heat during vaporization is considered in conjunction with formula (15), since with no change in temperature ΔK is equal to zero,

$$\Delta Q = \Delta P + \Delta W$$

During vaporization, there is a great increase in volume. This causes the molecules to be further and further separated which insists upon energy being supplied to overcome the mutual attraction existing between the molecules. This energy then is set up within the substance as an increase in internal potential energy, ΔP. Likewise energy must be provided to do the external work, ΔW, that must accompany an expansion or increase in volume. The above formula makes it clear that these things to be done during vaporization use all the heat energy, ΔQ, that is supplied during the process.

On the basis of unit mass, the amount of heat added to produce a saturated liquid and hence the saturation temperature of that liquid depends upon the given pressure. If insufficient heat is added to the liquid to produce its saturation temperature at the given pressure, the liquid is said to be Non-Saturated.

If just enough heat is added to a given mass of saturated liquid to completely vaporize it, the vapor formed is called Dry Saturated Vapor. It has the same temperature and the same pressure, but a much larger volume than the liquid from which it was produced. The amount of heat added to completely vaporize the given mass at the given pressure depends on that pressure.

If not enough heat is added to a given mass of saturated liquid to completely vaporize it, the vapor thus formed is called Wet Saturated Vapor. It has the same temperature and the same pressure as the liquid from whence it came. Its volume will be larger than the volume of its liquid but not as large as it would have been if carried to the dry saturated state. The reason for this is that wet saturated vapor contains within itself small particles of its saturated liquid still unvaporized. The amount of heat required to produce a certain quantity of wet saturated vapor depends not only on the pressure but also on the weight of unvaporized liquid carried by it.

Superheated Vapor. When a pound of dry saturated vapor is further heated at constant pressure, an increase in temperature results. Hence we have a pound of vapor at a temperature which is above the saturation temperature for the given pressure. Such vapor is said to be Superheated. The heat required to superheat the vapor depends upon its specific heat, C_p, which is highly variable, and upon the range in temperature, that is, the number of degrees through which
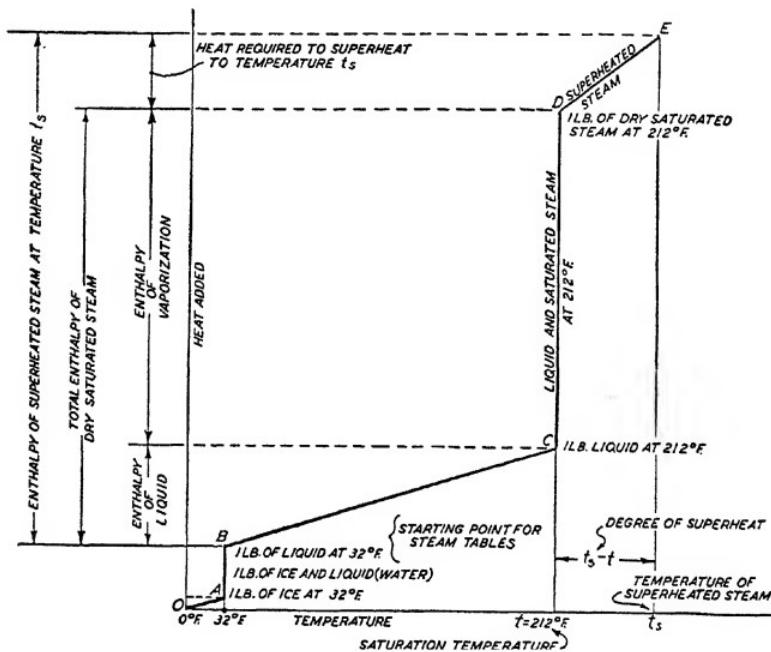
Fig. 25

it is superheated, or the number of degrees its temperature is raised above the temperature of saturated vapor at the same pressure.

Steam. One of the most important vapors we have to deal with is steam or water vapor. And the study of it furnishes additional information relative to the performance of other vapors and their liquids.

Fig. 25 shows graphically that which happens when heat is added to 1 pound of water originally in its solid state at 0° F. The heats added are plotted along the vertical axis, while the Fahrenheit temperatures are plotted along the horizontal axes. The pressure

is considered constant at 14.7 pounds per square inch absolute, otherwise no definite values such as the saturation temperature could be used.

Starting at the origin, O, heat is added to 1 pound of ice at 0° F. As this heat is added, the temperature of the ice rises as shown by line, OA, until at A sufficient heat has been added to raise the temperature to 32° F. At point A, there exists 1 pound of ice, still in the solid state, at 32° F.

The gradual addition of another quantity of heat causes the ice to melt or change its physical state to that of a liquid along the line AB, where we have a mixture of ice and water (solid and liquid) always at 32°, as is evident by the line AB being vertical. From A to B enough heat has been added to completely melt the original pound of ice and at B we have 1 pound of water (liquid) at 32° F. This heat added from A to B is all used to change the physical state from a solid to a liquid and it is known as the Latent Heat of Fusion.

This point, B, is in reality a new starting point, for the heats added and the resultant heat contents or enthalpies of steam and its liquid are reckoned from this point. Although it is customary to thus assume the heat content of water at 32° F. equal to zero, the student should not infer from this that there is no heat in water at this point.

From point B, the new starting point, to point C, another quantity of heat is gradually added to the pound of liquid at 32° F. and at the constant pressure of 14.7 pounds per square inch absolute. This causes the temperature of the water to rise with little change in volume until at point C, the temperature has reached 212° F., the boiling point temperature or saturation temperature for this pressure. The particular quantity of heat used along line BC to produce the saturated liquid condition at C differs in value for all pressures at which it may be added, just as the resulting saturation temperature is different for various pressures. This quantity of heat will be designated as h_f and is called the Heat of Liquid or (more recently) the Enthalpy of the Liquid.

Another quantity of heat is now added to the saturated liquid from C to D. Its addition changes the pound of water to a pound of dry saturated vapor, the state at point D. A very large increase in volume results but, as Fig. 25 clearly shows, there is no change in

temperature. The quantity of heat used here from C to D to effect completely this change of physical state is called the Latent Heat of Evaporation, the Heat of Vaporization, or (more recently) the Enthalpy of Vaporization. It will be designated by the symbol, h_{fg}. Its value like that of h_f depends upon the pressure. By the time the pound of water originally at point B has been turned into dry saturated steam at point D, two different quantities of heat have been added. The sum of these two quantities of heat is known as the Total Enthalpy of the saturated steam and is designated by the symbol, h_g.

With the steam in the dry saturated condition at D, suppose still more heat is added to it. The temperature will again increase with this heat added, as is shown by the line DE, and the greater this quantity of heat added, the farther upward will the line extend and the higher will be the temperature. Steam at any higher temperature than that of saturated steam at the same pressure is called Superheated steam. Such steam is shown graphically at E and its Fahrenheit temperature is given on the temperature axis as t_s, the symbol which will be used to designate the temperature of superheated steam. The temperature of saturated steam will be designated by t. If two such types of steam are at the same pressure, the quantity, $(t_s - t)$ is said to be the number of degrees of superheat. If the heat required to superheat (Note this heat in Fig. 25) through a given number of degrees of superheat is added to the total enthalpy of saturated steam at the same pressure, the result is the total enthalpy of the superheated steam.

Saturated Steam Table. The exact amounts of heat energy that are required to produce saturated as well as superheated vapors under given conditions, have been experimentally determined together with the relations existing between pressures, volumes, temperatures, etc. All such information relative to a vapor when properly tabulated is known as a vapor table. (See Table V).

The saturated steam table in this chapter is an abridged form of the one prepared by Joseph H. Keenan and published by the American Society of Mechanical Engineers. While it is prepared specifically for dry saturated steam, it may be used in problems dealing with wet saturated steam and also in problems dealing with superheated steam, as will be seen later on in this chapter.

In column 1 of our steam tables, we find given the pressure, p, in pounds per square inch absolute, with the exception of those below 1 pound per square inch which are given in inches of mercury.

Column 2 gives the Fahrenheit temperature, t, at which saturated steam (wet or dry) must be if at the pressure indicated opposite it in column 1. Thus to find the temperature at which saturated steam must be if its pressure is 100 pounds per square inch absolute, locate 100 in column 1 and read the corresponding temperature directly opposite it in column 2. In this case it is found to be 327.83° F. This not only means that saturated steam at a pressure of 100 pounds per square inch absolute is always at a temperature of 327.83° F., but also that if saturated steam is at this temperature it must be at a pressure of 100 pounds per square inch absolute. This makes it possible to prepare a saturated steam table with the temperatures given in column 1 and the corresponding pressures in column 2. Such a steam table is called a temperature table, while the one to which we are referring is a so-called pressure table.

Column 3 gives the specific volume, v_f, in cubic feet, which is the volume of 1 pound of the saturated liquid at the pressure of column 1 and the temperature of column 2. Thus the volume of 1 pound of saturated water at a pressure of 60 pounds per square inch absolute and a temperature of 292.71° F. is 0.01735 cubic feet.

Column 4 gives the increase in volume that results when 1 pound of saturated liquid, at the pressure and temperature of columns 1 and 2 respectively, is completely vaporized to 1 pound of dry saturated steam at that same pressure and temperature.

Column 5 gives the specific volume, v_g, in cubic feet, which is the volume of 1 pound of dry saturated steam at the pressure and temperature of columns 1 and 2. v_g of this column is the sum of v_f and v_{fg} in columns 3 and 4 for the same pressure and temperature.

Column 6 is the value of the heat of liquid, or the enthalpy of the liquid, h_f, in British thermal units. This is the number of British thermal units required to raise 1 pound of water from a temperature of 32° F. to the saturation temperature of column 2 if at the pressure of column 1.

Column 7 is the latent heat of evaporation, or the enthalpy of vaporization, h_{fg}, of 1 pound of dry saturated steam. This is the number of British thermal units required to change 1 pound of sat-

urated liquid at the pressure and temperature of columns 1 and 2 into dry saturated steam at the same pressure and temperature. Since the addition of this heat is accompanied by no change in temperature, there can be no increase in internal kinetic energy. Hence during vaporization, as explained under vapors in general,

$$\Delta K = 0$$

Therefore h_{gf}, the heat added during the complete vaporization of 1 pound of saturated liquid, must be used in increasing the internal potential energy and in doing the external work. Since the formation of steam is at constant pressure,

$$\Delta W = \frac{P}{J}(v_g - v_f) \text{ B.t.u. per lb.}$$

In that, $v_g = v_{fg} + v_f$, or $v_{fg} = $ the volume change during vaporization $= v_g - v_f$

$$\Delta W = \frac{P}{J} \times v_{fg} \text{ B.t.u. per lb.}$$

If the general energy equation is written for the process of vaporization at constant pressure, h_{fg} replaces ΔQ, and we have

$$h_{fg} = \Delta P + \frac{r}{J} \times v_{fg} \text{ B.t.u. per lb.} \tag{131}$$

Column 8 is the heat content or total enthalpy, h_g, of 1 pound of dry saturated steam. This is the number of British thermal units required to change 1 pound of water at 32° F. into 1 pound of dry saturated steam at the pressure and temperature given in columns 1 and 2. It is also equal to the sum of the corresponding enthalpy of the liquid of column 6 and the corresponding enthalpy of vaporization of column 7. Notice from the table that at a pressure of 80 pounds per square inch absolute, $h_g(1,182.4) = h_f(281.9) + h_{fg}(900.5)$. Thus for any pressure,

$$h_g = h_f + h_{fg} \tag{132}$$

Generation of Steam in Boiler. Steam is generated in what is known as a steam boiler, a name which is generally applied to that combination of units, consisting in the main of the following:

(1) The boiling vessel, a container, generally cylindrical in form, holds the water to be vaporized. The water occupies about two-

thirds the volume of the vessel. The remainder of the volume receives and momentarily stores the steam that rises from the water.

(2) The surface of water through which the steam passes as it enters the steam space is known as the disengaging surface.

(3) The furnace, in which the latent heat energy of fuel is liberated by combustion.

(4) The pump, which forces water into the boiling vessel as it is needed to replace that which has been vaporized, thus maintaining the water level nearly constant.

(5) The pipe lines, for carrying water to the boiler and the steam from the boiler.

(6) The boiler setting, which supports boiler, forms the side walls of the furnace, and aids in directing the heated products of combustion so that they can come into contact with the so-called heating surface of the boiling vessel, which is the surface of the boiler that is exposed to water or steam on the one side and to the heated gases on the other.

Heat is liberated by the combustion of fuel in the furnace. Some of this heat is radiated directly to the boiler heating surface, while the remainder is taken up by the products of combustion. These in leaving the furnace pass over and along the heating surface of the boiler on their way to the stack or chimney. The heat thus absorbed by the heating surface is conducted through the shell of the boiler where it is used in heating and vaporizing the water within the boiling vessel.

The bubbles of steam, as they form, rise rapidly through the liquid on their way to the steam space. In disengaging themselves from the liquid at the water level or disengaging surface, they carry along with them into the steam space small unvaporized particles of the liquid. These minute particles of liquid are of course at the same temperature as the steam and hence no transfer of heat can take place between them and the steam by which they are held in suspension. Therefore this mixture is very stable. Such a mixture of saturated steam and its saturated liquid is called wet saturated steam. The less the amount of unvaporized and saturated liquid held in suspension, the more nearly will it approach the dry saturated condition. It should be noted that steam formed in contact with its

liquid is always wet saturated steam, the degree or extent of its wetness depending on the design and operation of the boiler.

When the steam leaves the boiling vessel, it is sometimes piped to a series of coils which are located in the pathway of the hot gases or products of combustion. Heat transmitted through these coils from the heated gases will first vaporize the unvaporized portion of the wet saturated steam (for heat cannot be added to a vapor as long as there is any liquid present) and will next raise the temperature of the steam above the saturation temperature for the given pressure. Such steam is then superheated steam and the coils used are known as a superheater. Superheated steam is always dry steam; saturated steam, from a practical standpoint, is always wet to at least some small degree; and dry saturated steam is a theoretical line of demarcation between the wet saturated and superheated steam realms. It follows also that wet steam is always saturated steam.

Quality of Wet Saturated Steam. If in 100 parts by weight of wet saturated steam, 4 parts by weight are unvaporized water held in suspension or mechanically entrained, the actual vaporized portion will be the difference between 100 parts and 4 parts or 96 parts. The latter expressed as a per cent of the whole is called the Quality of the steam, which then, in the above, is 96 per cent. Its decimal equivalent is represented in formulas by the letter x, so that in the above, $x = 0.96$.

The unvaporized portion of the wet steam is spoken of as the Wetness or the Priming of the steam. So when given in per cent by weight,

$$\text{Priming} + \text{quality} = 100\%$$

Total Enthalpy and Volume of Wet Saturated Steam. The enthalpy and volume of dry saturated steam are given directly in the steam table, for the latter, as has been stated, is a table specifically for dry saturated steam. But with the quality known, this table and the quality permit the finding of both the total enthalpy and volume of 1 pound of wet steam at a given pressure.

Since 1 pound of water must be brought up to boiling point temperature before any of it can be vaporized or turned into steam, it is evident that the enthalpy per pound of wet saturated steam at a given pressure must be made up in part of all of the enthalpy

(heat content) of the liquid, h_f, at that pressure. In addition to this, since not all of each pound of wet steam is vaporized, it follows that not all of the h_{fg} British thermal units will be required during its vaporization but only x times h_{fg} British thermal units per pound, where x, as has been stated, is the quality of the vaporized part. Therefore, for wet saturated steam at a given pressure,

$$\text{the total enthalpy} = h_f + x \cdot h_{fg} \text{ B.t.u. per lb.} \quad (133)$$

It will be noted that this formula applies also in the case of dry saturated steam. For the quality of the latter is 100 per cent which makes x, its decimal equivalent, equal to unity.

The volume of wet saturated steam is equal to the volume of its saturated liquid plus the increase in volume of its vaporized part. Since its vaporized part is represented by x, the increase in volume during vaporization is $x \cdot v_{fg}$ cubic feet per pound. Therefore

$$\text{the volume of 1 lb. of wet steam} = v_f + x \cdot v_{fg} \text{ cu. ft. per lb.} \quad (134)$$

If the steam is fairly dry, its volume per pound may be expressed approximately as $x \cdot v_g$ cubic feet.

Total Enthalpy of Superheated Steam. The total enthalpy of superheated steam can be obtained directly from a steam table for this kind of steam. In the absence of such a table, the dry saturated steam table may be used if the mean specific heat of superheated steam for the given conditions is known. It is evident from Fig. 25 that a pound of superheated steam not only contains the enthalpy of the liquid, h_f, and the enthalpy of vaporization, h_{fg}, both of which can be obtained from the saturated steam table, but also the heat required to superheat. Like any heat added, ΔQ, at constant pressure,

$$\text{the heat to superheat} = MC_p(t_s - t) \text{ B.t.u. for } M \text{ lb.}$$
$$= C_p(t_s - t) \text{ B.t.u. per lb.}$$

where

C_p = mean specific heat at constant pressure

t_s = temperature of the superheated steam at the given pressure

t = temperature of saturated steam at the given pressure

Therefore from the above and as shown graphically in Fig. 25, for superheated steam,

$$\text{the total enthalpy} = h_f + h_{fg} + C_p(t_s - t) \text{ B.t.u. per lb.}$$
$$= h_g + C_p(t_s - t) \text{ B.t.u. per lb.} \quad (135)$$

Example. Find the total enthalpy and volume of 10 pounds of

dry saturated steam at a pressure of 60 pounds per square inch absolute. What is the temperature of this steam?

Solution. In column 1, locate 60 lbs. per sq. in. absolute. Directly opposite in column 5, obtain $v_g = 7.172$ cu. ft. per lb. and in column 8, obtain $h_g = 1,177.0$ B.t.u. per lb.

The total enthalpy $= 10 \times 1,177.0$ B.t.u. $= 11,770$ B.t.u. *Ans.*

The total volume $= 10 \times 7.172$ cu. ft. $= 71.72$ cu. ft. *Ans.*

Opposite 60 lbs. of column 1, in column 2, $t = 292.71°$ F. *Ans.*

Example. What is the density of the steam of the preceding example?

Solution. Since density is equal to the reciprocal of the specific volume,

$$\text{density} = \frac{1}{7.172} = 0.139 \text{ lb. per cu. ft. } Ans.$$

Example. Find the temperature, volume, and total enthalpy of 1 pound of wet saturated steam whose quality is 97 per cent and whose pressure is 150 pounds per square inch absolute.

Solution. Locate 150 lbs. in column 1, directly opposite in column 2, $t = 358.43°$ F. *Ans.*

From columns 3 and 4 obtain $v_f = 0.01808$ cu. ft. and $v_{fg} = 2.992$ cu. ft. respectively
From formula (134)
the volume of 1 lb. of wet steam $= 0.01808 + 0.97 \times 2.992$
$$= 2.9203 \text{ cu. ft. } Ans.$$

or approximately
the volume of 1 lb. of wet steam $= x \cdot v_g$ cu. ft.
$$= .97 \times 3.01 \text{ (from column 5)}$$
$$= 2.9197 \text{ cu. ft., a very close}$$
$$\text{approximation}$$

Directly opposite 150 lbs., in column 6, obtain $h_f = 330.44$ B.t.u. per lb. and in column 7 obtain $h_{fg} = 863.1$ B.t.u. per lb.
Applying formula (133)
total enthalpy $= 330.44 + 0.97 \times 863.1 = 1,167.647$ B.t.u. per lb. *Ans.*

Example. What is the total enthalpy of 1 pound of superheated steam at a pressure of 100 pounds per square inch absolute and a temperature of 450° F. if $C_p = 0.53$?

Solution. Here $t_s = 450°$ F.; t, temperature of saturated steam at 100 lbs. per sq. in. abs., $= 327.83°$ F., $h_g = 1,186.6$ B.t.u. per lb. Using formula (135)

total enthalpy $= 1,186.6 + 0.53(450 - 327.83)$
$$= 1,186.6 + 64.75 = 1,251.35 \text{ B.t.u. per lb. } Ans.$$

Example. Ten pounds of steam are at a temperature of 300° F. and a pressure of 14.7 pounds per square inch absolute. Is the steam saturated or superheated?

Solution. Locating 14.7 lbs. per sq. in. abs. in column 1, we find in the temperature column directly opposite 14.7 lbs., a temperature of 212° F. Therefore if this steam is saturated steam, its temperature at this pressure would be 212° F. Since the given temperature, 300° F., is higher than that which saturated steam would have if at the same pressure, the steam in this example is superheated. *Ans.*

Example. From the data of the preceding example, find
(a) the number of degrees of superheat,
(b) the heat content or total enthalpy of the steam assuming $C_p = 0.47$.

Solution. (a) the number of degrees of superheat $= t_s - t$
Here $t_s = 300°$ F., $t = 212°$ F.
Hence, the number of degrees of superheat $= 300° - 212° = 88°$, *Ans.*

(b) Opposite 14.7 lbs. of column 1, find in column 8, $h_g = 1,150.2$ B.t.u. per lb.
Substituting the known values in formula (135)

total enthalpy per lb. $= 1,150.2 + 0.47(300 - 212)$
$$= 1,191.56 \text{ B.t.u.}$$

Hence the total enthalpy for 10 lbs. will be
$$10 \times 1,191.56 \text{ B.t.u.} = 11,915.6 \text{ B.t.u. } Ans.$$

Example. A closed tank contains 30 cubic feet of dry saturated steam at a pressure of 110 pounds per square inch absolute. How many pounds of steam does the tank contain?

Solution. At this pressure, $v_g = 4.044$ cu. ft. per lb. Therefore
30 cu. ft. $\div 4.044$ cu. ft. per lb. $= 7.418$ lbs. *Ans.*

Interpolation. A condensed steam table does not include all of the pressures of steam with which one might wish to deal, and of

course the more the table is condensed, the fewer will be the pressures within a given range and the greater will be the differences or gaps between consecutive pressures. If a pressure is not directly included in a steam table, it naturally follows that the corresponding temperature, volumes, and enthalpies are not available. It is evident that such a missing pressure would fall between some two consecutive pressures of the table and hence the corresponding data for it would lie between the data as given in the table for these two consecutive pressures. In other words, the temperature, volumes, and enthalpies for dry saturated steam at a pressure of say 46 pounds will lie between the corresponding values as given for 40 and 50 pounds. For example, h_g for a pressure of 46 pounds per square inch absolute will lie somewhere between 1,169.2 B.t.u. per pound and 1,173.5 B.t.u. per pound. The method by which it is located between the latter is known as interpolation, which is simply an application of ratio and proportion. The method of interpolation is carried out with a steam table in the same identical manner as with a table of logarithms. The student must however be careful to note when interpolating whether the value he is seeking lies between given values which are increasing with the pressures or decreasing with the pressures. For instance, the enthalpies of the liquid increase but the enthalpies of vaporization decrease as the pressures increase.

The Non=Saturated Liquid. The non-saturated liquid has been defined indirectly in this chapter by defining a saturated liquid as one at saturation temperature for the given pressure. The non-saturated liquid then must be one at a temperature below saturation temperature for the given pressure. As an example of the latter, water at a temperature of 80° F. and at a pressure of 14.7 pounds per square inch is non-saturated. In fact it can be at any temperature below 212° F. and still be non-saturated providing of course its pressure is 14.7 pounds per square inch absolute.

Further Information on Enthalpy of Liquid. The enthalpy of liquid of a non-saturated liquid can be assumed in most cases equal to the enthalpy of liquid, h_f, of a saturated liquid at the same temperature. Hence if the temperature of a liquid (saturated or non-saturated) is known, its enthalpy will be the value of that h_f of the steam table which lies opposite the given temperature. In this case a temperature table is more easily used than a pressure table.

Since the mean specific heat of water between 32° F. and 212° F. is equal to one, the enthalpies of liquid of water at temperatures not much in excess of 212° F. can be readily obtained by subtracting 32 from the temperature. Thus the heat of liquid of water at 70° F. is equal to $70-32=38$ B.t.u. per pound. Likewise the heat of liquid of water at 212° F. is equal to $212-32=180$ B.t.u. per pound. (Compare these results with the values of h_f that are given in the steam table.)

Boiler Horsepower. The rate at which a boiler is operating or working is measured in terms of a unit known as the boiler horsepower. This unit has no connection with the unit of power of the same name which is used in heat engines.

Boilers are operated under varying conditions of pressure, temperature, quality of steam, and feedwater temperature, all of which play a part in the number of pounds of water evaporated by them per hour. Hence in order to have their ratings furnish a means by which their performances can be compared, it is necessary that the unit in which they are rated be defined or set up on the basis of some standard conditions, which would have the effect of making all boilers, when rated, operate theoretically under the same standard conditions. These have been set by the American Society of Mechanical Engineers (A.S.M.E.) in their definition of a boiler horsepower, which definition states that a boiler horsepower is the evaporation of 34.5 pounds of water per hour from feedwater at 212° F. into dry saturated steam at 212° F. and under a pressure of 14.7 pounds per square inch absolute. From this it becomes evident that the standard conditions are as follows: pressure, normal atmospheric; temperature, 212° F.; quality of steam, 100%; and temperature of feedwater, 212° F. Under the latter conditions, the evaporation of 1 pound of water into dry saturated steam would require 970.2 British thermal units. For only h_{fg} British thermal units need be used since the feedwater is assumed to be at saturation temperature for the given standard pressure. This value, 970.2 British thermal units, is known as the Unit of Evaporation.

The heat which the boiler absorbs per hour from the products of combustion under actual working conditions is the heat used in heating and evaporating the water in the boiler, or in other words, the heat added by the boiler to the feedwater in producing the steam

generated per hour. This heat can be determined from the data procured during a so-called boiler test and if this heat absorbed per hour is divided by 970.2, the result thus obtained would be the Equivalent Evaporation per hour which is the number of pounds of water that the boiler would evaporate per hour under the standard conditions noted above if it used in its evaporation the heat absorbed per hour under the actual working conditions. From the definition of the boiler horsepower, it now becomes evident that if the equivalent evaporation per hour is divided by 34.5, the result is the horsepower of the boiler in so-called boiler horsepower units.

The heat absorbed by a boiler per pound of steam generated is the heat content or total enthalpy of the steam minus the enthalpy of liquid of the feedwater. The latter heat is in the water as it enters the boiler. Therefore the boiler cannot be credited with it. Let Q_a represent the heat absorbed per hour, h_{f1} represent the total enthalpy per pound of the feedwater, and M represent the number of pounds of water evaporated per hour, which is the number of pounds of steam generated per hour. Then for dry saturated steam

$$Q_a = M(h_f + h_{fg} - h_{f1}) = M(h_g - h_{f1}) \text{ B.t.u. per hr.} \qquad (136)$$

for wet saturated steam,

$$Q_a = M(h_f + x \cdot h_{fg} - h_{f1}) \text{ B.t.u. per hr.} \qquad (137)$$

and for superheated steam,

$$Q_a = M[h_f + h_{fg} + C_p(t_s - t) - h_{f1}] \text{ B.t.u. per hr.}$$
$$= M[h_g + C_p(t_s - t) - h_{f1}] \text{ B.t.u. per hr.} \qquad (138)$$

Division of Q_a, the heat absorbed per hour under the actual working conditions of the boiler, by the unit of evaporation, 970.2 British thermal units, will give the equivalent evaporation per hour. Thus

$$\text{Equivalent evaporation per hour} = \frac{Q_a}{970.2} \text{ lbs. per hr.} \qquad (139)$$

in which Q_a will be replaced by its value in formulas (136), (137); or (138), depending on what kind of steam is generated by the boiler.

Since for every 34.5 pounds of equivalent evaporation per hour we have 1 boiler horsepower, b. hp., from formula (139)

$$\text{b. hp.} = \frac{Q_a}{970.2} \div 34.5 = \frac{Q_a}{970.2 \times 34.5} \qquad (140)$$

Example. How much heat is required to change 50 pounds of water at 70° F. into superheated steam at a temperature of 450° F. and a pressure of 140 pounds per square inch absolute, if the specific heat of superheated steam under these conditions is equal to 0.56?

Solution. Here $h_g = 1{,}192.4$ B.t.u. per lb., $h_{f1} = 70 - 32 = 38$ B.t.u. per lb., $C_p = 0.56$, $t_s = 450°$ F., $t = 353.03°$ F., $M = 50$ lbs. Substituting these values in formula (138)

$$Q_a = 50[1{,}192.4 + 0.56(450 - 353.03) - 38]$$
$$= 50 \times 1{,}208.7 = 60{,}435 \text{ B.t.u.} \quad Ans.$$

Example. A boiler generates 3,500 pounds of wet saturated steam per hour from feedwater at 80° F. The gauge shows a pressure of 130.3 pounds per square inch. If the quality of the steam is 98 per cent, find

    (a) the heat absorbed by the boiler per hour in the production of the steam,

    (b) the equivalent evaporation in pounds per hour,

    (c) the boiler horsepower developed.

Solution. (a) $p = 130.3 + 14.7 = 145.0$ lbs. per sq. in. absolute. h_f at 145 lbs. per sq. in. abs. $= 327.59$ B.t.u. per lb. h_{fg} at 145 lbs. per sq. in. abs. $= 865.4$ B.t.u. per lb. h_{f1} at 80° F. $= 80 - 32 = 48.0$ B.t.u. per lb. $x = 0.98$ and $M = 3{,}500$ lbs. Substituting these values in formula (137)

$$Q_a = 3{,}500(327.59 + 0.98 \times 865.4 - 48.0)$$
$$Q_a = 3{,}500 \times 1{,}127.682 = 3{,}946{,}887 \text{ B.t.u. per hr.} \quad Ans.$$

(b) The equivalent evaporation per hour is equal to the heat absorbed by the boiler per hour divided by 970.2. Hence evaluating in formula (139),

$$\text{Equivalent evaporation per hr.} = \frac{3{,}946{,}887}{970.2}$$
$$= 4{,}068.1 \text{ lb. per hr.} \quad Ans.$$

(c) Evaluating in formula (140)

$$\text{b. hp.} = \frac{4{,}068.1}{34.5} = 117.9, \quad Ans.$$

The Efficiency of a Boiler. It is the office of the boiler as a whole to release all of the latent heat energy of the fuel through combustion

in the furnace and then to absorb this heat energy and introduce it into water and steam at the boiler proper, or boiling vessel. Since in general

$$\text{efficiency} = \frac{\text{output}}{\text{input}}$$

the input in the case of the boiler is the heat energy of the number of pounds of fuel actually fired per hour. In other words, the boiler is charged with all the heat energy in the fuel that is fired, whether or not the heat energy is all released in the furnace. The output of the boiler is the heat absorbed by the boiling vessel per hour and hence is that part of the heat content of the steam that is furnished by the boiler. The latter is evidently Q_a of formulas (136), (137), or (138), the Q_a selected depending upon the kind of steam produced. The efficiency of a boiler when thus obtained is called the overall efficiency in that it includes the efficiency of boiler, furnace, and grate combined.

To express this overall efficiency as a formula, let

e_b = overall efficiency of the boiler

w = weight of fuel fired, in pounds per hour

H_c = heat energy or calorific value of 1 pound of fuel as fired, in British thermal units per pound.

Then

$$e_b = \frac{Q_a}{w \times H_c} \tag{141}$$

Example. It is required to find the overall efficiency of a boiler working under the following conditions:

Atmospheric pressure = 14.7 lbs. per sq. in. absolute

Steam Pressure = 150 lbs. per sq. in. gauge

Quality of steam = 99%

Temperature of feedwater = 160° F.

Water fed to boiler per hour = 5,600 lbs.

Coal fired per hour = 610 lbs.

Calorific value of coal as fired in B.t.u. per lb. = 12,200.

Solution. $p = 150 + 14.7 = 164.7$ lbs. per sq. in. abs., $M = 5,600$ lbs., $h_f = 338.29$ B.t.u. per lb., $h_{fg} = 856.7$ B.t.u. per lb., $x = 0.99$, $h_{f1} = 160 - 32 = 128$ B.t.u. per lb.

Substituting these values in formula (137),

$$Q_a = 5,600(338.29 + 0.99 \times 856.7 - 128)$$

$$Q_a = 5,600 \times 1,058.42 = 5,927,152 \text{ B.t.u. per hour}$$

Applying formula (141), in which Q_a is given above, $w = 610$ lbs., and $H_c = 12,200$ B.t.u. per lb., we have

$$e_b = \frac{5,927,152}{610 \times 12,200} = 0.796 \text{ or } 79.6\% \quad Ans.$$

Determination of Quality. The device used for determining the quality of steam is called a steam calorimeter. There are several
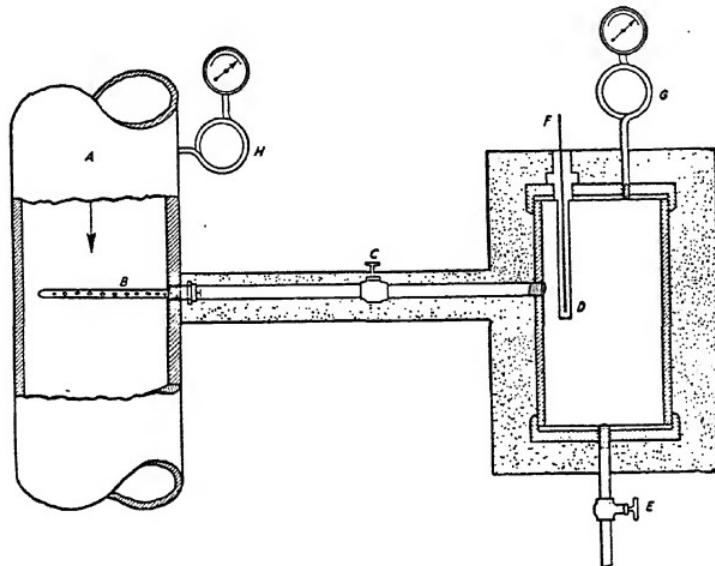
Fig. 26

types of steam calorimeters in use, one of which, the throttling calorimeter, is shown in Fig. 26. Here A is the steam line in which the pressure is registered by the pressure gauge, H. D is the calorimeter into which a thermometer well is introduced carrying the thermometer F, which registers the temperature within the calorimeter. The pressure in the calorimeter is given by the pressure gauge G. All parts of the calorimeter should be thoroughly lagged by a covering of good insulating material, such as hair felt.

Steam is taken from the steam line, where it is wet saturated, by a properly designed perforated sampling tube, B. This tube is

connected to a pipe which carries the steam through a throttling valve, C, into the calorimeter. From there the steam leaves through the discharge valve, E. The function of the throttling valve, C, is to reduce the pressure of the steam, as it enters the calorimeter, without loss of heat content. Hence the pressure, p_2, in the calorimeter is less than the pressure, p_1, in the steam line while the enthalpy, h_2, of the steam in the calorimeter is equal to the enthalpy, h_1, of the steam in the steam line. The result is that the steam becomes superheated as it enters the calorimeter, and the temperature, t_2, of the superheated steam as given by the thermometer, F, becomes higher than the temperature, t_1, of the steam when in the steam line. It will be noted that the subscript, $_1$, is here used to indicate conditions in the steam line, while the subscript, $_2$, is used to indicate conditions in the calorimeter. In the steam line, from formula (133), the total enthalpy,

$$h_1 = h_{f1} + x \cdot h_{fg1}$$

In the calorimeter, from formula (135), the total enthalpy,

$$h_2 = h_{g2} + C_p(t_2 - t_1)$$

Since $h_1 = h_2$, their equals are equal to each other. Hence

$$h_{f1} + x \cdot h_{fg1} = h_{g2} + C_p(t_2 - t_1)$$

Transposing h_{f1},

$$x \cdot h_{fg1} = h_{g2} + C_p(t_2 - t_1) - h_{f1}$$

Dividing both members by h_{fg1},

$$x, \text{ (the quality)} = \frac{h_{g2} + C_p(t_2 - t_1) - h_{f1}}{h_{fg1}}$$

Since the pressure in such a throttling calorimeter is about atmospheric, C_p is always equal to 0.47, which substituted in the above formula gives

$$x, \text{ (the quality)} = \frac{h_{g2} + 0.47(t_2 - t_1) - h_{f1}}{h_{fg1}} \tag{142}$$

Example. Steam at a gauge pressure of 140 pounds per square inch in the steam line is passed through a throttling calorimeter, in which the gauge pressure is 1 pound per square inch and the temperature is 372° F. Determine the quality, assuming atmospheric pressure to be normal.

Solution. Here $p_1 = 140 + 14.7 = 154.7$ lbs. per sq. in. abs., $p_2 = 1 + 14.7 = 15.7$ lbs. per sq. in. absolute, h_{f1}, (in the steam line) $= 332.99$ B.t.u. per lb., h_{fg1}, (in the steam line) $= 861.0$ B.t.u. per lb., t_1, (in the steam line) $= 360.8°$ F., h_{g2}, (in the calorimeter) $= 1{,}151.4$ B.t.u. per lb., t_2 (in the calorimeter) $= 372°$ F.

Substituting these values in (**142**), we have

$$x = \frac{1{,}151.4 + 0.47(372.0 - 360.8) - 332.99}{861.0}$$

$$= 0.957 \text{ or } 95.7\% \quad Ans.$$

Heat Rejected by a Vapor. Consideration has been given so far to the addition or absorption of heat in the formation of saturated and superheated vapors. It is possible of course to have vapors or their liquids reject heat as well as absorb it. In fact if a process, in which absorption of heat has taken place, is reversed or carried in the opposite direction, the same amount of heat, as was previously absorbed, will now be rejected. And if several different heats, such as the enthalpy of liquid, the enthalpy of vaporization, and the heat to superheat were involved during the absorption process, the reversed process would cause these heats to be rejected in the reverse of the order in which they were absorbed. As an example, if superheated steam is cooled at constant pressure, its heat of superheat will be rejected first. During the rejection of this heat of superheat, the temperature of the superheated steam will decrease until finally saturation temperature is reached and the steam is dry and saturated. A further rejection of heat at constant pressure will remove the enthalpy of vaporization, during the removal of which the vapor, remaining at the saturation temperature, will gradually change into its liquid or be condensed. The rejection of the entire enthalpy of vaporization will bring about the complete condensation of the steam (or vapor in general), and the vapor will be returned to its saturated liquid state. Further cooling at constant pressure will now be accompanied by a lowering of the temperature and the rejection of the enthalpy of liquid. Thus the heats are rejected in the reverse of the order in which they were absorbed or added.

THERMODYNAMICS

TABLE V

Properties of Saturated Steam[1]

1	2	3	4	5	6	7	8
Absolute pressure, b. per sq. in. p	Temp., deg. F. t	Specific volume cu. ft. per lb.			Total heat B.t.u. per lb.		
		Sat. liquid V_f	Evap. V_{fg}	Sat. vapor V_g	Sat. liquid h_f	Evap. h_{fg}	Sat. vapor h_g
½″ Hg	58.83	0.01603	1256.9	1256.9	26.88	1058.8	1085.7
¾″ Hg	70.44	0.01605	856.5	856.5	38.47	1052.5	1091.0
1″ Hg	79.06	0.01607	652.7	652.7	47.06	1047.8	1094.9
1½″ Hg	91.75	0.01610	445.3	445.3	59.72	1040.8	1100.6
2″ Hg	101.17	0.01613	339.5	339.5	69.10	1035.7	1104.8
2½″ Hg	108.73	0.01616	275.2	275.2	76.63	1031.5	1108.1
3″ Hg	115.08	0.01618	231.8	231.8	82.96	1027.9	1110.8
1.0	101.76	0.01614	333.8	333.9	69.69	1035.3	1105.0
2.0	126.10	0.01623	173.94	173.96	93.97	1021.6	1115.6
3.0	141.49	0.01630	118.84	118.86	109.33	1012.7	1122.0
4.0	152.99	0.01630	90.72	90.74	120.83	1005.9	1126.8
5.0	162.25	0.01641	73.59	73.61	130.10	1000.4	1130.6
6.0	170.07	0.01645	62.03	62.05	137.92	995.8	1133.7
7.0	176.85	0.01649	53.68	53.70	144.71	991.7	1136.4
8.0	182.87	0.01652	47.38	47.39	150.75	988.1	1138.9
9.0	188.28	0.01656	42.42	42.44	156.19	984.8	1141.0
10.0	193.21	0.01658	38.44	38.45	161.13	981.8	1143.0
11.0	197.75	0.01661	35.15	35.17	165.68	979.1	1144.8
12.0	201.96	0.01664	32.40	32.42	169.91	976.5	1146.4
13.0	205.88	0.01666	30.06	30.08	173.85	974.1	1147.9
14.0	209.56	0.01669	28.05	28.06	177.55	971.8	1149.3
14.696	212.00	0.01670	26.80	26.82	180.00	970.2	1150.2
16.0	216.32	0.01673	24.75	24.76	184.35	967.4	1151.8
18.0	222.40	0.01678	22.16	22.18	190.48	963.5	1154.0
20.0	227.96	0.01682	20.078	20.095	196.00	959.9	1156.0
22.0	233.07	0.01685	18.363	18.380	201.25	956.6	1157.8
24.0	237.82	0.01689	16.924	16.941	206.05	953.4	1159.5
26.0	242.25	0.01692	15.701	15.718	210.54	950.4	1161.0
28.0	246.41	0.01695	14.647	14.664	214.75	947.7	1162.4
30.0	250.34	0.01698	13.728	13.745	218.73	945.0	1163.7
40.0	267.24	0.01712	10.480	10.497	235.93	933.3	1169.2
50.0	281.01	0.01724	8.496	8.514	249.98	923.5	1173.5
60.0	292.71	0.01735	7.155	7.172	261.98	915.0	1177.0
70.0	302.92	0.01744	6.186	6.203	272.49	907.4	1179.9
80.0	312.03	0.01754	5.452	5.470	281.90	900.5	1182.4
90.0	320.27	0.01763	4.874	4.892	290.45	894.2	1184.6
100.0	327.83	0.01771	4.408	4.426	298.33	888.2	1186.6
110.0	334.79	0.01779	4.026	4.044	305.61	882.7	1188.3
120.0	341.26	0.01786	3.707	3.725	312.37	877.4	1189.8
130.0	347.31	0.01794	3.433	3.451	318.73	872.4	1191.2
140.0	353.03	0.01801	3.198	3.216	324.74	867.7	1192.4
150.0	358.43	0.01808	2.992	3.010	330.44	863.1	1193.5
160.0	363.55	0.01814	2.812	2.830	335.86	858.7	1194.5
170.0	368.42	0.01821	2.653	2.671	341.03	854.4	1195.4
180.0	373.08	0.01827	2.511	2.529	345.99	850.3	1196.3
190.0	377.55	0.01833	2.383	2.401	350.77	846.3	1197.0
200.0	381.82	0.01839	2.267	2.285	355.33	842.4	1197.8
210.0	385.93	0.01844	2.162	2.180	359.76	838.6	1198.4
220.0	389.89	0.01850	2.066	2.084	364.02	835.0	1199.0
230.0	393.70	0.01856	1.9778	1.9964	368.14	831.4	1199.6
240.0	397.40	0.01861	1.8970	1.9156	372.13	827.9	1200.1
250.0	400.97	0.01867	1.8223	1.8410	376.02	824.5	1200.5

[1]Abstracted from "Steam Tables and Mollier Diagram," by Prof. J. H. Keenan, 1930 edition by permission of The American Society of Mechanical Engineers.

PROBLEMS

1. Is it possible for saturated steam at a pressure of 20 pounds per square inch absolute to have a temperature of 250° F.?

2. If wet saturated steam and dry saturated steam have the same pressure, how do their specific volumes compare?

3. What is the temperature of superheated steam if the pressure is 250 pounds per square inch absolute and the number of degrees of superheat is equal to 90? *Ans.* 490.97° F.

4. If 1 pound of dry saturated steam at an absolute pressure of 100 pounds per square inch is condensed at that pressure to the saturated liquid state, how much heat will be given up? What is the temperature before condensation? What change in temperature occurs during condensation?

5. When 1 pound of saturated liquid at 30 pounds per square inch absolute is completely evaporated at that pressure,

(a) how many British thermal units are required?

(b) what is the change in volume?

(c) what is the temperature of the steam produced?

Ans. (a) 945.0 B.t.u. (b) 13.728 cu. ft. (c) 250.34° F.

6. Why is superheated steam always dry?

7. Find the temperature, total volume, and total enthalpy of 10 pounds of wet saturated steam at a pressure of 100 pounds per square inch absolute and quality of 95 per cent.

Ans. 327.83° F., 42.05 cu. ft., 11,421+ B.t.u.

8. What is the density of dry saturated steam at normal atmospheric pressure? *Ans.* 0.0373 lb. per cu. ft.

9. How much heat is required to change 10 pounds of water at 70° F. into wet saturated steam at a pressure of 90 pounds per square inch absolute and a quality of 97 per cent? *Ans.* 11,198+ B.t.u.

10. What is the heat content, or total enthalpy, of the 10 pounds of steam in the preceding problem? *Ans.* 11,578+ B.t.u.

11. What is the total enthalpy of 1 pound of superheated steam at a pressure of 14.7 pounds per square inch absolute and a temperature of 312° F., if $C_p = 0.47$? *Ans.* 1,197.2 B.t.u. per lb.

12. A boiler generates 5,600 pounds of wet saturated steam per hour at a gauge pressure of 170 pounds per square inch. The feedwater temperature is 180° F. and the quality of the steam is 99 per cent. Required

(a) the heat absorbed by the boiler per hour

(b) the equivalent evaporation per hour

(c) the boiler horsepower developed.

Ans. (a) 5,824,896 B.t.u. per hr. (b) 6,003.8 lbs. per hr. (c) 174.0 b.hp.

13. It is required to find the overall efficiency of a boiler working under the following conditions:

Steam pressure = 180 lbs. per sq. in. absolute.

Steam temperature = 495° F.

Feedwater temperature = 200° F.

Water fed to boiler per hour = 40,000 lbs.

Coal fired per hour = 4,700 lbs.

B.t.u. per pound of coal as fired = 12,000.

C_p, mean specific heat under these conditions $= 0.57$.

Ans. 77.9%.

14. Steam, at a gauge pressure of 100 pounds per square inch in the steam line, is passed through a throttling calorimeter in which the gauge pressure is 2 pounds per square inch and the temperature is 352° F. The atmospheric pressure is 14.6 pounds per square inch absolute. Determine the quality of the steam. *Ans. 96.7%.*

LOGARITHMS

This is a day of efficiency and time-saving devices. Anyone who can devise some means for saving time on a job is well paid for his services. Competition is so great in business that time-saving devices are looked for continually. And as it is in business so it is in our study. We need every help we can get which will make our time count to the greatest advantage.

A shortcut method, called logarithms (pronounced log'a-rĭthms), has been devised which will not only save time in multiplying large numbers together, but it can also be used to divide, to extract the square root, the cube root, or any other root; to square a number, cube it, or figure out the exact amount for any other power of the given number; or it can be used for any combination of these operations. Take, for example, the following problem.

Find the value of $\sqrt{\dfrac{3678^4 \times .03257^2}{1679^3 \times 1.345^5}}$

The symbol in front of the problem is called the **square root** or **radical sign.** It means that the square root of the combined operations under the symbol is to be extracted. To find the square root of a given number means to find a number which when multiplied by itself will equal the given number. For instance, the square root of 9 is 3, for 3 times 3 equals 9.

The first number in the numerator is 3678. It has a small figure above it. This figure is called an **exponent** and means that the number is taken as many times as the exponent indicates. Each number in this problem has an exponent, and therefore each number is taken as a factor as many times as its exponent indicates.

To solve this problem we would have to multiply the first number in the numerator (3678) four times (that is, $3678 \times 3678 \times 3678 \times 3678$); then we would have to multiply the second number (.03257) two times (that is, $.03257 \times .03257$); and then we would have to multiply the results of these two operations together, which

would give us the combined numerator. Similarly, for the denomina-tor we would have to multiply the first number in the denominator (1679) three times (that is, $1679 \times 1679 \times 1679$); we would have to multiply the second number in the denominator (1.345) five times; and then we would have to multiply these two results together to get the combined denominator. After we have done this, we would have to divide the combined numerator by the combined denominator. We would now have one number, which is the com-bination of all the terms under the **square root sign.** The final operation would be to extract the square root of this number.

You, no doubt, already appreciate how long a problem this would be to work out by arithmetic. When you have learned how to use the logarithmic tables, we will show you in detail just how to work a similar problem by logarithms and you will see how short it is. This short method also takes away the chance of many errors which are constantly being made in the operations of multiplying and dividing, therefore we feel sure that you will want to learn this method right away.

The invention of logarithms has been accorded to John Napier, who was a baron of Scotland. The tables, however, which we are using at the present time are called the Briggs Tables. Henry Briggs was a professor of geometry in London, England. He often visited Mr. Napier in order to get his ideas on this subject of logar-ithms. Then Briggs spent a large part of his time from 1615 to 1628 in compiling his tables.

The fact that it took one man a great many years to calculate and produce these tables is sufficient proof that it was a real job, and because a man was willing to spend years to produce a tool with which to make your work in Mathematics easier and shorter, you should be glad to use it. The important thing at this time, however, is for you to **learn how** to use this tool.

Read the following table carefully and note the exponents:

$10^1 = 10$ and the logarithm of 10 is 1
$10^2 = 100$ and the logarithm of 100 is 2
$10^3 = 1000$ and the logarithm of 1000 is 3
$10^6 = 1000000$ and the logarithm of 1000000 is 6
$10^x = N$ and the logarithm of N is x

In the last line x **is the index of the power to which** 10 **must be raised in order to equal N, or x is the logarithm of N to the base 10.**

Therefore, *the common logarithm of a number is the exponent to which* 10 *must be raised to give the number.*

The Logarithmic Tables give a systematic list of exponents of 10 with their corresponding numbers.

If a decimal exponent is used, then the logarithm has a decimal part, which is written in decimal form.

Thus $10^{1.6} = 39.81+$ and the logarithm is 1.6.

Thus, it is seen that when the number is an exact power of 10, the logarithm has no decimal part; and when the number is not an exact power of 10, the logarithm has a decimal part. These decimal parts are the parts which required such a long time for Mr. Briggs to calculate. You can multiply a number two or three times quite easily, but you cannot multiply it $2\frac{1}{2}$ or $2\frac{1}{3}$ times.

There are two parts to every logarithm. The first part, which is to the left of the decimal point, as figure 2 in 2.342817, is called the **characteristic.** The second part, which is to the right of the decimal point, as figures .342817, is called the **mantissa.** These are large words but you will soon learn to handle them easily. Characteristic is pronounced char′ac-ter′is-tic. Mantissa is pronounced man-tis′sa.

CHARACTERISTIC OF A LOGARITHM

A characteristic may be either positive or negative. Its value depends entirely on the location of the decimal point in the original number. To get the characteristic for numbers which have figures to the left of the decimal point the following rule will apply:

Count the number of figures to the left of the decimal point and subtract 1 from this total and you have the characteristic of the number.

Some numbers are given in tabulated form in Table I to show you how this rule operates.

In the same way as shown in Table I, there can be any number of figures to the **left** of the decimal point and by **subtracting one** from the number of figures, you get the characteristic. These

TABLE I

Number	Figures to left of Decimal Point	Minus	1	Equals	Positive Characteristic
2.34567	1	—	1	=	0
23.4567	2	—	1	=	1
234.567	3	—	1	=	2
2345.67	4	—	1	=	3
23456.7	5	—	1	=	4

characteristics are all **positive**. Cover answers with a sheet of paper, then pick out the characteristics for the following numbers:

Number	Answer	Number	Answer
1.23456	0	34,567,890.	7
654,321.0	5	230,000.00	5
897.654	2	8,760,007.2	6
37.09843	1	92,007,000.	7
2768.91	3	1.0007239	0
7.0	0	900.	2
100.	2	100,000,000.	8

When the figures are all to the **right** of the decimal point, the characteristic is **negative**. When the first figure to the **right** of the decimal point is **not a cipher**, the characteristic is a **minus one**

TABLE II

Number	Ciphers to right of Decimal Point	Plus	1	Equals	Negative Characteristic
.234567	0	+	1	=	−1 or $\bar{1}$
.023456	1	+	1	=	−2 or $\bar{2}$
.002345	2	+	1	=	−3 or $\bar{3}$
.0000234	4	+	1	=	−5 or $\bar{5}$
.0000023	5	+	1	=	−6 or $\bar{6}$

(−1), regardless of the number of other figures to the right. When only the first figure to the right of the decimal point is a **cipher**, the

characteristic is a **minus 2** (-2), and for each **additional cipher** to the right of the decimal point add **one** to the next preceding characteristic and place a minus sign in front or above the result. This is illustrated in Table II.

In the same way as shown in Table II, there can be any number of ciphers to the **right of the decimal point,** and by adding one to the number of ciphers and placing a $-$ sign in front or above this sum, you get the characteristic. These characteristics are all **negative.** Cover answers with a sheet of paper, then pick out the characteristics for the following numbers:

Number	Answer	Number	Answer
0.3	$\bar{1}$	0.010203	$\bar{2}$
.0005	$\bar{4}$	.001002	$\bar{3}$
00.6789	$\bar{1}$	.0000009	$\bar{7}$
.00803	$\bar{3}$	00.0002003	$\bar{4}$
.00001	$\bar{5}$	000.0000823	$\bar{5}$

The complete rule for obtaining either a positive or a negative characteristic may be stated as follows:

When the number is one or greater than one (1), the characteristic is positive and is one less than the number of figures to the left of the decimal point. When the number is less than one, the characteristic is negative and is one more than the number of ciphers between the decimal point and the first significant figure.

The characteristic of a number cannot be given in the tables because it depends entirely on the location of the decimal point.

MANTISSA OF A LOGARITHM

The mantissa is entirely independent of the decimal point. This is just the opposite of the characteristic. The mantissa is always positive and is always a decimal number. In other words it is the decimal part of a logarithm.

If you have two or more quantities composed of the same significant figures in the same order, it does not make any difference with the mantissa how many ciphers are to the right or to the left of the significant figures. The mantissa is the same in every case.

Let us study the Logarithmic Tables and find out just what is meant by the above statement. The following illustration is the first horizontal line on page 2 of the Logarithmic Tables.

LOGARITHMIC TABLES

N	0	1	2	3	4	5	6	7	8	9	D
100	000000	000434	000868	001301	001734	002166	002528	003029	003461	003891	432

The first vertical column in the Logarithmic Tables is headed by the letter N, which stands for the word **number.** This column contains the numbers whose mantissae are given in the horizontal lines corresponding with the numbers. Thus in the above illustration 100 is shown under N and the various mantissae for 100 with the following additional tenths (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) are shown in the horizontal line with 100 and in the vertical columns to correspond to the tenths given. To explain further, the mantissa for 100.0 is .000000 and the characteristic is 2, as shown on page 2 of the text. The mantissa for 100.1 is .000434, as found in the column headed by 1. The mantissa for 100.2 is .000868, as found in the column headed by 2. Thus as the tenths are added to 100, the mantissa increases until under the column headed by 9, the mantissa for 100.9 is given as .003891.

The equally spaced steps, one tenth apart, are for aiding the reader to obtain accurate results quickly. In the Logarithmic Tables the second number in column N is 101. If the intervening 10 steps between 100 and 101 were not given, it would take considerable time to calculate them.

As the numbers in column N progress to the bottom of the page, one unit is added as each line is passed. Thus from the first mantissa in column 0 to the last mantissa of column 9, a complete set of mantissae are given for each 1/10 interval for the numbers given in column N.

The Logarithmic Tables do not show any decimal points. As the mantissae are entirely decimal, the decimal point will be at the left of each left-hand figure.

In column N the decimal point can be placed at any place to suit the user. For instance, the first number may be 0.10, 1.00, 10.0,

100, 1000, 10000, or any other multiple of ten. The mantissa is the same in any case as the only significant figure is 1. The only difference is in the characteristic, as previously shown.

In the last column to the right in the Logarithmic Tables, headed **D**, the differences between any two adjoining mantissae in each horizontal line are given. Since there are ten mantissae in each line, the differences are not always the same; therefore, column **D** gives the average difference for each line. This variation does not make much difference ordinarily. This column is given to save you the labor of subtracting and also insures accurate work. The use for these differences as given in column **D** will be explained later.

HOW TO FIND THE LOGARITHM OF A NUMBER

From the accompanying illustration, copied from page 4 of the Logarithmic Tables, we will show you how to find the mantissa for any given number. Take number 220 in column N. Let us follow this line right across horizontally and see what the different mantissae are for 220, for 220.1, for 220.2, for 220.9, for 221, etc. See Table III.

LOGARITHMIC TABLES

N	0	1	2	9	D
220	.342423	.342620	.342817	.344196	197
221	.344392	.344589	.344785	.346157	196
222	.346353	.346549	.346744	.348110	195

TABLE III

Number	Mantissa	Column found in	Characteristic
220.0	.342423	0	2
220.1	.342620	1	2
220.2	.342817	2	2
220.9	.344196	9	2
221.0	.344392	0	2
222.2	.346744	2	2

In the vertical column under 0, first line, we find the mantissa .342423. This is in line with number 220 in column N and as there

are no tenths added in the 0 column, this is the proper mantissa for number 220.0. There are three figures to the left of the decimal point; by subtracting 1, according to Table I, we get the characteristic 2. Therefore, the complete logarithm for 220.0 is 2.342423.

In the next vertical column under 1, first line, we find the mantissa .342620. This mantissa is larger than the one in column 0 in the same line because we have added one-tenth (0.1) to our number 220.0, making it 220.1. The characteristic has not changed, so our complete logarithm for 220.1 is 2.342620.

In the next vertical column under 2, first line, we find the mantissa .342817. This has increased over the preceding mantissa because we have added one-tenth to our number 220.1, making it 220.2. The characteristic, however, remains 2 for we still have the same number of integers to the left of the decimal point in the number, so the logarithm for 220.2 is 2.342817.

As we progress to the right across the table we add 0.1 to the number shown in the next preceding column, and when we reach the column under 9, the number becomes 220.9 and the mantissa given is .344196.

If we add 0.1 to this number 220.9, we have 221.0, which is the number given in the second line under N, and the mantissa is the second in the column under 0.

Thus you see the table progresses in 0.1 steps from the number 220 until the bottom of the page is reached in the column under 9. On this page 4 of the Logarithmic Tables, the last number under N is 250, so the mantissa for 250.9 is the last one in the column under 9. Thus the complete logarithm for 250.9 is 2.399501. Therefore, to find the mantissa for any number from 200 to 250.9 you would use page 4 of the Logarithmic Tables.

These same numbers (220, 220.9, 221, etc.) may have the decimal point changed either to the right or to the left without changing the mantissae at all, but the characteristics will be changed. This is illustrated in Table IV.

In Table IV we have used the numbers as given in Table III, but have changed the decimal points in order to show you how the characteristics change but the mantissae are the same and do not change. Apply the rule which we gave you concerning the characteristic and refer to Tables I and II, and you will understand

TABLE IV

Number	Logarithm	Characteristic				
2.200	0.342423	1	−	1	=	0
22.01	1.342620	2	−	1	=	1
220.2	2.342817	3	−	1	=	2
2209.	3.344196	4	−	1	=	3
0.221	$\bar{1}$.344392	0	+	(−1)	=	−1 or $\bar{1}$
0.0221	$\bar{2}$.344392	−1	+	(−1)	=	−2 or $\bar{2}$
.00221	$\bar{3}$.344392	−2	+	(−1)	=	−3 or $\bar{3}$

Table IV. To illustrate, note that the last three numbers in Table IV have the same consecutive figures (221), but the decimal point has been moved one figure more to the left in each case. Therefore, while these numbers have the same mantissa, the characteristic has become negative and changed according to the number of ciphers to the right of the decimal point.

FINDING NUMBER THAT CORRESPONDS TO A LOGARITHM

In using the Logarithmic Tables it is also necessary to know how to find the number when the logarithm is given. Take, for example, logarithm 3.203848. To find the number which corresponds to this logarithm, we just reverse the process for finding the logarithm for the number.

Take the first three figures of the mantissa, which are 203, and turn to the Logarithmic Tables. Follow down the first vertical column 0 until we find nearly the same mantissa as the logarithm contains. Then follow across the horizontal line when necessary to get the same, or nearest to but less, mantissa than the one in the problem. We will find this mantissa in the tenth line of column 9, page 3. In this particular case, we find the exact mantissa which we have in our logarithm (Step 1).

The number which corresponds to this mantissa is given in the same line by the three figures in column N, which are 159, and we add on to the right end figure 9 from the column under 9. This gives the number 1599.

It is better to use a smaller mantissa than a larger one. This is shown later.

LOGARITHMIC TABLES

Page	Line	N	9	D
3	Tenth	159	.203848	272

Annex, or place, after 159 the figure 9 which makes the number 1599. $3+1=4$, so point off four figures to left of the decimal point.

To find where to point off the figures for the decimal point, we **reverse** the operations used in finding the characteristic.   In our problem we have a characteristic of 3, to which we add 1, which makes 4.   Begin at the left and count off four figures and place the decimal point.

Having a positive characteristic, the rule is: *Add one to the given characteristic to find the number of figures to the left of the decimal point.*

Having a negative characteristic, the rule is: *Subtract one from the negative characteristic to find the number of ciphers to the right of the decimal point.*

To illustrate these rules a number of logarithms and the way the corresponding numbers are pointed off for both the positive and the negative characteristic are given in Table V.

TABLE V—Locating the Decimal Point

Logs. +Characteristic	To Characteristic add one			Figures to left of point	Number
0.301030	0	+ 1	= 1	One	2.00
1.204120	1	+ 1	= 2	Two	16.00
2.161368	2	+ 1	= 3	Three	145.00
3.203848	3	+ 1	= 4	Four	1599.00
5.687013	5	+ 1	= 6	Six	486421.4 (Prob. I)

Logs. —Characteristic	From characteristic subtract one			Ciphers to right of point	Number
$\bar{1}$.518514	1	— 1	= 0	None	.330
$\bar{2}$.521138	2	— 1	= 1	One	.0332
$\bar{3}$.653502	3	— 1	= 2	Two	.004503
$\bar{4}$.660865	4	— 1	= 3	Three	.000458
$\bar{4}$.331690	4	— 1	= 3	Three	.00021462+

Let us now take a logarithm with a minus characteristic and follow it through in the same way as we did the logarithm with the positive characteristic. For example: We will take the last one in Table V. The logarithm has a minus 4 characteristic, and the mantissa is 331690. The minus sign is placed above the characteristic so that it will not be interpreted as a minus logarithm, for a minus sign in front of a number refers to all of the figures in the number following it. Take the first three figures of the mantissa, which are 331. Follow down through the column 0 on Page 4 of the Logarithmic Tables until we find the mantissa which contains the figures 330, which are the nearest to 331 in this column. In line 15, corresponding to number 214 in column N, we find these figures and by following this line over to vertical column 6, we find the nearest mantissa to the one we have and a little less. You will note by looking under column 7 at the next mantissa that it is quite a little bit over. (Step 2.)

Step 2

<div align="center">LOGARITHMIC TABLES</div>

Page	Line	N	6	7	D
4	15	214	.331630	.331832	202

As before, we have three figures, 214, under the vertical column N, and to the right of that we annex figure 6 from column 6, and that gives the four significant figures 2146. To this we must annex the difference of the mantissa in the table and our logarithm for the mantissa is a little less than the mantissa in the problem.

Step 3 $\overline{4}$.331690 our logarithm
                       .331630 our logarithm in the table
          Difference =   60

In Step 3 we subtract the mantissa in column 6 from our logarithm. It gives us a difference of 60. We now have the question of how much to annex to the number 2146 for this difference of 60 in the mantissae. You will note the difference between the mantissae in columns 6 and 7 (Step 2) is the value given in column D, or 202. In other words, if we would use column 7 instead of column 6,

the mantissa would have to be 202 more. As we have only 60 more than in column 6, we divide 60 by 202, which equals 29+, or the fractional part which we annex to our number. This is shown in Step 4.

Step 4 $\dfrac{60}{202} = 29+$

      Annexing 29+ to 2146 we get 214629+
4−1=3 (Table V). Placing 3 ciphers to right of decimal point
= .000214629+   Answer

MULTIPLICATION

In order to multiply two numbers by the use of logarithms, we find the logarithms of the two numbers and then add these two logarithms together. This sum is the logarithm of the product of the two numbers. Then we turn to the Logarithmic Tables and find the number which corresponds to this logarithm. We will take two of the numbers and their logarithms from Table III to illustrate.

NOTE: In problems use log, abbreviation for logarithm, and mant., abbreviation for mantissa.

Problem 1. Multiply 220.2 by 2209

<table>
<tr><td colspan="2" align="center">Instruction</td><td colspan="2" align="center">Operation</td></tr>
<tr><td colspan="2">Step 1</td><td colspan="2">Step 1</td></tr>
<tr><td colspan="2">Find log of the first number in problem.
Do this by finding the mantissa of the number.
Next find the characteristic of the number.
Combine and you have the log of the number.</td><td>Find log of 220.2
Mantissa of 220.2
Characteristic of 220.2
Log of 220.2</td><td>= .342817
=2.
<u></u>
=2.342817</td></tr>
<tr><td colspan="2">Step 2</td><td colspan="2">Step 2</td></tr>
<tr><td colspan="2">Find log of the second number in problem.
Do this by finding the mantissa of the number.
Next find the characteristic of the number.
Combine and you have the log of the number.</td><td>Find log of 2209
Mantissa of 2209
Characteristic of 2209
Log of 2209</td><td>= .344196
=3.
<u></u>
=3.344196</td></tr>
</table>

Step 3

To multiply numbers, add the logs of those numbers together.

Step 3

Add log of 220.2 and log of 2209 together.

From Step 1, log of 220.2 = 2.342817
From Step 2, log of 2209 = 3.344196
Log of product   = $\overline{5.687013}$

Step 4

Find number that corresponds to this logarithm.
Do this by finding the number in Logarithmic Tables that has a mantissa the same as the mantissa in your problem, or slightly LESS.
Subtract this mantissa from the given mantissa.

Step 4

Find number that corresponds to given mantissa.

From Step 3, mantissa = .687013
Mantissa of 4864   = .686994
Difference   = $\overline{19}$
486 from column N
  4 from column 4
$\overline{4864}$ = first four figures of number

LOGARITHMIC TABLES

Page	Line	N	4	5	D
9	37	486	.686994	.687083	89

Step 5

Find difference between mantissa in adjacent columns on same horizontal line in the log tables or use column D.

Step 5

Mantissa of 4865   = .687083
Mantissa of 4864   = .686994
Difference   = $\overline{89}$
Column D   =   89

Step 6

Divide difference in Step 4 by difference in Step 5.

Step 6

Divide 19 by 89

$\underline{89\ (\ 19.0\ )\ .214}$
$17\ 8$
$\overline{1\ 20}$

$\overline{310}$

Step 7

Place the quotient of Step 6 at the right-hand end of the number found from the table, Step 4.

Step 7

Number in Step 4   = 4864
Quotient in Step 6 = $214-$
Result   = $\overline{4864214}$

Step 8	Step 8
Locate decimal point. Number of figures to left of decimal point is characteristic +1. Begin at left of result in Step 7 and count six figures and place point.	Characteristic of log is $5+1=6$ (Table V) Decimal point is between figure 1 and figure 4 Thus 486421.4 Therefore 220.2 multiplied by 2209 is 486421.4

The problem will now be given in condensed form to show the actual work.

Multiply 220.2 by 2209.

Step 1	Log of 220.2	= 2.342817
Step 2	Log of 2209	= 3.344196
Step 3	Log of product	= $\overline{5.687013}$
Step 4	Mant. of 4864	= .686994
	Difference	= $\overline{19}$
Step 5	Difference	= 89
Step 6	$19 \div 89$	= .214
Step 7	214 annexed to 4864	= 4864214
Step 8		486421.4 Answer

In order that you may understand all the details, we will describe this first problem carefully.

We have taken the two logs from Table II, set them down under each other, and added them and thus obtained the logarithm of a product. Now to find the number which corresponds to this logarithm of the product, we reverse the process for finding the logarithm. We turn to our Logarithmic Tables, and look for the mantissa which corresponds to the mantissa of the product.

To do this, take the first three figures of the mantissa of the product (in this case 687), and look for the same figures in a column in the Logarithmic Tables. You will find these figures on page 9 in the horizontal line where column N equals 486. By following across horizontally until you get to the vertical column 4, you will find the same mantissa which we have shown you in the solution, Step 4.

The figures 486 in column N are the first three figures of your new number, and the figure 4, at the head of column 4, is the fourth figure of your number.

These four figures we have placed in Step 4, in front of the mantissa which we have just found. This is the **nearest lower mantissa** to the one which we have in our logarithm for the product of the two numbers. We now subtract this mantissa from the one above it, as shown in Step 4. The difference is 19. This means that our number is a little larger than this mantissa, so now we must do what we call interpolate (pronounced ĭn-tûr′pō-lāte). If you look in column **D**, Step 4 of our problem, you will find there is a difference between the mantissa in column **4** and the one in column **5** of 89. So we must divide the difference of the mantissae (19) by the difference (89) found in column **D**, to get the amount that we are to annex to the number 4864. This gives us the figures 214 to be annexed to our number, as shown in Step 7. We annex these three figures to the other four and we now have seven figures for our number.

To point off this number properly, we use the rule regarding the finding of the decimal point when given a positive characteristic. We have the characteristic of 5 in the logarithm of the product of the numbers shown in Step 3. To locate the decimal point, we begin at the left-hand figure and count off 5+1 figures, then place the decimal point as shown in Step 8. (See Table V.)

If you wish to check this problem to see whether it is right or not, you can multiply 2000 by 200, which are the round figures involved, and you will find there are six figures to the left of the decimal point. This proves that the problem is solved properly.

Now if you are to multiply the two numbers out in longhand to see whether it is correct or not, you will find that there is a slight difference in the decimal figures only. In other words your exact number would end in .8 instead of .4. You see your result is almost exactly what we obtained by the use of logs. Also you will notice that you have done very little actual figuring to get this product by logs.

In order to multiply 3, 4, 5, or 6 numbers together, all that is necessary is to add the logarithms of those numbers together, and finish the problem in the same way as Problem 1. Thus you see you can multiply 5 or 6 numbers together by the use of logarithms almost as easily as you can two. You would, of course, have the

extra logarithms to look up, otherwise the problem would be just the same as one operation for multiplication.

To show how easy it is to multiply several numbers together by logs, we will work out two problems in detail.

Problem 2. Multiply: $37 \times .46 \times .053 \times 7.9 \times 940$.

LOGARITHMIC TABLES

Page	N	0	Characteristic					Table
7	370	.568202	2	−	1	=	1	I
9	460	.662758	0	+	1	=	$\bar{1}$	II
10	530	.724276	1	+	1	=	$\bar{2}$	II
15	790	.897627	1	−	1	=	0	I
18	940	.973128	3	−	1	=	2	I

Log of 37.   1.568202 ⎫

Log of   .46 $\bar{1}$.662758   Add the mantissae column by column and

Log of   .053 $\bar{2}$.724276   hold the number to be carried over from the

Log of   7.9   0.897627   last column.

Log of 940.   2.973128

Log of product = 3.825991 ⎬ From the sum of the positive characteristics, plus what is carried over from adding the mantissae, subtract the sum of the minus characteristics which = $+3 + 3 - 3 = +3$ characteristic.

Mant. of 6698 =   .825945 ⎬ Mantissa for first 4 figures of answer.

Difference of mantissae   46

LOGARITHMIC TABLES

Page	N	8	D
13	669	.825945	65

$46 \div 65 = 71-$ (71−) annexed to 6698 = 669871− Characteristic $3+1=4$ Point off four figures, starting at the left, and it gives 6698.71 Answer

Problem 3. Multiply $250.9 \times 36.72 \times .9875 \times .04756$

LOGARITHMIC TABLES

Page	N	0	2	5	6	9	D
4	250	. . . .	. . . .	. . . .	. . . .	.399501	. . . . . .
7	367	. . . .	.564903	. . . .	. . . .	. . . .	. . . . .
19	987	. . . .	. . . .	.994537	. . . .	. . . .	. . . . .
9	475	. . . .	. . . .	. . . .	.677242	. . . .	. . . . .
8	432	. . . .	. . . .	. . . .	636087	. . . .	100

Log of 250.9 = 2.399501 ⎫
Log of 36.72 = 1.564903 ⎬ Add logs
Log of   .9875 = $\overline{1}$.994537 ⎱ for
Log of   .04756 = $\overline{2}$.677242 ⎭ product

	Characteristic		Table
3	− 1	= 2	1
2	− 1	= 1	I
0	+ 1	= $\overline{1}$	II
1	+ 1	= $\overline{2}$	II

Log of product = 2.636183 +2+3−3=2 characteristic (like Problem 2).

Mant. of 4326 = .636087 =Mantissa for first 4 figures of answer.

Difference = 96

$96 \div 100 = 96$ 96 annexed to 4326 = 432696 Characteristic $2+1=3$

Point off three figures, starting at the left, and it gives 432.696 Answer.

These problems illustrate the small amount of figuring required to solve them by logs when compared to arithmetic.

In our previous problems in multiplication we had only four figures in each number. Now we will illustrate a problem in which there are six figures in each number. This problem will be worked out in detail in order that you may be able to follow it all the way through.

Problem 4.　Multiply 369.875 by 4863.45

Instruction	*Operation*

Step 1

Find log of first number in prob=
lem.
Do this by finding mantissa of first
four figures of number; then find
amount to be added for the other
two figures. Next find the character-
istic. Combine to get log of number.

Step 1

Find log of 369.875
Mantissa of 3698 　　　　　.567967
Part added for 75 　　　　　　　885
Characteristic 　　　　=2.
Log of 369.875 　　　　=2.5680555

.75×118 =88.5 =added part
3− 1 =2 characteristic (Table I)

LOGARITHMIC TABLES

Page	N	8	9	D
7	369	567967	568084	118

Step 2

Find log of second number in
problem.
Do this by finding mantissa of first
four figures of number; then find
amount to be added for the other
two figures. Next find the character-
istic.
Combine to get log of number.

Step 2

Find log of 4863.45
Mantissa of 4863 　　=.686904
Part added for 45 　　=　　　40
Characteristic 　　　　=3.
Log of 4863.45 　　　=3.686944

.45×89 = 40. + =added part
4− 1 =3 Characteristic (Table I)

LOGARITHMIC TABLES

Page	N	3	4	D
9	486	.686904	.686994	89

Step 3

To multiply numbers, add the
logs of those numbers together.

Step 3

Add logs of 369.875 and 4863.45
Step 1 gives log 　　　　2.5680555
Step 2 gives log 　　　　3.686944
Log of product 　　　=6.2549995

Step 4

Find number that corresponds to this logarithm.
Do this by finding the number in the Logarithmic Tables that has a mantissa the same as the mantissa of the product or slightly less.
Subtract the mantissae.

Step 4

Find the number that corresponds to mantissa of product

Step 3 mantissa	=	.2549995
Mantissa for 1798	=	.254790
Difference	=	2095

From column N	=	179
From column 8	=	8
First four figures	=	1798

LOGARITHMIC TABLES

Page	N	8	9	D
3	179	.254790	255031	242

Step 5

Find difference between mantissa in same horizontal line in the Logarithmic Tables, or use column D

Step 5

Mantissa of 1799	=	.255031
Mantissa of 1798	=	.254790
		241
Column D	=	242

Step 6

Find part to annex
Divide difference in Step 4 by difference in Step 5.

Step 6

Divide 2095 by 242

```
242 ( 2095 ) 865+
      1936
      1590
      1452
      1380
      1210
       170
```

Step 7

Place the quotient at right-hand end of the number found from the Logarithmic Tables in Step 4.

Step 7

Number in Step 4	=	1798
Quotient in Step 6	=	865+
		1798865+

Step 8

Locate decimal point.
Number of figures to left of decimal equals characteristic plus one.
Begin at left and count seven figures, then place point

Step 8

Characteristic of log of product = 6

6+1 = 7 (Table V)
Place decimal point at right end of the number   1798865.+
The product of 369.875×4863.45 =
1798865.+   Answer

Below is shown the solution without explanation.

Step 1 Mant. of 3698   =   .567967
Mant. of   75   =   885
Log   of 369.875 = 2.5680555

Step 2 Mant. of 4863   =   686904
Mant. of   45   =   40
Log   of 4863.45   = 3.686944

Step 3 Log   of 369.875 = 2.5680555
Log   of product = 6.2549995

Step 4 Mant. of 1798   =   .254790
Difference   =   2095

Step 5 Column D   =   242

Step 6 2095 ÷ 242 = 865 +

Step 7 865 annexed to 1798 = 1798865 +
1798865. +   Answer

Description of Solution. We first find the logarithms of the numbers. We find the mantissa of the first four figures of the first number by turning to page 7, as illustrated in Step 1. Under column 8 we find the mantissa. We have now the two figures 7 and 5 to account for. The 75 is 75/100 of 1 unit in the column for figure 8, the fourth figure of our number 3698. In other words, if we would add 25 to 75 we would have 100 or 1 to add to 8. Thus, to get the amount which we are to add to this mantissa we have 75/100 of the difference between the mantissae in columns 8 and 9. Column D gives the difference 118, so we multiply 118 by .75, and point off accordingly, which gives 88.5 This is added to our mantissa, which gives the total mantissa for the whole number 369.875. We find the characteristic as shown in Table I.

We will now find the mantissa for the first four figures of the next number on page 9 of the Logarithmic Tables, as illustrated in Step 2. Under column 3 we find the proper mantissa. As in the first case, we have two figures left to find the amount we are to add on. This is found in the same way as before. This is added on in the same way and we find the characteristic in the same way as in Step 1.

Having our logarithms of the two numbers, we add them together in Step 3 for the log of the product, as we did in Problem 1. We have to find out what the corresponding number is for this logarithm as before. We will find the first four figures on page 3 of Logarithmic Tables as illustrated in Step 4. In column 8 we find the next less mantissa for the first four figures. This mantissa we subtract from the logarithm of the product and have a difference of 2095. This difference divided by the value in column D in Step 4 gives the value 865 (Step 6), which is the amount we annex to the four figures previously found (Step 7).

Next we place the decimal point in the right place by Table V. To the characteristic of 6 we add 1, which makes seven figures to the left of the decimal point. (Step 8.) This is the answer for the product of our two numbers.

While we have gone into quite a bit of detail to describe the operation to you, there is little actual figuring. When you compare this with the amount of time that would be required to multiply this out in the ordinary way, you will agree with us that logs can save you a lot of time besides eliminating the chances of errors.

Six-place tables are only accurate to sixth figure of answer.

<div align="center">Practice Problems</div>

In order to give you practice in using the Logarithmic Tables, several practice problems in multiplication are given for you to solve by means of logarithms.

Follow the methods illustrated in the four problems already worked out. It is not necessary to send in the solutions of these practice problems unless you fail to get the answers given here. If you send them in, show all your work.

1. Multiply 820×40×293   9610422. Answer
2. Multiply 423.1×12.34×65   339367.9 Answer
3. Multiply 9.99×.777×67.8   526.278 Answer
4. Multiply 68.52×.0019×371.8   48.4039 Answer
5. Multiply 254.76×4.3219   1101.05 Answer
6. Multiply .007342×.09837   .000722233 Answer
7. Multiply 2700000×39000000×980000   103,194,000,000,000,-
                                       000,000 Answer
8. Multiply 8100.07×3002.58   24,321,112+Answer

DIVISION

In order to divide two numbers by the use of logarithms, we find the logarithms of the two numbers and then subtract these two logarithms instead of adding them as we did in multiplication. To illustrate this, we will use the same two numbers we did in Problem 1 and divide them.

Problem 5. Divide 220.2 by 2209

Instruction	*Operation*

Step 1

Find log of the first number in problem.
Do this by finding the mantissa of the number. Next find the characteristic.
Combine the two and you have the log of the number.

Step 1

Find log of 220.2
Mantissa of 220.2　　= .342817
Characteristic of 220.2 = 2.
Log of 220.2　　　　= 2.342817
(See Table I)

Step 2

Find log of the second number.
Do this by finding the mantissa of the number. Next find the characteristic.
Combine the two and you have the log of the number.

Step 2

Find log of 2209
Mantissa of 2209　　　.344196
Characteristic of 2209　= 3.
Log of 2209　　　　= 3.344196

Step 3

To divide one number by another subtract the logs of the numbers.
After 1 is borrowed from 2 to make 13 in the top line, instead of changing the 2 to 1, add 1 to the 3 in the bottom line, subtract the smaller number from the larger, and place a −sign in front of the difference:

$$+2-(3+1)=-2 \text{ or } \bar{2}$$

Step 3

Subtract log of 2209 from log of 220.2
From Step 1 log of 220.2 = 2.342817
From Step 2 log of 2209　= 3.344196
Log of quotient　　　= $\bar{2}$.998621

Finding the Number that Corresponds to the Logarithm

Step 4

Find the first four figures of the number that corresponds to this logarithm.
Do this by finding the number in Logarithmic Tables that has a mantissa the same as the mantissa in your problem, or slightly less.
Subtract the mantissae.

Step 4

Find number that corresponds to mantissa.
From Step 3 log　　= 2.998621
Mantissa of 9968　= .998608
Difference　　　= 　　13

Page	N	8	9	D
19	996	998608	998652	44

Figure 8 from Column 8 annexed to $996 = 9968$.

Step 5

Find difference between man=
tissa in adjacent columns on
same horizontal line in the
Logarithmic Tables or use Col=
umn D.

Step 5

Mantissa of 9969	=	.998652
Mantissa of 9968	=	.998608
Difference	=	44
Column D	=	44

Step 6

Divide difference in Step 4 by
difference in Step 5.

Step 6

Divide 13 by 44

$$\underline{44\ (\ 13.0\ )\ .29+}$$
$$8\,8$$
$$\overline{4\,20}$$
$$3\,96$$
$$\overline{}$$

Step 7

Place the quotient at the right=
hand end of the number found
from the Table, Step 4.

Step 7

Number in Step 4	= 9968
Quotient in Step 6	= 29
	996829

Step 8

Locate decimal point.
Number of ciphers to right of deci-
mal point is negative characteristic
minus one, or -2 less $1 = -1$

Step 8

Characteristic of log is -2 less 1
$= -1$
Decimal point is to left of one
cipher $= .0996829$
Thus $220.2 \div 2209 = .0996829$ Ans.

Below we show how simple the problem is when the explanation
is omitted.

Step 1	Log of 220.2	= 2.342817
Step 2	Log of 2209	= 3.344196
Step 3	Log of quotient	= $\overline{2}$.998621
Step 4	Mant. of 9968	= .998608
	Difference	= 13
Step 5	Difference	= 44
Step 6	$13 \div 44 = 29+$	
Step 7	29 annexed to $9968 = 996829$	
Step 8	.0996829 Answer	

Since we are dividing one number by a larger one our result, of course, will be a decimal number. In that case, we will have a negative characteristic as described in Table II.

You notice that the larger characteristic is subtracted from the smaller one. In order to do this we need to add two to the smaller characteristic. Thus, we will have as a result a minus 2 as a characteristic for the difference.

We now proceed the same as we did with the problem in multiplication to find the corresponding number. We find the next mantissa nearest to the one of the quotient as shown in Step 4. Subtract these two mantissae and you have a difference of 13. Now turn over to column **D** where we find the difference 44, Step 5. We divide 13 by 44 as we did in our multiplication problem and we have the result 29. This is annexed to your number as indicated by the location of your mantissa in your table and we have the result in Step 7.

In order to point this off properly, refer to Table V. Here we have a negative characteristic of 2, and we subtract 1, which is just the reverse of finding the characteristic. That gives us one cipher to the right of the decimal, so point it off as in Step 8.

If you want to check this result, you can divide by the ordinary way and see how near our log method is correct. In actual figures, however, we have done very little mathematical work. It takes quite a little time to describe this log method to you, but after you learn it once, you will find it comes easy.

Practice Problems

Solve the following problems by logarithms. Do not send in the solutions unless you fail to get the answers given. If you send them in, show all your work.

1. Divide 847.6 by 24.99   33.9176 Answer
2. Divide 7256.2 by 879.26   8.2526   Answer
3. Divide 276.543 by 912.34   .303113 Answer
4. Divide 0.9783 by .1234   7.92787 Answer
5. Divide .00879 by .0092   .955435 Answer
6. Divide 8321000 by 2135000   3.89743 Answer
7. Divide 850.06 by 50.082   16.9733 Answer
8. Divide .00097 by .0023   .421739 Answer

You will find that the knowledge of logarithms will come in very handy when you take up your advanced studies. In these texts there are a number of problems in which four, five, or six numbers are multiplied together and sometimes one or two divided. Both of these operations can be done at the same time by logarithms. In other words, you can multiply and divide at the same time. We will illustrate by a problem. Let us use three of the numbers, of which we already have the logarithms in our first table.

MULTIPLICATION AND DIVISION

Problem 6. Find the result $\dfrac{220.2 \times 22.01}{2209}$

Step 1

Log of 220.2	= 2.342817	⎫ Add logs for product of number
Log of 22.01	= 1.342620	⎭
Log of product	= 3.685437	⎫ Subtract logs for quotient of
Log of 2209	= 3.344196	⎭ numbers
Log of quotient	= 0.341241	
Mant. of 2194,		
Step 2	= .341237	
Difference		

Step 2

LOGARITHMIC TABLES

Page	N	4	5	D
4	219	.341237	.341435	198

Step 3

4 ÷ 198 = 02

Annexing 02 to 2194 = 219402

Step 4

Characteristic 0 + 1 = 1   Point off one figure to left of decimal
(Table V) 2.19402 Answer

In this problem, add the first two logarithms. This gives the logarithm of the product. Subtract the logarithm of the third num-

ber from the logarithm of the product. This gives the quotient. Now all we have to do is to find the corresponding number for this logarithm. In the Logarithmic Tables, as shown in Step 2, you will find the next mantissa nearest to the one we have for the quotient. The difference is 4. We divide the difference by the difference in column **D**, Step 2, which is 198. The result is 02. We annex 02 to the right-hand end of our number 2194 and we get 219402, which is the whole number we are looking for. To find where to put the decimal point, we refer to Table V. Having a zero characteristic, we add one and this gives us one figure to the left of the decimal.

Thus you see we finished these two operations almost as quickly as we did the one before. The only difference is the extra step of subtracting one of the logarithms from the sum of the other two. Thus you see the use of logarithms will save you quite a bit of time from what is necessary to work this kind of a problem by arithmetic.

Practice Problems

Solve the following problems by logarithms. Do not send in the solutions unless you fail to get the answers given. If you send them in, show all your work.

1. $\dfrac{67432 \times .02035}{700.09}$ 1.96009 Answer

2. $\dfrac{25.729 \times 1.0025}{35.678}$ .722947 Answer

3. $\dfrac{29000 \times 670000 \times 5300}{80029}$ 1,286,770,000 Answer

ROOTS AND POWERS

In the introductory discussion and in the lesson on Powers and Roots we explained the ordinary use of the square root or radical sign and exponents. This same sign is often used for other roots beside the square root. For instance, $\sqrt[3]{7235}$ means that the cube root of the number under the radical sign is to be obtained. We may also use any other figure in place of the 3 to indicate any root of the number under the radical sign. $\sqrt[4]{256}$ indicates that the fourth

root of 256 is to be obtained. The result equals 4, for $4 \times 4 \times 4 \times 4 = 256$. Also, $\sqrt[5]{243}$ indicates the fifth root of 243, which equals 3. Or $\sqrt[6]{64}$ indicates the sixth root of 64, which equals 2.

Ordinarily, there is a single figure used as an exponent for a number. The exponent indicates the power to which the number is to be increased. These exponents can be either fractional or decimal. $853^{2/3}$ is an illustration of a fractional exponent. It is read 853 to the 2/3rds power. This indicates that we are to raise 853 to the second power and extract the cube root. Thus, the numerator of the exponent indicates the power and the denominator indicates the root. $275^{7/5}$ is read 275 to the 7/5ths power. In this exponent the numerator is larger than the denominator. Thus it is seen that any kind of a fraction may be used as an exponent.

$724^{1.6}$ is an illustration of a decimal exponent. It is read 724 to the one and six-tenths power. This type of exponent may be either part or wholly decimal.

We have a great variety of exponents, but we can solve all problems containing these indicated operations by logarithms. Some of these operations are impossible by arithmetic. Therefore, you see how valuable logarithms are in this particular work.

In order to solve a problem in which we want to find the root, we find the logarithm of the number and then divide the logarithm by the exponent of the root of the number. For instance, if we want the square root of 75, we find the logarithm of 75 and divide the log by 2, which gives us the logarithm of the square root of the number. To find the cube root, we divide by 3; to find the fourth root, we divide by 4, etc.

To find the power of a number, we find the logarithm of the number and multiply it by the exponent of the power to which we want to raise the number. This gives the log of the power. You will note that this is just the reverse of finding the root.

For instance, if we want to find the square of 229, we find the logarithm of 229, and then multiply it by 2. That gives us the logarithm of the square of the number. If we multiply the logarithm of any given number by any indicated power, we obtain the logarithm of that power for that number. Fractional or decimal exponents are handled in a similar manner. We will work out two problems to illustrate how this work is done.

Problem 7. Solve $99^{9/7}$

Step 1

Log of 99	:	1.995635	—Logarithmic Tables, page 19
		9	Multiply by 9 to obtain the log of 9th power of number
		7)17.960715	Divide by 7 to obtain the log of the 7th root
Log of $99^{9/7}$		2.5658164+	
Mant. of 3679		.565730	
Difference		864	

Step 2

LOGARITHMIC TABLES

Page	N	9	D
7	367	565730	118

Step 3

$864 \div 118 = 73$

73 annexed to 3679 = 367973

Step 4

Characteristic $2 + 1 = 3$

Point off three figures to the left of the decimal point.

367.973 Answer

First, find the mantissa for 99. This is easily located on Page 19 of Logarithmic Tables. The characteristic is located by Table I.

To get the ninth power of this logarithm, you multiply it by 9; and to get the seventh root of that, you divide it by 7. That gives you the logarithm of the two operations. We next find the corresponding number to this log as we have done in the other problems.

In this problem you will see you have saved yourself a lot of time in working it out with logs. You can find the ninth power by arithmetic, but you could not find the seventh root that way.

Problem 8.   Solve 741.4$^{1.6}$

Step 1

Log of 741.4	= 2.870053	Log Table, Page 14. Character-istic, Table I.
Multiply 1.6	= 1.6	Multiply by exponent to get the log of the power.
	17220318	
	2870053	
Log of power	= 4.5920848	
Mant. of 3909	= 592066	See Step 2
Difference	= 188	

Step 2   LOGARITHMIC TABLES

Page	N	9	D
7	390	.592066	111

Step 3

188 ÷ 111 = 17 −

17 − annexed to 3909 = 390917 −

Step 4

Characteristic 4 + 1 = 5

Point off five figures to left of decimal point, Table V.

39091.7 Answer

We find the logarithm of 741.4 in the same way as we did in the other problems, then multiply by 1.6. That gives us the logarithm of the 741.4 to the 1.6 power. We find the nearest less mantissa in the Logarithmic Tables and subtract it as shown. Steps 3 and 4 are the same as in the other problems. This problem cannot be solved by arithmetic.

Practice Problems

Work out the following problems by logarithms. Do not send in the solutions unless you fail to get the given answers. If you send them in, show all of your work.

1. Find the value of $\sqrt[5]{476.92}$ 3.43312 Answer
2. Find the value of 12345.$^{9/3}$ 1,881,365,200,000+ Answer
3. Find the value of 9.645$^{3.5}$ 2786.48 Answer

DECIMAL NUMBERS WITH EXPONENTS

Decimal numbers have negative characteristics, as we have shown in previous discussions. When we find it necessary to solve problems having decimal numbers with **powers** or **roots**, it will be necessary to change the negative characteristic to a positive characteristic.

As an example let us find the log of .07854³

Step 1

Log of .07854³ = $\overline{2}.895091 \times 3$ Table II

In order to avoid negative characteristics, we can make them positive by adding 10 to the characteristics and subtracting 10 at the end of the logarithms. This is illustrated in Step 2.

Step 2
$$\begin{array}{r} \overline{2}.895091 \times 3 \\ +10 \qquad -10 \\ \hline (8.895091 - 10) \times 3 \end{array}$$

Multiplying as indicated, gives us Step 3. We can simplify, by subtracting 20 from each part of the characteristic as shown.

Step 3
$$\begin{array}{ll} 26.685273 - 30 & \quad 26 - 30 = 4 \\ 20 \qquad\quad 20 & \\ \hline 6.685273 - 10 & \quad +6 - 10 = 4 \end{array}$$

Now we can go back to the negative form by subtracting the 10 from the 6, which gives us a minus 4 characteristic with the mantissa shown in Step 4.

Step 4 $\overline{4}.685273$ Answer

This is the correct result, which cannot be obtained by multiplying directly the negative characteristics shown in Step 1.

Still another complication arises when you have the root instead of the power of a decimal number.

As an example let us find the log of $\sqrt[4]{.07854}$

This is the same number as used in the above problem, so we have the same logarithm and the negative characteristic, Steps 1 to 4.

Step 5 $(\overline{2}.895091 \div 4)$

In this case we have to divide by 4 in order to get the log of the root. Change the negative to the positive characteristic and we have Step 6.

Step 6 $(8.895091 - 10) \div 4$

In dividing such a logarithm we must have a -10 at the end after the division. Therefore, we must add to both parts of the characteristic to allow for this condition. In this case we add 30 as shown in Step 7. Dividing as indicated we get Step 8.

Step 7 $(8.895091 - 10) \div 4$

$$\frac{30 \qquad\quad -30}{4)\overline{38.895091 - 40}}$$

Step 8 $9.72377275 - 10$

Now we can put this back in a negative form as before, and $\overset{\smile}{9} - 10$ gives us -1 for a characteristic, as shown in Step 9. This is the correct logarithm for our problem.

Step 9 $\overline{1}.72377275$ Answer

This is the only method by which problems of this kind can be solved. We will solve a problem of this type to show you how it is done.

Problem 9. Solve: $\dfrac{(.07345)^3}{(.62547)^{1/2}}$

Log of .07345 $= \overline{2}.865992$ Logarithmic Tables, Page 14. Characteristic, Table II.

Change to positive characteristic $= 8.865992 - 10$ $(8 - 10 = -2)$

Multiply by 3 for log of power $= \underline{\qquad\qquad 3}$

This gives log of numerator $= 26.597976 - 30$

Mant. of 6254 $= .796158$ Logarithmic Tables, Page 12

Add for fifth figure 7 $= \underline{\qquad 48.3}$ $(7 \times 69 = 48.3)$

Log of .62547 $= \overline{1}.796206.3$ Characteristic, Table II

Change to $+$ characteristic $=$ $9.7962063 - 10$ $(9 - 10 = -1)$

Add 10 to both parts $=$ $\underline{10 \qquad\quad -10}$

Divide by 2 for root $= 2)\overline{19.7962063 - 20}$

This gives log of denominator $=$ $9.89810315 - 10$

Subtract log of denominator from log of numerator in order to divide the numbers.

Log of numerator $= 26.597976 - 30$
Log of denominator $= \ \ 9.89810315 - 10$
Log of quotient $= \overline{16.69987285 - 20}$
Change to negative characteristic $= \overline{4}.69987285 - (+16 - 20 = -4)$
Mant. of 5010 $= \ \ .699838$
Difference $= \ \ \ \ \ \ \ \ \ 3485$

<div align="center">LOGARITHMIC TABLES</div>

Page	N	0	D
10	501	699838	87

Divide problem difference by column D value. $34.85 \div 87 =$ 4+

Annex to other figures at right-hand end. Annexing 4 to 5010 = 50104

Table V, $4 - 1 = 3$

There will be three ciphers to right of decimal point.

.00050104 Answer

COMBINATION OF OPERATIONS

We are going to work out in detail a problem, which will illustrate four or five of the different operations at once. It is only a little different from the first problem illustrated, but is more complicated. The two numbers in the numerator both have simple exponents, while one number in the denominator has a fractional exponent and the other has a decimal exponent. And the radical is for the cube root of the total result of the other operations.

We have worked out this problem, giving the details of each operation so that you can follow it all the way through. Study this very carefully as it illustrates many of the different operations at once. It also shows you how much time and labor can be saved by using logarithms and how the chance for many mistakes is eliminated. Some of these operations cannot be solved by arithmetic.

After you have studied this problem through thoroughly so

that you understand it, you should be able to handle any kind of a problem which involves logarithms.

You must remember that you cannot add or subtract numbers after you get the logarithms of them. You must add or subtract by the ordinary methods before you get the logs. When you add the logarithms, you multiply the numbers; and when you subtract the logarithms, you divide the numbers.

Problem 10. Find value of: $\sqrt[3]{\dfrac{3678^4 \times 3.257^2}{1679^{2/3} \times 1.345^{1.5}}}$

Step 1

Find log of first number of denominator.
Do this by finding mantissa of number in Logarithmic Tables, page 3.
Next find characteristic of number and combine.
Next find log for power of number.
Next find log for root of number.

Step 1

Find log of $1679^{2/3}$

Mantissa of 1679		.225051
Characteristic, Table I	=	3.
Log of 1679	=	3.225051
Multiply by power		2
Divide by root		3)6.450102
Log of $1679^{2/3}$	=	2.150034

Step 2

Find log of second number of denominator.
Do this by finding mantissa of number in Logarithmic Tables, page 2.
Next find characteristic of number and combine.
Next find log for power of number.

Step 2

Find log of $1.345^{1.5}$

Mantissa of 1.345	=	.128722
Characteristic, Table I		
Log of 1.345		0.128722
Multiply by power		1.5
		643610
		128722
Log of $1.345^{1.5}$		0.193083

Step 3

Find log of denominator.
Do this by adding the two logs of the numbers for log of product of numbers.

Step 3

Add logs of $1679^{2/3}$ and $1.345^{1.5}$

Step 1 gives log		2.150034
Step 2 gives log		0.193083
Log of denominator	=	2.343117

Step 4

Find log of first number of numerator.
Do this by finding mantissa of number in Logarithmic Tables, page 7.
Next find characteristic of number and combine.
Next find log for power of number.

Step 4

Find log of 3678^4

Mantissa of 3678	=	.565612
Characteristic, Table I	=	3.
Log of 3678	=	3.565612
Multiply by power		4
Log of 3678^4	=	14.262448

Step 5

Find log of second number of numerator.
Do this by finding mantissa of number in Logarithmic Tables, page 6. Next find characteristic of number and combine.
Next find log for power of number.

Step 5

Find log of 3257^2

Mantissa of 3257	=	.512818
Characteristic, Table I		0.
Log of 3257		0.512818
Multiply by power		2
Log of 3257^2		1.025636

Step 6

Find log of numerator.
Do this by adding the logs of the two numbers for log of product.

Step 6

Add logs of 3678^4 and 3.257^2

Step 4, log of 3678^4	=	14.262448
Step 5, log of 3.257^2	=	1.025636
Log of numerator	=	15.288084

Step 7

Find log of all numbers under the radical sign.
Do this by subtracting log of denominator from log of numerator for quotient of numerator divided by denominator.

Step 7

Subtract log of denominator from log of numerator

Step 6, log of numerator	=	15.288084
Step 3, log of denominator	=	2.343117
Log of quotient	=	12.944967

Step 8

Find root of combined results under radical sign.
Do this by dividing log of quotient of combined numerator and denominator by indicated root of the radical sign.

Step 8

Find cube root of number whose log by Step 7 = 12.944967

Divide by 3		3)12.944967
Log of cube root	=	4.314989

Step 9

Find number that corresponds to log of Step 8.
Do this by finding the number in Logarithmic Tables that has a mantissa the same or a little less than mantissa of the log.
Subtract mantissa.

Step 9

Find number that corresponds to mantissa of Step 8 = .314989

Column N gives 206		
Column 5 gives		5
		2065
Mantissa for 2065	=	.314920
Difference		69

LOGARITHMIC TABLES

Page	N	5	D
4	206	314920	210

Step 10

Find amount to be annexed to the number from the difference of mantissa of Step 9.
Do this by dividing this difference by the difference shown in column **D**.

Step 10

Divide 69 by 210

$$\underline{210\ (\ 69.0\ )\ .328}$$
$$\underline{63\ 0}$$
$$\overline{6\ 00}$$
$$4\ 20$$
$$1\ 800$$
$$1\ 680$$

Step 11

Place quotient of Step 10 at right end of number found in Step 9 to get all figures of resulting number.

Step 11

Number in Step 9	2065
Quotient in Step 10	328
	2065328

Step 12

Locate decimal point.
Number of figures to left of decimal point equals characteristic plus one.

Step 12

Step 8 Characteristic =   4
Add one   1
                                 5

Count five figures from left end and place decimal point.

20653.28 Answer

Six-Place
Logarithmic Table

From 1 to 9999

N.	0	1	2	3	4	5	6	7	8	9	D.
100	00 0000	00 0434	00 0868	00 1301	00 1734	00 2166	00 2598	00 3029	00 3461	00 3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	01 0300	01 0724	01 1147	01 1570	01 1993	01 2415	424
103	01 2837	01 3259	01 3680	01 4100	4521	4940	5360	5779	6197	6616	420
104	7033	7451	7868	8284	8700	9116	9532	9947	02 0361	02 0775	416
105	02 1189	02 1603	02 2016	02 2428	02 2841	02 3252	02 3664	02 4075	02 4486	02 4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	03 0195	03 0600	03 1004	03 1408	03 1812	03 2216	03 2619	03 3021	404
108	03 3424	03 3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	04 0207	04 0602	04 0998	397
110	04 1393	04 1787	04 2182	04 2576	04 2969	04 3362	04 3755	04 4148	04 4540	04 4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
112	9218	9606	9993	05 0380	05 0766	05 1153	05 1538	05 1924	05 2309	05 2694	386
113	05 3078	05 3463	05 3846	4230	4613	4996	5378	5760	6142	6524	383
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	06 0320	379
115	06 0698	06 1075	06 1452	06 1829	06 2206	06 2582	06 2958	06 3333	06 3709	06 4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
117	8186	8557	8928	9298	9668	07 0038	07 0407	07 0776	07 1145	07 1514	370
118	07 1882	07 2250	07 2617	07 2985	07 3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	07 9181	07 9543	07 9904	08 0266	08 0626	08 0987	08 1347	08 1707	08 2067	08 2426	360
121	08 2785	08 3144	08 3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	09 0258	09 0611	09 0963	09 1315	09 1667	09 2018	09 2370	09 2721	09 3071	352
124	09 3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	09 6910	09 7257	09 7604	09 7951	09 8298	09 8644	09 8990	09 9335	09 9681	10 0026	346
126	10 0371	10 0715	10 1059	10 1403	10 1747	10 2091	10 2434	10 2777	10 3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	11 0253	338
129	11 0590	11 0926	11 1263	11 1599	11 1934	11 2270	11 2605	11 2940	11 3275	3609	335
130	11 3943	11 4277	11 4611	11 4944	11 5278	11 5611	11 5943	11 6276	11 6608	11 6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	12 0245	330
132	12 0574	12 0903	12 1231	12 1560	12 1888	12 2216	12 2544	12 2871	12 3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	13 0012	323
135	13 0334	13 0655	13 0977	13 1298	13 1619	13 1939	13 2260	13 2580	13 2900	13 3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	9879	14 0194	14 0508	14 0822	14 1136	14 1450	14 1763	14 2076	14 2389	14 2702	314
139	14 3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	14 6128	14 6438	14 6748	14 7058	14 7367	14 7676	14 7985	14 8294	14 8603	14 8911	309
141	9219	9527	9835	15 0142	15 0449	15 0756	15 1063	15 1370	15 1676	15 1982	307
142	15 2288	15 2594	15 2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	16 0168	16 0469	16 0769	16 1068	301
145	16 1368	16 1667	16 1967	16 2266	16 2564	16 2863	16 3161	16 3460	16 3758	16 4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	17 0262	17 0555	17 0848	17 1141	17 1434	17 1726	17 2019	17 2311	17 2603	17 2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	17 6091	17 6381	17 6670	17 6959	17 7248	17 7536	17 7825	17 8113	17 8401	17 8689	289
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
150	17 6091	17 6381	17 6670	17 6959	17 7248	17 7536	17 7825	17 8113	17 8401	17 8689	289
151	8977	9264	9552	9839	18 0126	18 0413	18 0699	18 0986	18 1272	18 1558	287
152	18 1844	18 2129	18 2415	18 2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	19 0051	281
155	19 0332	19 0612	19 0892	19 1171	19 1451	19 1730	19 2010	19 2289	19 2567	19 2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	20 0029	20 0303	20 0577	20 0850	20 1124	274
159	20 1397	20 1670	20 1943	20 2216	20 2488	2761	3033	3305	3577	3848	272
160	20 4120	20 4391	20 4663	20 4934	20 5204	20 5475	20 5746	20 6016	20 6286	20 6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	21 0051	21 0319	21 0586	21 0853	21 1121	21 1388	21 1654	21 1921	267
163	21 2188	21 2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	21 7484	21 7747	21 8010	21 8273	21 8536	21 8798	21 9060	21 9323	21 9585	21 9846	262
166	22 0108	22 0370	22 0631	22 0892	22 1153	22 1414	22 1675	22 1936	22 2196	22 2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	23 0193	256
170	23 0449	23 0704	23 0960	23 1215	23 1470	23 1724	23 1979	23 2234	23 2488	23 2742	255
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	24 0050	24 0300	250
174	24 0549	24 0799	24 1048	24 1297	24 1546	24 1795	24 2044	24 2293	2541	2790	249
175	24 3038	24 3286	24 3534	24 3782	24 4030	24 4277	24 4525	24 4772	24 5019	24 5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	25 0176	245
178	25 0420	25 0664	25 0908	25 1151	25 1395	25 1638	25 1881	25 2125	25 2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	25 5273	25 5514	25 5755	25 5996	25 6237	25 6477	25 6718	25 6958	25 7198	25 7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26 0071	26 0310	26 0548	26 0787	26 1025	26 1263	26 1501	26 1739	26 1976	26 2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	26 7172	26 7406	26 7641	26 7875	26 8110	26 8344	26 8578	26 8812	26 9046	26 9279	234
186	9513	9746	9980	27 0213	27 0446	27 0679	27 0912	27 1144	27 1377	27 1609	233
187	27 1842	27 2074	27 2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	27 8754	27 8982	27 9211	27 9439	27 9667	27 9895	28 0123	28 0351	28 0578	28 0806	228
191	28 1033	28 1261	28 1488	28 1715	28 1942	28 2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	29 0035	29 0257	29 0480	29 0702	29 0925	29 1147	29 1369	29 1591	29 1813	29 2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	8853	9071	9289	9507	9725	9943	30 0161	30 0378	30 0595	30 0813	218
200	30 1030	30 1247	30 1464	30 1681	30 1898	30 2114	30 2331	30 2547	30 2764	30 2980	217
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
200	30 1030	30 1247	30 1464	30 1681	30 1898	30 2114	30 2331	30 2547	30 2764	30 2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	9630	9843	31 0056	31 0268	31 0481	31 0693	31 0906	31 1118	31 1330	31 1542	212
205	31 1754	31 1966	31 2177	31 2389	31 2600	31 2812	31 3023	31 3234	31 3445	31 3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	32 0146	32 0354	32 0562	32 0769	32 0977	32 1184	32 1391	32 1598	32 1805	32 2012	207
210	32 2219	32 2426	32 2633	32 2839	32 3046	32 3252	32 3458	32 3665	32 3871	32 4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	8380	8583	8787	8991	9194	9398	9601	9805	33 0008	33 0211	203
214	33 0414	33 0617	33 0819	33 1022	33 1225	33 1427	33 1630	33 1832	2034	2236	202
215	33 2438	33 2640	33 2842	33 3044	33 3246	33 3447	33 3649	33 3850	33 4051	33 4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	8456	8656	8855	9054	9253	9451	9650	9849	34 0047	34 0246	199
219	34 0444	34 0642	34 0841	34 1039	34 1237	34 1435	34 1632	34 1830	2028	2225	198
220	34 2423	34 2620	34 2817	34 3014	34 3212	34 3409	34 3606	34 3802	34 3999	34 4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	35 0054	194
224	35 0248	35 0442	35 0636	35 0829	35 1023	35 1216	35 1410	35 1603	35 1796	1989	193
225	35 2183	35 2375	35 2568	35 2761	35 2954	35 3147	35 3339	35 3532	35 3724	35 3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	9835	36 0025	36 0215	36 0404	36 0593	36 0783	36 0972	36 1161	36 1350	36 1539	189
230	36 1728	36 1917	36 2105	36 2294	36 2482	36 2671	36 2859	36 3048	36 3236	36 3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	37 0143	37 0328	37 0513	37 0698	37 0883	185
235	37 1068	37 1253	37 1437	37 1622	37 1806	37 1991	37 2175	37 2360	37 2544	37 2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	38 0030	181
240	38 0211	38 0392	38 0573	38 0754	38 0934	38 1115	38 1296	38 1476	38 1656	38 1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	178
245	38 9166	38 9343	38 9520	38 9698	38 9875	39 0051	39 0228	39 0405	39 0582	39 0759	177
246	39 0935	39 1112	39 1288	39 1464	39 1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	39 7940	39 8114	39 8287	39 8461	39 8634	39 8808	39 8981	39 9154	39 9328	39 9501	173
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
250	39 7940	39 8114	39 8287	39 8461	39 8634	39 8808	39 8981	39 9154	39 9328	39 9501	173
251	9674	9847	40 0020	40 0192	40 0365	40 0538	40 0711	40 0883	40 1056	40 1228	173
252	40 1401	40 1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	40 6540	40 6710	40 6881	40 7051	40 7221	40 7391	40 7561	40 7731	40 7901	40 8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	9933	41 0102	41 0271	41 0440	41 0609	41 0777	41 0946	41 1114	41 1283	41 1451	169
258	41 1620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
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261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	9956	42 0121	42 0286	42 0451	42 0616	42 0781	42 0945	42 1110	42 1275	42 1439	165
264	42 1604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
265	42 3246	42 3410	42 3574	42 3737	42 3901	42 4065	42 4228	42 4392	42 4555	42 4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	9752	9914	43 0075	43 0236	43 0398	43 0559	43 0720	43 0881	43 1042	43 1203	161
270	43 1364	43 1525	43 1685	43 1846	43 2007	43 2167	43 2328	43 2488	43 2649	43 2809	161
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272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
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281	8706	8861	9015	9170	9324	9478	9633	9787	9941	45 0095	154
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315	49 8311	49 8448	49 8586	49 8724	49 8862	49 8999	49 9137	49 9275	49 9412	49 9550	138
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366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
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368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
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393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	111
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462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	66 7453	66 7546	66 7640	66 7733	66 7826	66 7920	66 8013	66 8106	66 8199	66 8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	67 0060	67 0153	93
468	67 0246	67 0339	67 0431	67 0524	67 0617	67 0710	67 0802	67 0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	67 2098	67 2190	67 2283	67 2375	67 2467	67 2560	67 2652	67 2744	67 2836	67 2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
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475	67 6694	67 6785	67 6876	67 6968	67 7059	67 7151	67 7242	67 7333	67 7424	67 7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
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491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
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494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	69 4605	69 4693	69 4781	69 4868	69 4956	69 5044	69 5131	69 5219	69 5307	69 5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
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514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	71 1807	71 1892	71 1976	71 2060	71 2144	71 2229	71 2313	71 2397	71 2481	71 2566	84
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521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
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530	72 4276	72 4358	72 4440	72 4522	72 4604	72 4685	72 4767	72 4849	72 4931	72 5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
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537	9974	73 0055	73 0136	73 0217	73 0298	73 0378	73 0459	73 0540	73 0621	73 0702	81
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541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
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544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	73 6397	73 6476	73 6556	73 6635	73 6715	73 6795	73 6874	73 6954	73 7034	73 7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
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552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	74 4293	74 4371	74 4449	74 4528	74 4606	74 4684	74 4762	74 4840	74 4919	74 4997	78
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557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
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560	74 8188	74 8266	74 8343	74 8421	74 8498	74 8576	74 8653	74 8731	74 8808	74 8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	75 0045	75 0123	75 0200	75 0277	75 0354	75 0431	77
563	75 0508	75 0586	75 0663	75 0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	75 2048	75 2125	75 2202	75 2279	75 2356	75 2433	75 2509	75 2586	75 2663	75 2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	75 5875	75 5951	75 6027	75 6103	75 6180	75 6256	75 6332	75 6408	75 6484	75 6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
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574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	75 9668	75 9743	75 9819	75 9894	75 9970	76 0045	76 0121	76 0196	76 0272	76 0347	75
576	76 0422	76 0498	76 0573	76 0649	76 0724	0799	0875	0950	1025	1101	75
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578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
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583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
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590	77 0852	77 0926	77 0999	77 1073	77 1146	77 1220	77 1293	77 1367	77 1440	77 1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	77 4517	77 4590	77 4663	77 4736	77 4809	77 4882	77 4955	77 5028	77 5100	77 5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
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610	78 5330	78 5401	78 5472	78 5543	78 5615	78 5686	78 5757	78 5828	78 5899	78 5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
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613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	78 8875	78 8946	78 9016	78 9087	78 9157	78 9228	78 9299	78 9369	78 9440	78 9510	71
616	9581	9651	9722	9792	9863	9933	79 0004	79 0074	79 0144	79 0215	70
617	79 0285	79 0356	79 0426	79 0496	79 0567	79 0637	0707	0778	0848	0918	70
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622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	79 5880	79 5949	79 6019	79 6088	79 6158	79 6227	79 6297	79 6366	79 6436	79 6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
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633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
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636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
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641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
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644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	80 9560	80 9627	80 9694	80 9762	80 9829	80 9896	80 9964	81 0031	81 0098	81 0165	67
646	81 0233	81 0300	81 0367	81 0434	81 0501	81 0569	81 0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	81 2913	81 2980	81 3047	81 3114	81 3181	81 3247	81 3314	81 3381	81 3448	81 3514	67
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652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	81 6241	81 6308	81 6374	81 6440	81 6506	81 6573	81 6639	81 6705	81 6771	81 6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	81 9544	81 9610	81 9676	81 9741	81 9807	81 9873	81 9939	82 0004	82 0070	82 0136	66
661	82 0201	82 0267	82 0333	82 0399	82 0464	82 0530	82 0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
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665	82 2822	82 2887	82 2952	82 3018	82 3083	82 3148	82 3213	82 3279	82 3344	82 3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	82 6075	82 6140	82 6204	82 6269	82 6334	82 6399	82 6464	82 6528	82 6593	82 6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	82 9304	82 9368	82 9432	82 9497	82 9561	82 9625	82 9690	82 9754	82 9818	82 9882	64
676	9947	83 0011	83 0075	83 0139	83 0204	83 0268	83 0332	83 0396	83 0460	83 0525	64
677	83 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	83 2509	83 2573	83 2637	83 2700	83 2764	83 2828	83 2892	83 2956	83 3020	83 3083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	63
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	83 5691	83 5754	83 5817	83 5881	83 5944	83 6007	83 6071	83 6134	83 6197	83 6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	83 8849	83 8912	83 8975	83 9038	83 9101	83 9164	83 9227	83 9289	83 9352	83 9415	63
691	9478	9541	9604	9667	9729	9792	9855	9918	9981	84 0043	63
692	84 0106	84 0169	84 0232	84 0294	84 0357	84 0420	84 0482	84 0545	84 0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	62
695	84 1985	84 2047	84 2110	84 2172	84 2235	84 2297	84 2360	84 2422	84 2484	84 2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
700	84 5098	84 5160	84 5222	84 5284	84 5346	84 5408	84 5470	84 5532	84 5594	84 5656	62
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701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	84 8189	84 8251	84 8312	84 8374	84 8435	84 8497	84 8559	84 8620	84 8682	84 8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85 0033	85 0095	85 0156	85 0217	85 0279	85 0340	85 0401	85 0462	85 0524	85 0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	85 1258	85 1320	85 1381	85 1442	85 1503	85 1564	85 1625	85 1686	85 1747	85 1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	85 4306	85 4367	85 4428	85 4488	85 4549	85 4610	85 4670	85 4731	85 4792	85 4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	61
720	85 7332	85 7393	85 7453	85 7513	85 7574	95 7634	85 7694	85 7755	85 7815	85 7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
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725	86 0338	86 0398	86 0458	86 0518	86 0578	86 0637	86 0697	86 0757	86 0817	86 0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	86 3323	86 3382	86 3442	86 3501	86 3561	86 3620	86 3680	86 3739	86 3799	86 3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	86 6287	86 6346	86 6405	86 6465	86 6524	86 6583	86 6642	86 6701	86 6760	86 6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	86 9232	86 9290	86 9349	86 9408	86 9466	86 9525	86 9584	86 9642	86 9701	86 9760	59
741	9818	9877	9935	9994	87 0053	87 0111	87 0170	87 0228	87 0287	87 0345	59
742	87 0404	87 0462	87 0521	87 0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	87 2156	87 2215	87 2273	87 2331	87 2389	87 2448	87 2506	87 2564	87 2622	87 2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	87 5061	87 5119	87 5177	87 5235	87 5293	87 5351	87 5409	87 5466	87 5524	87 5582	58
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751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	87 7947	87 8004	87 8062	87 8119	87 8177	87 8234	87 8292	87 8349	87 8407	87 8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	88 0013	88 0070	88 0127	88 0185	57
759	88 0242	88 0299	88 0356	88 0413	88 0471	88 0528	0585	0642	0699	0756	57
760	88 0814	88 0871	88 0928	88 0985	88 1042	88 1099	88 1156	88 1213	88 1271	88 1328	57
761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	88 3661	88 3718	88 3775	88 3832	88 3888	88 3945	88 4002	88 4059	88 4115	88 4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	88 6491	88 6547	88 6604	88 6660	88 6716	88 6773	88 6829	88 6885	88 6942	88 6998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	88 9302	88 9358	88 9414	88 9470	88 9526	88 9582	88 9638	88 9694	88 9750	88 9806	56
776	9862	9918	9974	89 0030	89 0086	89 0141	89 0197	89 0253	89 0309	89 0365	56
777	89 0421	89 0477	89 0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	89 2095	89 2150	89 2206	89 2262	89 2317	89 2373	89 2429	89 2484	89 2540	89 2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	89 4870	89 4925	89 4980	89 5036	89 5091	89 5146	89 5201	89 5257	89 5312	89 5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	89 7627	89 7682	89 7737	89 7792	89 7847	89 7902	89 7957	89 8012	89 8067	89 8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	9821	9875	9930	9985	90 0039	90 0094	90 0149	90 0203	90 0258	90 0312	55
795	90 0367	90 0422	90 0476	90 0531	90 0586	90 0640	90 0695	90 0749	90 0804	90 0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	90 3090	90 3144	90 3199	90 3253	90 3307	90 3361	90 3416	90 3470	90 3524	90 3578	54
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800	90 3090	90 3144	90 3199	90 3253	90 3307	90 3361	90 3416	90 3470	90 3524	90 3578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	90 5796	90 5850	90 5904	90 5958	90 6012	90 6066	90 6119	90 6173	90 6227	90 6281	54
806	6335	6389	6443	6497	6551	6664	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	90 8485	90 8539	90 8592	90 8646	90 8699	90 8753	90 8807	90 8860	90 8914	90 8967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	53
812	9556	9610	9663	9716	9770	9823	9877	9930	9984	91 0037	53
813	91 0091	91 0144	91 0197	91 0251	91 0304	91 0358	91 0411	91 0464	91 0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	91 1158	91 1211	91 1264	91 1317	91 1371	91 1424	91 1477	91 1530	91 1584	91 1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	91 3814	91 3867	91 3920	91 3973	91 4026	91 4079	91 4132	91 4184	91 4237	91 4290	53
821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	91 6454	91 6507	91 6559	91 6612	91 6664	91 6717	91 6770	91 6822	91 6875	91 6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	91 9078	91 9130	91 9183	91 9235	91 9287	91 9340	91 9392	91 9444	91 9496	91 9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	92 0019	92 0071	52
832	92 0123	92 0176	92 0228	92 0280	92 0332	92 0384	92 0436	92 0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	· 1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	92 1686	92 1738	92 1790	92 1842	92 1894	92 1946	92 1998	92 2050	92 2102	92 2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	92 4279	92 4331	92 4383	92 4434	92 4486	92 4538	92 4589	92 4641	92 4693	92 4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	92 6857	92 6908	92 6959	92 7011	92 7062	92 7114	92 7165	92 7216	92 7268	92 7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	92 9419	92 9470	92 9521	92 9572	92 9623	92 9674	92 9725	92 9776	92 9827	92 9879	51
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851	9930	9981	93 0032	93 0083	93 0134	93 0185	93 0236	93 0287	93 0338	93 0389	51
852	93 0440	93 0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	93 1966	93 2017	93 2068	93 2118	93 2169	93 2220	93 2271	93 2322	93 2372	93 2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	93 4498	93 4549	93 4599	93 4650	93 4700	93 4751	93 4801	93 4852	93 4902	93 4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	93 7016	93 7066	93 7116	93 7167	93 7217	93 7267	93 7317	93 7367	93 7418	93 7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	93 9519	93 9569	93 9619	93 9669	93 9719	93 9769	93 9819	93 9869	93 9918	93 9968	50
871	94 0018	94 0068	94 0118	94 0168	94 0218	94 0267	94 0317	94 0367	94 0417	94 0467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	94 2008	94 2058	94 2107	94 2157	94 2207	94 2256	94 2306	94 2355	94 2405	94 2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	94 4483	94 4532	94 4581	94 4631	94 4680	94 4729	94 4779	94 4828	94 4877	94 4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	94 6943	94 6992	94 7041	94 7090	94 7139	94 7189	94 7238	94 7287	94 7336	94 7385	49
886	7434	7483	7532	7581	7630	7679	7723	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	94 9390	94 9439	94 9488	94 9536	94 9585	94 9634	94 9683	94 9731	94 9780	94 9829	49
891	9878	9926	9975	95 0024	95 0073	95 0121	95 0170	95 0219	95 0267	95 0316	49
892	95 0365	95 0414	95 0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	95 1823	95 1872	95 1920	95 1969	95 2017	95 2066	95 2114	95 2163	95 2211	95 2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	95 4243	95 4291	95 4339	95 4387	95 4435	95 4484	95 4532	95 4580	95 4628	95 4677	48
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
900	95 4243	95 4291	95 4339	95 4387	95 4435	95 4484	95 4532	95 4580	95 4628	95 4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	95 6649	95 6697	95 6745	95 6793	95 6840	95 6888	95 6936	95 6984	95 7032	95 7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	95 9041	95 9089	95 9137	95 9185	95 9232	95 9280	95 9328	95 9375	95 9423	95 9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	96 0042	96 0090	96 0138	96 0185	96 0233	96 0280	96 0328	96 0376	96 0423	48
913	96 0471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
915	96 1421	96 1469	96 1516	96 1563	96 1611	96 1658	96 1706	96 1753	96 1801	96 1848	48
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	96 3788	96 3835	96 3882	96 3929	96 3977	96 4024	96 4071	96 4118	96 4165	96 4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	· 5766	5813	5860	5907	5954	6001	6048	6095	47
925	96 6142	96 6189	96 6236	96 6283	96 6329	96 6376	96 6423	96 6470	96 6517	96 6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	96 8483	96 8530	96 8576	96 8623	96 8670	96 8716	96 8763	96 8810	96 8856	96 8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	97 0021	97 0068	97 0114	97 0161	97 0207	97 0254	97 0300	47
934	97 0347	97 0393	97 0440	0486	0533	0579	0626	0672	0719	0765	46
935	97 0812	97 0858	97 0904	97 0951	97 0997	97 1044	97 1090	97 1137	97 1183	97 1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
940	97 3128	97 3174	97 3220	97 3266	97 3312	97 3359	97 3405	97 3451	97 3497	97 3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	97 5432	97 5478	97 5524	97 5570	97 5616	97 5662	97 5707	97 5753	97 5799	97 5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	97 7724	97 7769	97 7815	97 7861	97 7906	97 7952	97 7998	97 8043	97 8089	97 8135	46
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
950	97 7724	97 7769	97 7815	97 7861	97 7906	97 7952	97 7998	97 8043	97 8089	97 8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	98 0003	98 0049	98 0094	98 0140	98 0185	98 0231	98 0276	98 0322	98 0367	98 0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	98 2271	98 2316	98 2362	98 2407	98 2452	98 2497	98 2543	98 2588	98 2633	98 2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	98 4527	98 4572	98 4617	98 4662	98 4707	98 4752	98 4797	98 4842	98 4887	98 4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	98 6772	98 6817	98 6861	98 6906	98 6951	98 6996	98 7040	98 7085	98 7130	98 7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	98 9005	98 9049	98 9094	98 9138	98 9183	98 9227	98 9272	98 9316	98 9361	98 9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	99 0028	99 0072	99 0117	99 0161	99 0206	99 0250	99 0294	44
978	99 0339	99 0383	99 0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	99 1226	99 1270	99 1315	99 1359	99 1403	99 1448	99 1492	99 1536	99 1580	99 1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	99 3436	99 3480	99 3524	99 3568	99 3613	99 3657	99 3701	99 3745	99 3789	99 3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	99 5635	99 5679	99 5723	99 5767	99 5811	99 5854	99 5898	99 5942	99 5986	99 6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	99 7823	99 7867	99 7910	99 7954	99 7998	99 8041	99 8085	99 8129	99 8172	99 8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

PRACTICAL MATHEMATICS
Section 10

EQUATIONS—FORMULAS

L e s s o n 1

For Step 1, keep in mind the contents of the Introduction. For Step 2, learn the meaning and purpose of formulas and equations and the rules governing the solution of equations. For Step 3, work the Illustrative Examples. For Step 4, work the Practice Problems.

An **equation** is a statement of equality between two quantities. You have already learned that when the sign of equality (=) is used, the quantities on one side of that sign equal or balance the quantities on the other side. Therefore, when the equality sign is used, you have an equation.

Expressed in other words this simply means that an **equation** is a means of showing that two numbers or two groups of numbers are equal to the same amount.

In proportion principles, which you have already studied, you learned that the ratio on one side of the equality sign must equal the ratio on the other side. For instance, in the proportion 3:6::4:8 the ratio $\frac{3}{6}$ is equal to the ratio $\frac{4}{8}$, because $\frac{3}{6}$ when reduced to lowest terms equals $\frac{1}{2}$ and $\frac{4}{8}$ reduced to lowest terms equals $\frac{1}{2}$.

This can be expressed or written $\frac{3}{6} = \frac{4}{8}$. This is said, therefore, to **balance.** At this point imagine that Fig. 1 is a rough drawing of a scale such as is used to weigh groceries.
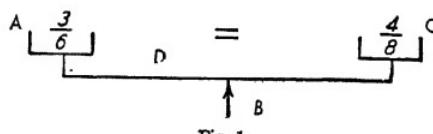
Fig. 1

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The weighing pans are A and C. The balance arm is D. At B is the balance point. In order for the arm D to be perfectly horizontal, the weights in both pans, A and C, must be equal. This simple balance principle can be used to illustrate equations.

In pan A we have $\frac{3}{6}$ and in pan C we have $\frac{4}{8}$. Because both of these fractions are equal (both being equal to $\frac{1}{2}$) we can think of them as being **balanced**. This doesn't mean that they weigh the same, but it does mean that they are **equal**. So we can now state the rule that all **equations must balance**.

With this in mind, it is easily seen that $\frac{3}{6} = \frac{4}{8}$ is an equation. Both fractions are equal to the same thing, or, in other words they balance. Other simple equations are illustrated in the multiplication tables. Thus, $8 \times 7 = 56$ is an equation, because 8×7 is 56. Or, if we put 8×7 in the A pan and 56 in the C pan, in Fig. 1, the scale would balance, because 8×7 is exactly the same as 56. Other forms of simple equations are as follows: $6 + 5 = 11$; $12 - 5 = 7$; $20 \div 5 = 4$; $.6 \times .4 = .24$; $\sqrt{4+5} = 1 + \sqrt{4}$; $3^2 - 2 = 2^3 - 1$. Thus, we see that equations may have various numbers on either side of the equal sign, connected by $+$, $-$, $\times$, $\div$, root, or power signs. But whatever combination of numbers are shown on either side of the equal sign, they must balance in order to become an equation. The student should make sure he understands the explanation of equations up to this point before going ahead.

A **formula** is an equation which contains or means a rule or principle. In electrical work we know that current equals electromotive force divided by resistance. This is really a rule or principle because it always holds true. We can write this rule in equation form as follows:

$$I = \frac{E}{R}$$

I means current. E means electromotive force. R means resistance. This is an equation because it balances. In Fig. 2, the letter I can be assumed as being in pan A and $\frac{E}{R}$ in pan C. This is also called **a formula**.

The only difference between an equation and a formula is that

$$\boxed{I} \qquad = \qquad \boxed{\frac{E}{R}}$$

A _____ B

Fig. 2

in an equation the numbers generally haven't any particular meaning, whereas if an equation is composed of letters that mean something such as $I = \dfrac{E}{R}$ then it is called a formula. In other words a formula is a rule. In formula $I = \dfrac{E}{R}$, we know this is a rule for finding the current when we already know the electromotive force and resistance.

7+2=4+5 is an equation because it balances. The numbers have no particular meaning. $I = \dfrac{E}{R}$ is a formula because the letters all mean definite things and because it is a rule for finding the current.

As far as balance is concerned, or as far as method of working is concerned, equations and formulas are the same.

Formulas are the same as rules. We can say—**The current is equal to the electromotive force divided by the resistance.** But, this is a long and time taking rule to write each time we want to show it, so in all types of engineering work where we have given such names as **Current, Electromotive Force,** and **Resistance,** we substitute letters for them so, whenever we see these letters, we know what they mean. This allows us to write rules, as above, in short and easily written formulas.

Now, when we have formulas written in a simple form we can substitute actual numerical values for them and "solve the problem." The function of these lessons is to teach you how to substitute actual values in formulas and to solve such problems.

As a further illustration of balance, study the following material in Fig. 3. Here we have shown six different examples by first stating the form in words and then showing it by figures and also showing the balance by means of sketches.

Two added to three is equal to five
In formula form this is written $2+3=5$

From five take three and the result is equal to two
In formula form this is written $5-3=2$

Multiply two by three and the result is equal to six
In formula form this is written $2\times3=6$

Six divided by three is equal to two
In formula form this is written $\frac{6}{3}=2$

Two raised to the second power is equal to four
In formula form this is written $2^2=4$

The square root of four is equal to two
In formula form this is written $\sqrt{4}=2$

$$2+3 \qquad = \qquad 5$$
$$5-3 \qquad = \qquad 2$$
$$2\times3 \qquad = \qquad 6$$
$$6\div3 \qquad = \qquad 2$$
$$2^2 \qquad = \qquad 4$$
$$\sqrt{4} \qquad = \qquad 2$$

Fig. 3

When we use the formula $I=\dfrac{E}{R}$, we must substitute actual values in place of the letters before we can work it out in order to find one or another of the parts such as I, E, or R. Whenever we have such an actual problem to solve, where we have to substitute real values for these letters, it is a simple matter to tell what I, E, or R means as will be explained later. When we have substituted real values in the formula, the equation must balance or it is not correct.

Suppose we know that $I=20$, $E=120$, and $R=6$. Then the formula $I=\dfrac{E}{R}$ becomes $20=\dfrac{120}{6}$ because we have substituted the numbers in place of letters, like this

$$\overset{20}{\cancel{I}} = \dfrac{\overset{120}{\cancel{E}}}{\underset{6}{\cancel{R}}}$$

Here we first wrote down the formula and then crossed out the I and put the 20 in its place. Then we crossed out the E and put the 120 in its place. Also we crossed out the R and put 6 in its place. Such a procedure is called **substituting**. In this case, where we

know the value, in numbers, of all the letters in the formula we quickly see that the resulting equation balances because

$$\frac{120}{6} = 120 \div 6 = 20.$$

So $20 = 20$, or the equation balances.

Now we come to the point where we can explain the real use of formulas and equations. (Remember that a formula is where only letters are given and an equation is where we have substituted actual number values for the letters.) In actual practice, when solving such formulas as $I = \dfrac{E}{R}$ we never know the value of more than two of the three letters. In other words there is always one letter the actual value of which we do not know. For example, we may know that the current is 40 and that the resistance is 12. But we do not know what the value of E is. The formula to start with is

$$I = \frac{E}{R}$$

Then we substitute the known values

$$\overset{40}{\cancel{I}} = \frac{E}{\cancel{R}} \\ {}_{12}$$

This becomes—

$$40 = \frac{E}{12}$$

The problem is then to find, by calculation, the value of E. Or, by another variation we may have

$$40 = \frac{240}{R}$$

Here we must find the value of R by calculation.

At this point the student can begin to see how equations and formulas are used to a great extent. In the following text material, we will study the means or ways of solving equations and formulas.

PRACTICAL MATHEMATICS

Lesson 2

For Step 1, recall the principles involved in formulas and equations. For Step 2, learn the method of solving formulas and equations by placing the unknown by itself on one side of the sign of equality. For Step 3, study the Illustrative Examples. For Step 4, work the Practice Problems.

In $2+3=5$ we have an equation. (It is called an **equation** because it isn't stating any rule and because the numbers do not mean anything in particular.) In this equation we know every number and the equation balances. But, suppose we had $2+?=5$. We do not at first know the value of the "?." We say to ourselves, "What number must be added to 2 to get 5?" As the equation stands, it does not balance. We are to find the missing number which added to 2 will make the equation balance. In this simple illustration we can immediately see that the missing number is 3. Then $2+3=5$ and the equation balances. Now, what we really did was to find the missing number which subtracted from 5 gave 2. Finding this value is solving the equation.

$$(1) \quad 2+?=5$$
$$\text{or}$$
$$(2) \quad 2=5-?$$
$$(3) \quad 2=5-3$$

In the third equation we have shown that 3 is the required number. To do this, we have **changed the equation around** a little, as will be explained further.

You learned how to calculate the area of a square or rectangular field in the book on Denominate Numbers. If the area of a rectangle is 36 and one side is 9 and the unknown side is ?, then $?\times9=36$, or $?=36\div9=4$. Thus you see an unknown number can be found if enough of the other numbers are known. This feature was also discussed in the lesson on Proportion. The number missing is called the **unknown.** The part on the left side of the $=$ sign in a formula is called the **first member** and the part on the right side of the $=$ sign is called the **second member.** Also in some cases the part on the left side of the equal sign is called the "left side of the equation" and the part on the right side of the equal sign is called the "right side of the equation."

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There are a few necessary rules to follow in solving equations and formulas. These will be given and illustrated after which we can go ahead with practical illustrations.

Rule (A). *The letters that occur in a formula or equation must be given such numerical values as will make both members numerically equal when these values are substituted in the formula.*

As an illustration of Rule (A)

Consider the equation $\qquad$ $12+6=2\times9$
Simplified, this becomes $\qquad$ $18=18$

Here it can be assumed that the values of the equation have been substituted, resulting in $12+6=2\times9$. Both members (that is first and second or left and right members) are equal because when simplified both members equal 18. To simplify, means to add the 12 and 6 and to multiply 2×9. Or, it can be assumed to mean performing what the signs indicate. The signs are $+$, $-$, $\times$, etc.

Rule (B). *The same quantity may be added to or subtracted from both members without unbalancing the equation.*

As an illustration of Rule (B)

(1) Add 10 to each side $\qquad$ $18+10=18+10$
And the equation is still true $\qquad$ or $28=28$

In other words the equation still balances.

(2) Subtract 10 from each side $\qquad$ $18-10=18-10$
The final result is still true $\qquad$ or $8=8$

In other words the equation still balances.

(1) We added 10 to each side of the simplified equation using Rule (B). It is easily seen that after this has been done the equation still balances.

(2) We subtracted 10 from 18 in both sides of the equation, Rule (B), and find after simplifying that $8=8$ so our equation still balances. It should be noted that in the above illustrative examples the amount changes such as $18=18$ and $8=8$, etc., but the main point is that no matter which of these examples we apply the equation *still balances.*

Rule (C). *Both members of a formula or equation may be multiplied or divided by the same quantity without destroying the balance.*

As an illustration of Rule (C)

(1) Multiply both sides by 7 Again the result is still true	$18 \times 7 = 18 \times 7$ or $126 = 126$
(2) Divide each side by 9 The result is still equal	$18 \div 9 = 18 \div 9$ or $2 = 2$

(1) We multiplied both sides of the simplified equation, Rule (C), by 7 and the equation still balances.

(2) We divided both sides of the same equation, Rule (C), by 9 and it still balances.

Rule (D). *Both members of a formula or equation may be raised to the same power without destroying the balance.*

As an illustration of Rule (D)

Suppose we again take the simplified equation given under Rule (A). We have $18 = 18$. Now raise both to the third power. $18^3 = 18^3$ or $5832 = 5832$. Also $18^4 = 18^4$, $18^7 = 18^7$, etc. The equation will still balance.

Rule (E). *The same root may be extracted in both members without destroying the equality.*

As an illustration of Rule (E)

We will take the same simplified equation $18 = 18$. If we take the square root of each side we have $\sqrt{18} = \sqrt{18}$ or $4.2426 = 4.2426$ or we could say $\sqrt[3]{18} = \sqrt[3]{18}$ or $\sqrt[63]{18} = \sqrt[63]{18}$. In all cases the equation balances.

Rule (F). *The order of the numbers or the order of the letters is not important if the proper sign is given to each.*

As an illustration of Rule (F)

We may change the location of the letters or numbers on the same side of the equation if we carry their signs with them.

(1) For example:	$(6 \times 8) - 6 - 20 = 22$
(2) Changing locations	$-6 + (6 \times 8) - 20 = 22$
(3) or	$-20 + (6 \times 8) - 6 = 22$

At (1) we have an equation that balances because $6 \times 8 = 48 - 6 = 42$ and $-20 = 22$. Thus $22 = 22$. The parentheses ( ) around the 6×8 mean that 6×8 must be multiplied before subtracting the -6. At (2) the second member or right side of the equation remains un-

changed. But the first member or left side of the equation has been changed insofar as the -6 being moved so it comes before the (6×8) instead of after it. The results are still correct because $-6 + (6 \times 8) = -6 + 48 = 42$. And $42 - 20 = 22$. In (3) the -20 has been moved but the equation still balances.

Remember that we can change the location of letters or numbers on the same side of the equation without changing signs. But if we move a number from the left side of the equation to the right side, or vice versa, it becomes a different matter as will be explained by the following rule.

Rule (G). *Numbers and letters can be moved or shifted from one member or side of an equation to the other member or side if their signs are changed from $+$ to $-$ or from $-$ to $+$.* This is called **transposition.** As an illustration of Rule (G)

The process of transposition is constantly used in the solution of formulas and equations. When there is no sign in front of a number or letter, $+$ is understood. Thus, 8 means $+8$; 12 means $+12$.

ILLUSTRATIVE EXAMPLES

1. Transpose the first number in the left member of the equation, $7 + 5 - 3 = 6 + 3$, to the right member.
Here we have

$$7 + 5 - 3 = 6 + 3$$

Transposing we get

$$+5 - 3 = 6 + 3 - 7$$

or

$$+2 = +2$$

The arrow shows how the transposition was done. Rule (G) says that the sign must be changed from $+$ to minus when a number is shifted from one side of the equation to the other. The 7 without any sign means $+7$. Following the rule we changed the sign to $(-)$ when we shifted the 7. Another rule meaning the same as Rule (G) is: *Whenever a number is moved or shifted from one side of the equation to the other, its sign must be changed to the opposite of what it was before the move or shift was made.*

2. Transpose the second and third numbers in the right member of the equation, $2 + 2 = 3 + 2 - 1$, to the left member.

Here we have

$$2+2=3+2-1$$

Transposing we get

$$2+2-2+1=3$$

or

$$3=3$$

The arrows show how the shift or transposing was done. The signs were changed to the opposite of what they were originally.

In both examples 1 and 2 the results balance perfectly.

Rule (H). *When transposing or shifting numbers from one side of the equation to the other side, the following two methods must be remembered.*

Method (1).

Any number or numbers being shifted or transposed from the first member or left side of the equation to the second member or right side of the equation must be annexed or written **after** what was originally on the right side of the equation. Thus in example 1, page 9, the 7 goes from the left side of the equation to the right side of the equation and is annexed or written after the 6+3.

Method (2).

Any number or numbers being shifted or transposed from the second member or right side of the equation to the first member or left side of the equation must be annexed or written **after** what is already in the first or left member of the equation. Thus, in example 2, page 10, the 2 and 1 are written after the 2+2.

To Sum Up

We can change or shift the numbers around on one side of an equation, or the other side, as much as we please just so long as we make sure their signs are not changed.

But, when we shift or transpose the numbers from one side of an equation to the opposite side, Methods (1) and (2) must be remembered and the plus or minus signs must be changed to the opposite of what they were before the shift or move was made.

These rules are not difficult if the student will remember them and refer to them often. The following practice problems should

now be worked out, but not until the student thoroughly understands all of Lesson 2 perfectly. This may require lots of time and hard study. Be sure to work *all* problems.

PRACTICE PROBLEMS

Work the problems without looking at the answers.
Transpose the last number in the right member of the equation to the left of the = sign:

1.	$2+4=5+1$	Ans. $2+4-1=5$
2.	$3+5=9-1$	Ans. $3+5+1=9$
3.	$7-2=8-3$	Ans. $7-2+3=8$
4.	$6-5=0+1$	Ans. $6-5-1=0$

Transpose the first number in the left member to the right member:

5.	$6-5=1-0$	Ans. $-5=1-0-6$
6.	$8+10=12+6$	Ans. $+10=12+6-8$
7.	$5-9+1=-3$	Ans. $-9+1=-3-5$

Lesson 3

For Step 1, keep in mind the meaning of the words "unknown," "term," "expression," and their relation to other numbers with which they are found. For Step 2, learn how letters are used to indicate certain values in formulas and how to solve a simple formula. For Step 3, work the Illustrative Examples. For Step 4, work the Practice Problems.

Having learned, in Lessons 1 and 2, what equations and formulas are and the rules for shifting letters or numbers around in equations, and how to transpose numbers, we can now go ahead another step and learn how to solve an equation when all but one of the quantities or parts are known. In Problems 1 and 2, pages 9 and 10, all quantities were known. In other words, there were no missing or unknown parts. In actual practice where formulas and equations are used in engineering work, for example, the equations are formed from standard formulas and in every case there is one part unknown as indicated on page 5. Thus, we see the real benefit of equations and of transposing. The basic use of equations is to find missing parts or unknown numbers, as will be shown later.

When all but one of the quantities of a formula or equation are given, we can often make use of the principle of transposition

to solve the equation. In the equation $2 + ? = 5$, we found that the value of "?" is 3, through a simple process of reasoning; but when the formulas are more complicated, we can make use of the principle of transposition.

ILLUSTRATIVE EXAMPLES

1. $? + 3\frac{12}{27} = 6$

Here we have the problem of finding the value of the "?."

Rule (I). *To find the missing number or to balance the equation proceed as follows: If the unknown number (represented by "?") is on the left side of the equation (or on the left side of = sign), let it remain there but move all other known numbers to the right side of the equation. If the missing number is on the right side of the equation, move it to the left side and move all known numbers so they will all be on the right side of the equation.* (Remember Rules (F) and (G) and Methods (1) and (2) following Rule (H)).

Now, to solve Problem 1 we write it out as given

$$? + 3\frac{12}{27} = 6$$

$$? = 6 - 3\frac{12}{27}$$

Then following first part of Rule (I) we leave the (?) where it is and move the $3\frac{12}{27}$ to the right side of the equation.
The arrow shows how the $3\frac{12}{27}$ was moved from the left to the right side of the equation. We changed the sign from $+$ to $-$ applying Rule (G). The $-3\frac{12}{27}$ was put after the 6 applying Method (1) following Rule (H).

Proof. Now if we actually subtract $3\frac{12}{27}$ we get $2\frac{15}{27}$. Thus the "?" becomes $2\frac{15}{27}$. Then

$$2\frac{15}{27} = 6 - 3\frac{12}{27}$$

and our equation balances and we have solved the problem. Now go back over Problem 1 again to make sure you understand it.

2. $6 = ? + 2\frac{15}{27}$

Here we have another equation in which we have a missing number which is represented by "?." To work Problem 2, we use the last part of Rule (I). Thus the "?" is moved to the left side of the equation and the 6 is moved to the right side.

We write the problem as given

$$6 = ? + 2\tfrac{1}{2}\tfrac{5}{7}$$

Then we follow Rule (I)

$$-? = 2\tfrac{1}{2}\tfrac{5}{7} - 6$$

The "?" becomes $-?$ applying Rule (G). The 6 becomes -6 also applying Rule (G). The -6 is placed after the $2\tfrac{1}{2}\tfrac{5}{7}$ applying Method (1) following Rule (H). Solving by arithmetic we find that $2\tfrac{1}{2}\tfrac{5}{7} - 6 = -3\tfrac{1}{2}\tfrac{2}{7}$. Then $-? = -3\tfrac{1}{2}\tfrac{2}{7}$ or $? = 3\tfrac{1}{2}\tfrac{2}{7}$. Thus the $3\tfrac{1}{2}\tfrac{2}{7}$ is the quantity or missing part we wanted. It might be explained that $2\tfrac{1}{2}\tfrac{5}{7} - 6$ is the same as $-6 + 2\tfrac{1}{2}\tfrac{5}{7}$.

Rule (J). *In cases like the above problem where* $-? = -3\tfrac{1}{2}\tfrac{2}{7}$, *we have a minus sign for both sides so in the final answer it becomes plus.*

We can also work Problem 2 in another way as follows:
Using our sign principles, we change the $2\tfrac{1}{2}\tfrac{5}{7}$ from the right to the left of the equal sign and leave the ? all by itself. Thus, $6 - 2\tfrac{1}{2}\tfrac{5}{7} = ?$
Solving this left side by Arithmetic, we get $3\tfrac{1}{2}\tfrac{2}{7} = ?$ or, in words, $3\tfrac{1}{2}\tfrac{2}{7}$ is the quantity we needed in place of the question mark in order to have all the facts or to balance the equation.
Here, to balance the equation, the number missing was on the right side of the equal sign, and we changed all the known numbers that were on the right to the left of the equal sign and thus left the question mark all by itself on the right side. We solved the left side by Arithmetic, and the answer was the value of the question mark.

Either way of solving Problem 2 is correct. The first method of solution uses the second part of Rule (I) whereas the second method of solution doesn't use Rule (I). The student is advised to become familiar with both methods of solution for problems such as Problem 2 where the unknown or missing quantity is on the right side of the equation to start with. When the missing quantity is on the left side of the equation to start with, as in Problem 1, then always use the method of solution given for that problem.

You have already noticed that when a number is all by itself and it is $+$, the sign is omitted. Also, when a number is the first of a group of numbers on one side of the equal sign, and it is $+$, the

sign is omitted. This is just a convention for simplicity. The $+$ sign is the only one that may be omitted, the $-$ sign must always be written, and the $+$ sign may be omitted only when, as already stated, the number is all by itself or is the first of the group.

$$+3-2=+1 \text{ is the same as } 3-2=1$$
$$-3+4=+1 \text{ is the same as } -3+4=1$$

PRACTICE PROBLEMS

Find the value of ? in each equation by the method of leaving the ? by itself. In Problems 1, 2, 4, and 5 use both methods of solution as explained for Problem 2, page 12.

1.	$5+3+1=8+?$	Ans. $?=1$
2.	$6+1+5=10+?$	Ans. $?=2$
3.	$?+7-2=8+0$	Ans. $?=3$
4.	$7-2+3=?+8$	Ans. $?=0$
5.	$2\frac{1}{2}+?=7$	Ans. $?=4\frac{1}{2}$
6.	$3\frac{1}{15}+?=4+1$	Ans. $?=1\frac{14}{15}$
7.	$1\frac{1}{3}-\frac{2}{3}+?=1+7-3-4$	Ans. $?=\frac{1}{3}$

Lesson 4

For Step 1, recall the method of substituting given values for letters in a simple formula. For Step 2, learn how to indicate the multiplication of two or more letters, and how this principle is used in formulas. For Step 3, work the Illustrative Examples. For Step 4, work the Practice Problems.

So far, we have used the question mark (?) for the missing number. Instead of the question mark, we could have used any other symbol or mark. The question mark was an arbitrary symbol selected by us. The symbol generally used is one of the letters of the alphabet. Suppose we select the letter x. Then, we should write the equation $5+3+1=8+?$ in this form: $5+3+1=8+x$. In the same way in the other equations, we could have substituted an x for the question mark.

Also, we have used the words "number" and "figure" to designate the quantities separated by $+$ and $-$ signs. The proper name is **term** and we shall use it from now on. Thus in the equation $5-3+2=8-x$, each one of the numbers, 5, 3, 2, 8, and the letter x, is called a term. A term must always have a $+$ or $-$ sign before

it either expressed or understood. (We have mentioned the cases where + may be omitted.) A group of related terms is usually called an **expression.** If a number is enclosed with other members within a parentheses, ( ), the part within the parentheses must be considered a single quantity or unit. The various terms cannot be dealt with separately. Thus, in the expression $3+(2+5-7)$, the part $(2+5-7)$ is a unit; and in $6-(3-8)$, the part $(3-8)$ is a unit. Similarly if several terms form the numerator or denominator of a fraction, each expression must be considered a single quantity. For example, in $4+\dfrac{5-3}{2+5}$, the 5 and the 3 of the numerator are inseparable, as are also 2 and 5 in the denominator. They are as much a unit as are 4 and 7 and 6 and 4 in the fraction $\frac{4+7}{6+4}$. You could not deal with the 4 apart from the 7 nor with the 6 apart from the 4.

So far we have used numbers only in our equations, with the exception of x, the term to be found. We could just as well have used the names of the figures instead. For instance, we could have stated $5+3+1=8+x$ as follows: Five + three + one = eight + x. Still further, we could have used the first letter of each one of the words instead of the word itself, thus, $F+T+O=E+x$, provided that we always kept in mind throughout the operations that each letter stood for the name of the number used, and naturally for the number itself.

This is just exactly what we do in writing formulas. We use letters to indicate names or quantities and connect them by the signs of operation, just as we did with our numerical practice formulas.

To solve these formulas with letters, we first use the numerical values of all the letters possible. We substitute these numerical values for the letters in the formula and then solve for the letter that remains, and for which we have no numerical value, in the same way that we solved for "?" in our practice formulas.

Note: The student should understand that formulas can be solved, even though the meaning of the letters is not known. So long as we know the values of a sufficient number of the letters, we can solve the formula. The meaning of the letters is given in the formulas that are discussed in this book, but that information is not necessary to the solution of the formula itself. It is necessary, however, in order to solve a written problem involving a formula.

In the many different branches of engineering, for example, we make use of formulas and equations very frequently. By actual tests, engineers have found that one formula will hold good for any condition. Just to illustrate this, we will use a formula connected with electrical engineering. It is not necessary that the student understand the principles of electricity to work with and understand these mathematical formulas. We will take the formula $I = \dfrac{E}{R}$.

We used this same formula on pages 2 and 5 in explaining what formulas were.

In the above formula, the relationship is always exactly the same. This was proven by actual tests many years ago. So, in electrical work, if we know the exact values of E and R, we can always find I. If we know the values of I and R, we can find E, and if we know the values of I and E, we can always find R. Thus, it can be seen that formulas save much time because without them we would have to make actual tests, at great expense of time and money, to find I, E, or R depending on which one was missing or unknown. We will try a few illustrations to see how the formulas work.

ILLUSTRATIVE EXAMPLES

1. If $E = 50$ and $R = 5$, what is the value of I?

Note: In electrical work this is a typical problem. It often happens that we know the value of E and R and desire to find the value of I.

To solve Problem 1, we take formula

$$I = \frac{E}{R}$$

The first step in solving Problem 1 is to substitute the known values for the letters of known value.

$$I = \frac{\overset{50}{\cancel{E}}}{\underset{5}{\cancel{R}}}$$

In place of the E we put 50 and in place of the R we put 5. Then we have

$$I = \frac{50}{5}$$

The next step is to perform all indicated operations. The 50 over 5 means 50 divided by 5. Whenever you see one number over another number, it indicates that the top number is to be divided by the lower number.

$$50 \div 5 = 10$$

2. If $R = 5$ and $I = 5$, what is the value of E?

The solution of Problem 2 is a little more difficult, but it requires only the things you have learned in Lessons 1, 2, and 3. The formula is

$$I = \frac{E}{R}$$

Substitute the known values for R and I in the formula

$$\overset{5}{\cancel{I}} = \frac{E}{\cancel{R}} \quad \text{or} \quad 5 = \frac{E}{5}$$

Now, we want to find the value of E. To do this we perform what is called "clearing of fractions." The $\frac{E}{5}$ is the fraction. Turn back to Rule (C) on page 7 and review it carefully. This rule says we can multiply both sides or members of an equation by the same quantity. In cases like this problem it is best to use as that quantity a number equal to the denominator of the fraction. So, we will multiply both sides of the equation by 5, and get

$$5 \times 5 = \frac{E}{5} \times 5$$

Now we can cancel

$$5 \times 5 = \frac{E}{\cancel{5}} \times \cancel{5}$$

because the upper 5 is the same as saying $\frac{5}{1}$. (If cancellation is not clear refer to Section 2.)

Now we have

$$5 \times 5 = E \quad \text{or} \quad 25 = E$$

3. If $I = 1$ and $E = 32$, what is the value of R?
The formula is

$$I = \frac{E}{R}$$

Substituting

$$\cancel{I} = \frac{\cancel{E}^{32}}{R} \quad \text{or} \quad 1 = \frac{32}{R}$$

Refer back to Rule (C) again. We can divide both sides of the equation by the same quantity without destroying the balance. Thus

$$\frac{1}{32} = \frac{32}{R \times 32}$$

$\frac{1}{32}$ is the same as $1 \div 32$. We put it in the form of a fraction so that we can cancel. When we divide $\frac{32}{R}$ by 32, we write it as shown because R is already the denominator of $\frac{32}{R}$ and not knowing the value of R we can't do anything more than indicate that R must be multiplied by 32. We use 32 as a divider because there is already a 32 in the equation and by using another one we can make cancellation possible.
Cancelling—

$$\frac{1}{32} \quad \frac{\overset{1}{\cancel{32}}}{R \times \overset{;}{}}$$

So

$$\frac{1}{32} = \frac{1}{R}$$

Now, if $\frac{1}{32} = \frac{1}{R}$ then 32 must $= R$. Ans. is 32 or $R = 32$.

In the formula $I = \frac{E}{R}$, there are three letters and any one of

the letters may be unknown. In Problems 1, 2, and 3 the method for solving for all three has been given and explained. No matter what the values of the letters in this formula are the method of solving is exactly the same as explained for the three problems. Therefore the student should memorize these three solutions the same as a rule.

PRACTICE PROBLEMS

1. If $E = 80$ and $R = 40$, what is I? Ans. $I = 2$
2. If $R = 15$ and $I = 15$, what is E? Ans. $E = 225$
3. If $I = 3$ and $E = 96$, what is R?   Ans. $R = 32$

ILLUSTRATIVE EXAMPLES

When you studied percentage, you learned three rules—one to find the percentage, one to find the base, and one to find the rate. Now that you know how to solve formulas, you may use only one of the rules and get all the parts from it. The formula is this:

$$\text{Percentage} = \frac{\text{Rate} \times \text{Base}}{100} \quad \text{or in symbols: } P = \frac{R \times B}{100}$$

1. Find P, when $R = 15$ and $B = 5000$
The formula is

$$P = \frac{R \times B}{100}$$

Note: The figure 100 is called a **constant**. No matter whether we are solving to find P, R, or B this constant remains in the same position. In Percentage you learned that per cents were in terms of 100. That is why the 100 is used.

To go on with the problem, the first step is substitution. Substituting the given values for R and B

$$P = \frac{\overset{15}{\cancel{R}} \times \overset{5000}{\cancel{B}}}{100}$$

or

$$P = \frac{15 \times 5000}{100}$$

Next we can cancel

$$P = \frac{15 \times \overset{50}{\cancel{5000}}}{\cancel{100}}$$

Then $P = 15 \times 50$ or 750.

    2. When P is 300, B is 15000, what is R?

Solution

Instruction	*Operation*

Step 1

Write the formula

Step 1

$$P = \frac{R \times B}{100}$$

Substitute the given values for P and B in the formula

$$300 = \frac{R \times 15000}{100}$$

Step 2

To free the equation from fractions, multiply both members by 100, since 100 is the denominator of the fraction. Rule (C).

Step 2

$$300 \times 100 = \frac{R \times 15000 \times 100}{100}$$

Step 3

Perform the cancellation

Step 3

$$300 \times 100 = \frac{R \times 15000 \times \cancel{100}}{\cancel{100}}$$

$$30000 = R \times 15000$$

Step 4

If 15000 times R is 30000,
$R = 30000 \div 15000$
R is 2% Ans.

Step 4

$$30000 \div 15000 = 2$$

    3. Given: $P = 250$ and $R = 10$. Find the value of B.

Solution

Instruction	*Operation*

Step 1

Formula is

Step 1

$$P = \frac{R \times B}{100}$$

Substitute the given values for P and R in the formula

$$250 = \frac{10 \times B}{100}$$

Step 2

Step 2

Multiply both members of the equation by 100 to clear of fraction. Rule (C).

$$250 \times 100 = \frac{10 \times B \times 100}{100}$$

Step 3

Step 3

Perform the cancellation

$$250 \times 100 = \frac{10 \times B \times \cancel{100}}{\cancel{100}}$$

$$25000 = 10 \times B$$

Step 4

Step 4

If 10 times B is 25000, B =
25000 ÷ 10
B = 2500   Ans.

$$25000 \div 10 = 2500$$

Note: $B \times 10$ means B taken 10 times (or 10 B's); $B \times 150$ means B taken 150 times (or 150 B's). We may omit the sign $\times$ between the figure and the letter and indicate the multiplication by placing the figure first and the letter next without any sign between. Then 10 times B would be written as $10B$; 150 times B as $150B$; 50 times X as $50X$, and so on.

PRACTICE PROBLEMS

1. Given formula $P = \dfrac{R \times B}{100}$ R is 9,

B is 3000.   Find P.   270   Ans.

2. Using the formula for Problem 1, find R when $P = 30$ and $B = 1000$.   3%   Ans.

Note: The student may have already noticed that the letter R has appeared in two different formulas. This is perfectly all right because R has a different meaning in each formula. In electrical work R always means one thing and in percentage work R always means rate.

Since letters stand for values or numbers, we may indicate the multiplication of two or more letters by placing them beside each other without any sign between them. For example, take the formula for the horsepower of a steam engine which is $HP = \dfrac{PLAN}{33000}$. This means that the values of P, of L, of A, and of N are to be

multiplied together and then this result is to be divided by 33000, to get the amount of horsepower, or the value of HP. The 33000 is a **constant** and appears whenever this formula is used. It is part of the formula and was created at the time the formula was made by actual test.

ILLUSTRATIVE EXAMPLES

1. Find HP when $P = 75$, $L = 1.33$, $A = 78.54$, and $N = 180$.

Substituting these values in the formula $HP = \dfrac{PLAN}{33000}$, we have

$$HP = \frac{\overset{75}{\cancel{P}}\ \overset{1.33}{\cancel{L}}\ \overset{78.54}{\cancel{A}}\ \overset{180}{\cancel{N}}}{33000}$$

or

$$HP = \frac{75 \times 1.33 \times 78.54 \times 180}{33000}$$

All of these numbers must be multiplied starting at the left.

$75 \times 1.33 = 99.75$
$99.75 \times 78.54 = 7834.3650$
$7834.3650 \times 180 = 1410185.7000$

This figure 1410185.7000 is then divided by 33000.

$$1410185.7000 \div 33000 = 42.7329$$

Thus $HP = 42.7329$

2. In the formula $HP = \dfrac{PLAN}{33000}$, $HP = 80$, $P = 75$, $A = 160$, $N = 180$. What is the value of L?

Solution

Instruction	*Operation*
Step 1	**Step 1**
Write the formula	$HP = \dfrac{PLAN}{33000}$
Substitute the given values in the formula	$80 = \dfrac{75 \times L \times 160 \times 180}{33000}$

Step 2

Multiply the known numbers in numerator together and divide by 33000

Step 2

$$(75 \times 160 \times 180) \div 33000 = 65.45$$

Then, $80 = 65.45 \times L$

Step 3

If 65.45 times L is 80, then L equals $80 \div 65.45$
$L = 1.22$ Ans.

Step 3

$$80 \div 65.45 = 1.22$$

3. What is the value of A, if $HP = 80$, $P = 75$, $L = 1$, $N = 180$, in the formula $HP = \dfrac{PLAN}{33000}$?

Substitute the given values

$$80 = \frac{75 \times 1 \times A \times 180}{33000}$$

Step 1

Multiply the known numbers in the numerator together and divide by 33000

Step 1

$$(75 \times 1 \times 180) \div 33000 = .409$$

Then, $80 = .409 \times A$

Step 2

If .409 times A is 80, then A equals $80 \div .409$
$A = 196$ Ans.

Step 2

$$80 \div 409 = 196 -$$

PRACTICE PROBLEMS

In the following problems, the formula is $HP = \dfrac{PLAN}{33000}$

1. Find HP when $P = 78$, $L = 1.5$, $A = 113$, and $N = 170$.　　　　　　　　　　　　　　Ans. 68.11

2. Find P when $HP = 80$, $L = 1.22$, $A = 160$, and $N = 180$.　　　　　　　　　　　　　　Ans. 75+

3. Given $HP = 100$, $L = 2$, $A = 1$, and $N = 150$. Find value of P.　　　　　　　　　　　Ans. 11000

L e s s o n 5

For Step 1, bear in mind the meaning of the square of a number and review the extraction of square root. For Step 2, learn the meaning of the word "constant" as used in formulas; study the relation of the parts of a number containing the square of a letter. For Step 3, work the Illustrative Examples. For Step 4, work the Practice Problems.

Neglecting the resistance of the air, the formula for finding the space traveled by a body falling from rest through space for a certain length of time is

$$S = \tfrac{1}{2} g t^2$$

In this formula the letter g is equal to a value that is always the same. It is another example of a constant. As explained, constants frequently occur in formulas, and their values are always given when a solution of the formula is required.

We are now going a step in advance in learning about formulas and equations and assign real values to the letters, and solve problems where we are required to reason out the proper values.

In the above formula, S is the space in feet, t is the time in seconds, and g is the constant and represents the acceleration due to gravity or the increase in speed due to a pull from the earth. The value of g is always 32.16 feet per second. That is, every succeeding second that the body falls, it falls 32.16 feet faster than in the previous second. The $\tfrac{1}{2}$ in the formula is for the purpose of reducing the amount of travel to an average figure. An object, such as a piece of iron, if dropped from the top of a building gradually gains speed. From the point where it starts, it is or has been at rest or motionless so at the end of the first second it will not have traveled 32.16 feet, but only one-half of that amount or the average. The student need not worry about this average in learning how to use the formula, but it has been briefly explained in case any questions should arise.

We notice that the $\tfrac{1}{2}$, the g, and the t^2 are side by side without any sign between them, therefore, we know that they are all multiplied together. The power sign in a formula affects only the letter or number or parentheses above which it is found, so the t, only, is squared. The formula might be written $S = \tfrac{1}{2} \times g \times t^2$. Therefore, to solve the formula for S, we square the value of t and then multiply the result by the other values.

ILLUSTRATIVE EXAMPLES

1. In the formula $S = \frac{1}{2}gt^2$, what is S if the value of t is 25 seconds?

Solution

Instruction	*Operation*

Step 1

Write the formula

Step 1

$$S = \frac{1}{2} \times g \times t^2$$

or

Substitute the given values in the formula

$$S = \frac{1}{2} \times \overset{32.16}{\cancel{g}} \times \overset{(25)^2}{\cancel{t^2}}$$

or

$$S = \frac{1}{2} \times 32.16 \times (25)^2$$

The 25 is placed in ( ) so no mistake will be made whereby some other numbers might be included in the squaring

Step 2

Cancel, and square 25

Step 2

$$S = \frac{1}{2} \times \overset{16.08}{\cancel{32.16}} \times 625$$

$$S = 16.08 \times 625$$

Step 3

Perform the multiplication
$S = 10050$ Ans.

Step 3

$$16.08 \times 625 = 10050.00$$

2. Using the same formula as in Problem 1, find t if $S = 5280$ feet.

Solution

Instruction   *Operation*

Step 1   **Step 1**

Write the formula $S = \dfrac{1}{2} \times g \times t^2$

Substitute the given values in
the formula $5280 = \dfrac{1}{2} \times 32.16 \times t^2$

Step 2   **Step 2**

$$16.08$$

Perform the cancellation $5280 = \dfrac{1}{\cancel{2}} \times \cancel{32.16} \times t^2$

$$5280 = 16.08 \times t^2$$

Step 3   **Step 3**

If 16.08 times t^2 is 5280, t^2 will
equal 5280 divided by 16.08

Perform the division $5280 \div 16.08 = 328.35$

$t^2 = 328.35$

Step 4   **Step 4**

If the square of t is 328.35, t will
be the square root of 328.35.
Rule (e). Find the square root
of 328.35 $\sqrt{328.35} = 18.1$

18.1 sec. Ans.

3. A weight is let drop from an airplane at a height of 600 feet. In what time will the weight strike the ground?

Solution

Instruction   *Operation*

Step 1   **Step 1**

Write the formula $S = \frac{1}{2}gt^2$

In our problem, S, the space the
weight falls, is 600 feet. g is
always 32.16. Substitute these $600 = \frac{1}{2} \times 32.16 \times t^2$
values in the formula

Step 2   **Step 2**

                                                        16.08
Perform the cancellation $600 = \frac{1}{2} \times \cancel{32.16} \times t^2$

                                                 $600 = 16.08 \times t^2$

Step 3   **Step 3**

If 16.08 times t^2 is 600, then t^2
will equal 600 divided by 16.08

Perform the division $600 \div 16.08 = 37.31+$
$t^2 = 37.31+$

Step 4   **Step 4**

Use rule (e). Take the square
root of both sides of equation
6.1 sec. Ans.

PRACTICE PROBLEMS

1. In the formula $S = \frac{1}{2}gt^2$, find t when $S = 25000$ feet.
                                                        Ans. 39.4 sec.

2. A stone is dropped from the top of a building. It reaches
the pavement in 3.2 seconds. How high is the building?
                                                        Ans. 164.6+ ft.

3. How many seconds will it take for a weight to fall to the
ground from a height of 50 feet?   Ans. 1.7+ sec.

Lesson 6

In the study of electricity many formulas are used. One of these was used in Lesson 3. Another is discussed here—the formula for multiple circuits in parallel.

When there are two resistances in parallel, the formula is

$$R = \frac{1}{\dfrac{1}{a} + \dfrac{1}{b}}$$

in which R represents the total resistance and a and b the individual resistances.

When there are three resistances in parallel, the formula is

$$R = \frac{1}{\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}}$$

In general, for any number of resistances in parallel, the formula is

$$R = \frac{1}{\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} + \dfrac{1}{d} + \dfrac{1}{e} +}$$

using as many different letters as there are individual resistances. The dots are used to show that the letters are continued to as many as are desired. They do not affect any problem where the values of the letters are given.

It must be borne in mind, as taught in Lesson 3, that in this formula all the fractions in the denominator taken together form a unit and must be combined into one fraction before any other operations are performed. We cannot deal with these fractions apart from each other, because they represent a single quantity.

ILLUSTRATIVE EXAMPLE

1. In the formula $R = \dfrac{1}{\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}+\dfrac{1}{d}+\dfrac{1}{e}}$ find R when,

a is 2, b is 3, c is 4, d is 5, and e is 6.

Solution

Instruction	*Operation*
Step 1	**Step 1**
Substitute the given values in the formula	$R = \dfrac{1}{\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{5}+\dfrac{1}{6}}$
Step 2	**Step 2**
Combine the fractions in the denominator	$\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{5}+\dfrac{1}{6}=\dfrac{87}{60}$ $R=\dfrac{1}{\dfrac{87}{60}}$

Note: The $\frac{87}{60}$ is obtained after finding the L.C.D. for the fractions. We find 60 is the L.C.D. After expressing all the fractions in terms of this L.C.D., they are added and equal $\frac{87}{60}$. (For review, refer to Section 3.)

Step 3	**Step 3**
$\dfrac{1}{\dfrac{87}{60}}$ means 1 divided by $\dfrac{87}{60}$ so perform the indicated division $R = .689$ Ans.	$1 \div \dfrac{87}{60} = 1 \times \dfrac{60}{87} = \dfrac{60}{87} = .689$ Remember the rule for dividing fractions given in Section 4. Also to get .689 we divided 60 by 87. (For review, refer to Section 5.)

1. In the formula used in the Illustrative Example, find R when $a=3$, $b=5$, and $c=2$. Ans. .97−

2. Using the same formula, find R when a is 5, b is 3, c is 7, and d is 5. Ans. 1.14+

3. What is the joint resistance of four wires when connected in parallel if their separate resistances are 3, 5, 6, and 8 ohms, respectively? Ans. 1.2+

Lesson 7

For Step 1, bear in mind the meaning of a group of numbers enclosed in a parentheses. For Step 2, study the given formula that contains more than one kind of parentheses. For Step 3, work the Illustrative Examples. For Step 4, work the Practice Problems.

The following formula leads us into the study of **parentheses.** We can see from it that many parentheses may be used inside of another parentheses. The outside parentheses act as parentheses of all the others inside of them. The parentheses inside must be worked first and finally the outside parentheses may be removed.

$$S = \frac{[(n-x)+(n-x)^2-(x-1)+(x-1)^2]P}{2n} + \frac{r(x-1)+(x-1)^2}{2n}$$

Here we have a long formula wherein several letters are used. These letters represent mechanical factors which we are not concerned with as we are learning how to solve equations. In this formula we will substitute actual values just as we did in the previous lessons.

Note: Both large and small letters like E and e are used in formulas and have different meanings.

ILLUSTRATIVE EXAMPLES

1. Suppose in the above formula we know that

$$n=6$$
$$x=4$$
$$r=1000$$
$$P=2000$$
$$S=?$$

Then our problem is to find the value of S. We do this by substituting actual values in the formula and then solving as will be explained.

Step 1. Substituting.

$$S=\frac{\left[(\overset{6}{n}-\overset{4}{x})+(\overset{6}{n}-\overset{4}{x})^2-(\overset{4}{x}-1)+(\overset{4}{x}-1)^2\right]\overset{2000}{P}}{2\overset{6}{n}}+\frac{\overset{1000}{r}\;(\overset{4}{x}-1)+(\overset{4}{x}-1)^2}{2\overset{6}{n}}$$

Here we crossed off the letter n in each case and put a 6 in place of each n. We did this because $n=6$. Then we crossed off the letter x in each case and put a 4 in place of each x. Then we crossed off the P and put 2000 in its place and crossed off the r and put 1000 in its place. This is called *substituting*. We might use this same formula in solving a great many different problems, and in each different problem the values of n, x, r, etc. would all be different, but we would substitute the value just as shown under Step 1.

Now we can write the formula over again, but this time we will use the values of the letters just as we substituted them above.

$$S=\frac{\left[(6-4)+(6-4)^2-(4-1)+(4-1)^2\right]2000}{2\times 6}+\frac{1000(4-1)+(4-1)^2}{2\times 6}$$

A careful study of this will show that it is exactly the same as the formula on page 30 except that the letters have been replaced by their actual values. When you start to work out formulas in actual practice, the values of the letters will be given, or the problems stated in such a manner that the letter values won't be hard to find.

In the two denominators, which form part of this formula, we have 2×6. In the original formula (page 30) these denominators were $2n$. Then we substituted the 6 for the n in each case. We then put a "times" sign between the 2 and 6 or otherwise anyone would think it was 26 (twenty-six) instead of 2×6. We can put two letters or a number and letter side by side without a times sign between them when we want to indicate they are to be multiplied Thus—

<div align="center">

nn means $n\times n$

$2n$ means $2\times n$

</div>

But when we substitute a figure such as 6, in place of the n (as in the second case) we must put a "times" sign between the 2 and 6.

Step 2. Perform operations inside of ( ) parentheses.

Taking the first set of parentheses, starting on left-hand end, we have $(6-4)$. Performing what is indicated we subtract 4 from 6 and have 2. Thus the $(6-4)$ becomes (2).

The next set of parentheses is $(6-4)^2$. Subtracting 4 from 6 we have 2. Then the $(6-4)^2$ becomes $(2)^2$. The square sign stays in its same position.

In like manner

$$(4-1) = (3)$$
$$(4-1)^2 = (3)^2$$

Now, the numerator of the first part of the equation (the formula becomes an equation after substitution takes place) is

$$[(2)+(2)^2-(3)+(3)^2]2000$$

In the second part of the equation, we form what is indicated within the parentheses and see that

$$(4-1) = (3)$$
$$(4-1)^2 = (3)^2$$

The numerator of the second part of the formula then becomes

$$1000(3)+(3)^2$$

Now we can write the equation

$$S-\frac{[(2)+(2)^2-(3)+(3)^2]2000}{12}+\frac{1000(3)+(3)^2}{12}$$

The number 12 in the two denominators is obtained by multiplying 2×6 in each case.

Step 3. The [ ] bracket or parentheses still holds the other parentheses inside, therefore, we must solve everything inside of the bracket before we can remove the bracket.

$$S=\frac{[2+4-3+9]2000}{12}+\frac{3000+9}{12}$$

Here we took away the ( ) and at the same time squared numbers where squaring was indicated. Also we multiplied the 1000 by the 3 in the second part of the equation.

Step 4. Combine the numbers inside the bracket

$$S = \frac{[12]\,2000}{12} + \frac{3009}{12}$$

The numbers $[2+4-3+9]$, shown in Step 3, equal 12, because $2+4=6-3=3+9=12$. The bracket is left on. Also $3000+9$, from Step 3 equals 3009.

Step 5. Perform the indicated operations

$$S = \frac{24000}{12} + \frac{3009}{12}$$

$$S = \frac{27009}{12} = 2251 \quad \text{Ans.}$$

In Step 4 we had $[12]2000$.
Worked out this is $12 \times 2000 = 24000$.
Then adding 24000 and 3009 we get 27009.
Thus

$$S = \frac{27009}{12} \text{ or } 2251 \quad \text{Ans.}$$

In this manner we have solved the problem and found the value of S to be 2251.

Summary. The student is advised to study Problem 1 again very thoroughly. Then look through the following solution step by step to see if Problem 1 is clear.

Solution Problem 1

Formula

$$S = \frac{[(n-x)+(n-x)^2-(x-1)+(x-1)^2]P}{2n} + \frac{r(x-1)+(x-1)^2}{2n}$$

$$n=6 \quad x=4 \quad r=1000 \quad P=2000$$

Find S

$$S = \frac{[(6-4)+(6-4)^2-(4-1)+(4-1)^2]2000}{2\times6} + \frac{1000(4-1)+(4-1)^2}{2\times6}$$

$$S = \frac{[(2)+(2)^2-(3)+(3)^2]2000}{12} + \frac{1000(3)+(3)^2}{12}$$

$$S = \frac{[2+4-3+9]2000}{12} + \frac{3000+9}{12}$$

$$S = \frac{[12]2000}{12} + \frac{3009}{12}$$

$$S = \frac{24000}{12} + \frac{3009}{12}$$

$$S = \frac{27009}{12} = 2251 \quad \text{Ans.}$$

The above solution for Problem 1, is the same as was given on page 30. But, we have left out all the explanations and shown only the main steps in the solution.

The student should study through this outline of the solution and make sure he understands each step thoroughly. If one is not sure of this outline as given, go back and review the explained solution on page 30.

When the student is sure he understands the problem, he should close the book and try to work the problem without looking in the book.

2. Using the above formula on page 30, find P when $n=8$, $x=5$, $r=1500$, and $S=5000$.

Step 1. Substitute the given values in the formula

$$5000 = \frac{[(8-5)+(8-5)^2-(5-1)+(5-1)^2]P}{2\times8} + \frac{1500(5-1)+(5-1)^2}{2\times8}$$

At this point, substituting was carried on in the same manner as Step 1 in Problem 1 on page 31. The student is advised to do this work on scrap paper and make sure he understands Step 1 of this problem.

Step 2. Perform the operations inside the ( ) parentheses.

$$5000 = \frac{[(3) + (3)^2 - (4) + (4)^2]P}{16} + \frac{1500(4) + (4)^2}{16}$$

This step is done exactly the same as Step 2 on page 32. For example, in this problem, the first set of ( ) is $(8-5)$. We know that $8 - 5 = 3$. So we have (3) as in Step 2. The 16's are obtained by multiplying 2×8.

Step 3. Solve the parts within the bracket

$$5000 = \frac{[3 + 9 - 4 + 16]P}{16} + \frac{6000 + 16}{16}$$

This comes from Step 2. We squared all the numbers where squaring was indicated. Then we multiplied 1500 by 4 to get 6000.

Step 4. Combine the parts inside the bracket

$$5000 = \frac{[24]P}{16} + \frac{6016}{16}$$

From Step 3 we have

$$[3 + 9 - 4 + 16]$$

This equals

$$3 + 9 = 12 - 4 = 8 + 16 = [24]$$

The 6016 is obtained by adding 16 to 6000.

Step 5. Transpose, so as to get the term containing P by itself.

At this point the student should go back to page 9 and review through to page 34 and make sure that he thoroughly remembers transposition. In transposing, you will recall, we move certain terms of an equation around so as to get the unknown term on one side of the equation by itself. The term $\frac{[24]P}{16}$ in Step 4, being already to the right of the equal sign can be left in that position. The term 5000 is already on the left side of equation so it doesn't move. The term $\frac{6016}{16}$ is on the right side so when it is moved to the left side its sign must be changed. (Rule G)

Now we have

$$5000 - \frac{6016}{16} = \frac{24P}{16}$$

The term $\dfrac{6016}{16}$ is the only one that was moved.

But the 5000 really means $\dfrac{5000}{1}$ when expressed as a fraction.

We cannot subtract $\dfrac{6016}{16}$ from $\dfrac{5000}{1}$ because the two fractions do not have the same denominator. (In Section 3 you had addition and subtraction of fractions.) We could find the L.C.D. and change both fractions so that they would be expressed in terms of L.C.D. explained in Section 3. But we have an easier method. If we multiply both numerator and denominator of the $\dfrac{5000}{1}$ by 16, we get

$\dfrac{16 \times 5000}{16}$ This doesn't change its value a bit as can be proven by multiplying 16 by 5000 and dividing the product by 16.

Now our equation becomes

$$\frac{16 \times 5000}{16} - \frac{6016}{16} = \frac{24P}{16}$$

Step 6. Combine the parts in left member of equation

$$
\begin{array}{ccc}
\dfrac{80000}{16} & \dfrac{6016}{16} & \dfrac{24P}{16} \\[2mm]
& \dfrac{73984}{16} & \dfrac{24P}{16}
\end{array}
$$

In this step the 80000 is obtained by multiplying 16 by 5000. The 73984 is obtained by subtracting $\dfrac{6016}{16}$ from $\dfrac{80000}{16}$.

Thus $80,000 - 6016 = 73984$. (Recall or look up in Section 3 the rule for subtracting fractions.)

Step 7. Apply Rule (C). Multiply both members of the equation by 16.

In order to be able to finish our problem without complicated calculations, we can follow Rule (C). We multiply by 16 because

the equation already has the number 16 in it and we want to make cancellation possible. Thus we have

$$\frac{73984}{16} \times 16 = \frac{24P}{16} \times 16$$

The number 16 can be written $\frac{16}{1}$. So we can cancel.

$$\frac{73984}{\not{16}} \times \frac{\not{16}}{1} = \frac{24P}{16} \times \frac{\not{16}}{1}$$

Then

$$73984 = 24P$$

and

$$P = 73984 \div 24$$
$$P = 3082 + . \quad \text{Ans.}$$

Summary. Once more the student is strongly advised to study the above detailed and explained solution again to make sure he understands it. Then look through the following solution step by step of Problem 2 to see if it is clear.

Solution Problem 2

Formula

Same as for Problem 1.

$$n = 8 \quad x = 5 \quad r = 1500 \quad S = 5000 \quad \text{Find } P$$

$$5000 = \frac{[(8-5)+(8-5)^2-(5-1)+(5-1)^2]P}{2 \times 8} + \frac{1500(5-1)+(5-1)^2}{2 \times 8}$$

$$5000 = \frac{[(3)+(3)^2-(4)+(4)^2]P}{16} + \frac{1500(4)+(4)^2}{16}$$

$$5000 = \frac{[3+9-4+16]P}{16} + \frac{6000+16}{16}$$

$$5000 = \frac{[24]P}{16} + \frac{6016}{16}$$

$$5000 - \frac{6016}{16} = \frac{24P}{16}$$

$$\frac{16 \times 5000}{16} - \frac{6016}{16} = \frac{24P}{16}$$

$$\frac{80000}{16} \quad \frac{6016}{16} \quad \frac{24P}{16}$$

$$73984 \quad 24P$$
$$16 \qquad 16$$
$$\frac{73984}{16} \times 16 = \frac{24P}{16} \times 16$$
$$73984 = 24P$$
$$P = 73984 \div 24$$
$$P = 3082+. \quad \text{Ans.}$$

Study through the above main steps in solution to Problem 2 to make sure you understand all parts. If you do not understand all parts, review the detailed explanation starting on page 34.

As an example of how a student should check his knowledge, the following sample questions are given.

(a) Where do the two denominators of 16 come from?

(b) How is the [24]P obtained?

(c) Why are both sides of the equation multiplied by 16?

(d) Why is 16 used rather than 18?

The student should ask himself many such questions.

When the student feels that he fully understands the problem, he should try to solve it without the help of the book.

Note: In the formula we have been using in this lesson, the values of r, n, and x cannot be found without the use of Algebra.

PRACTICE PROBLEMS

Using the same formula as in the Illustrative Examples

1. Find S when $n = 4$, $x = 3$, $r = 500$, and $P = 2000$. Ans. 1125.5

2. If $S = 3000$, $n = 8$, $x = 5$, and $r = 1000$, find value of P.
Ans. 1832.6+

Since there are great numbers of different formulas used in engineering and mechanical work of all kinds, it would be impossible to give instruction on the method of solving every possible formula. Many of them cannot be solved without a working knowledge of Algebra and Trigonometry.

However, if the student understands the principles discussed and illustrated in this book and will use judgment and common sense, he will be able to solve many formulas of importance.

PRINCIPLES FOR READY USE

1. A formula is like a weight scale; the part on one side of the equal sign must have the same resultant value as the part on the other side. There must be a balance.

2. In a formula, any number or term may be transposed from the left to the right or from the right to the left of the equal sign if, in doing so, the sign of the number or term so transposed is changed to the opposite sign. The sign of a number or term is always in front of the number or term and is a part of it.

3. To find the unknown number, or to balance the formula, if the number belongs on the left side of the equal sign, change all the known numbers that are on the left to the right of the equal sign and thus leave the unknown number all by itself on the left side. Solve the right side by arithmetic, and the answer is the value of the unknown number. Reverse the procedure when the unknown number is on the right side.

4. Omit the $+$ sign in front of a number when it is all by itself or when it is the first of a group of numbers on one side of the equal sign.

5. If a certain number has a $+$ or a $-$ sign in front of it but is enclosed in parentheses with other numbers or if it is above or below the dividing line in a fraction with other numbers, the operations indicated between the numbers must be performed before that number can be separated from the others.

6. If we add or subtract the same number from both members of an equation or formula or multiply or divide both members by the same number, the equality or balance of the formula does not change.

7. Multiplication between a number and a letter or between letters is indicated by placing them beside each other without any sign between them.

8. A number or letter beside a root sign when there is no sign between, indicates multiplication.

9. If we square the quantities on both sides of the equal sign or raise them to the same power, no matter what that power is, or if we extract the same root, the equality is not destroyed.

10. If two or more numbers, or two or more letters, or numbers and letters, or radical signs and letters, or radical signs and numbers, are to be multiplied together, and then the product raised to a power, each of the quantities to be multiplied together may be raised to the indicated power first and then the results multiplied.

11. In some formulas there appear letters used twice in the same formula such as T_1 and T_2. In such cases T_1 might mean temperature on the outside of a building and T_2 the temperature on the inside of the building.

12. A power sign in the formula affects only the letter or number or parentheses above which it is found.

13. A constant is a letter that always has the same known value.

14. When we have quantities of any kind anywhere connected by $+$ and $-$ signs, we may group them as one quantity by placing parentheses around all of them including their signs, and then dealing with the parentheses as if it were a single quantity or unit. Also, multiplication between quantities in parentheses is indicated by placing them beside each other without any sign between the parentheses.

15. Many parentheses may be used inside of other parentheses. The outside parentheses act as parentheses of all the other parentheses inside of it. The operations in the inside parentheses must be worked first and finally the outside parentheses may be removed.
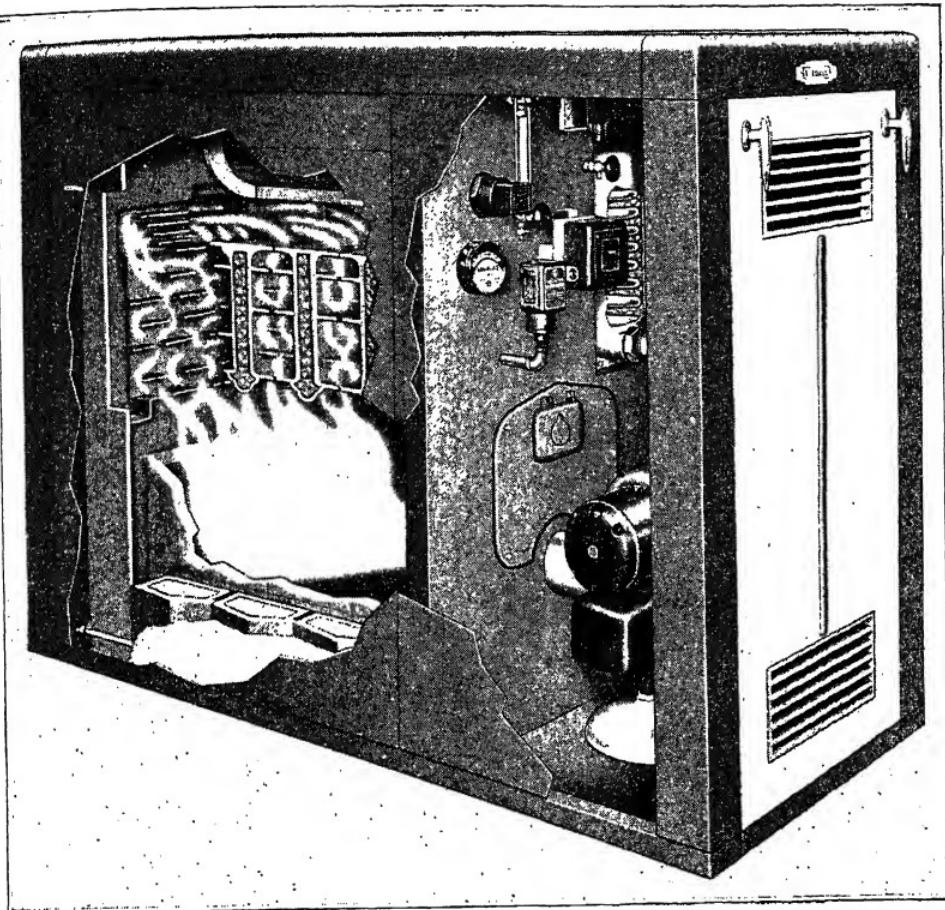
CUTAWAY VIEW OF OIL-FIRED BOILER

Courtesy of Gilbert & Barker Mfg. Co., Springfield, Mass.

SOLIDS
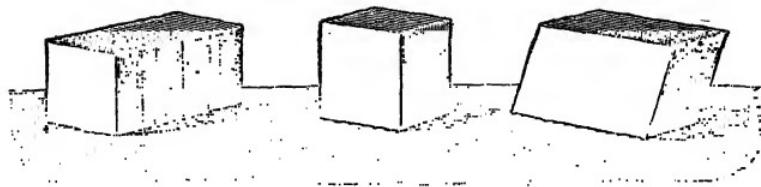
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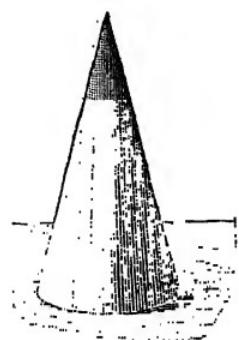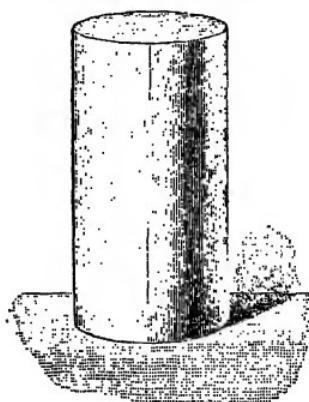
Cone   Cylinder
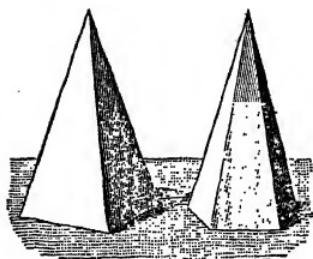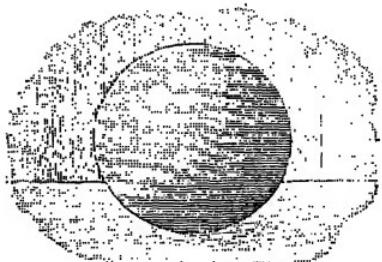
Pyramids   Sphere
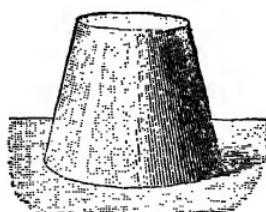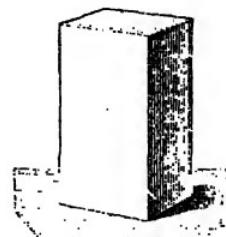
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PRACTICAL MATHEMATICS

Section 13

MENSURATION—Part II

L e s s o n 1

For Step 1, keep in mind what you learned in Section 12 relative to areas, because solids are bounded by areas. For Step 2, learn the names and definitions of various solids and the method of finding the areas of prisms. For Step 3, work the Illustrative Examples. For Step 4, solve the Practice Problems.

SOLIDS

In Section 12, we learned that mensuration is the process of computing the length of lines, the area of surfaces, and the volume of solids. We studied plane surfaces, which are surfaces with only two dimensions. In Section 13, we shall study **solids.** Solids are objects with three dimensions—length, breadth, and thickness. They are of many shapes, the most common of which are prisms, cylinders, pyramids, cones, and spheres. We shall learn how to find the surface area and volume or contents of such objects. It will be necessary to keep the definitions and principles of Section 12 constantly in mind during the study of this text. Frequent reference to them may be necessary.

On the opposite page is shown a group of typical solids. The student should become familiar with these before starting the study of the following text.

PRISMS

A **prism** is a solid whose ends (top and bottom) are equal, similar, and parallel polygons. These ends are called the **bases** of the prism. The sides of the prism are parallelograms. Figs. 1 to 6 are illustrations of prisms.

A prism is named triangular, rectangular, pentagonal, hexagonal, or octagonal according as its bases are triangles, rectangles, pentagons, hexagons, or octagons.

The sides of a prism are called **lateral faces.** It is evident that there will be as many of these lateral faces as there are sides in one of the bases.

The **altitude** of a prism is the perpendicular distance between its two bases. When the bases are perpendicular to the faces, the altitude equals the edge of a lateral face, as in Figs. 1, 3, and 4.

A **right prism** is one whose lateral faces are perpendicular to the bases, as illustrated in Figs. 1, 3, and 4.
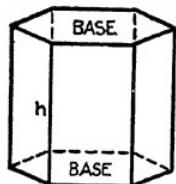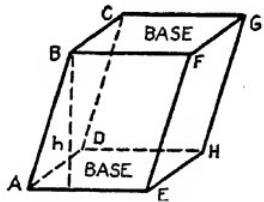
Fig. 1. Right Prism   Fig. 2. Parallelopiped   Fig. 3. Rectangular Parallelopiped
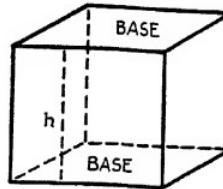
An **oblique prism** is one whose lateral faces are not perpendicular to the bases, Fig. 2.

A **parallelopiped** is a prism whose bases are parallelograms, Figs. 2 and 3. If all the edges are perpendicular to the bases, it is called a **right parallelopiped**, Fig. 3.

A **rectangular parallelopiped** is one whose bases and faces are all rectangles, Fig. 3.
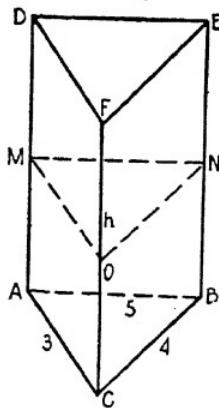
Fig. 4. Right Triangular Prism   Fig. 5   Fig. 6. Cube
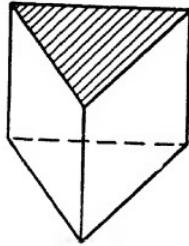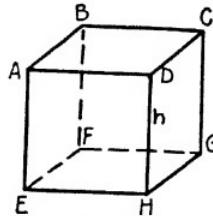
A **cube** is a parallelopiped whose bases and faces are all equal squares, Fig. 6.

A **cross section** of a prism is a section that is perpendicular to the edges of the prism. In Fig. 4, the plane MNO is the cross section.

To understand just what a cross section is, imagine that the prism in Fig. 4 had been sawed through following the dotted lines connecting *MNO*. That would cut the prism into two pieces. If the upper piece was moved away, the top of the bottom piece would represent the cross section. This is illustrated in Fig. 5. The shaded area is the cross section.

The **lateral area** of a prism is the combined area of all its faces.

In Fig. 4, the sides *DACF*, *FCBE*, and *EBAD* form the faces. Thus, to find the lateral area of this prism, it is necessary to add the areas of all three faces together.

The **total area** of a prism is the combined area of the lateral faces and the bases.

The bases, in Fig. 4, are *DFE* and *ACB*. Add the areas of these two bases to the areas of all the sides to find total area.

An **edge** is the line where two lateral faces meet, as *FE* or *GH* in Fig. 2.

AREAS OF RIGHT PRISMS

Since the faces or sides of a right prism are all rectangles, the area of one face is found by multiplying its base by its altitude or, in other words, the area of the rectangle is the product of its two adjacent sides. You will recall this fact from Section 12.
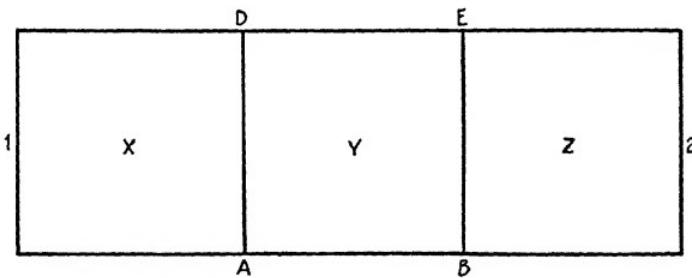
[Fig. 7

Refer to Fig. 4. Imagine that the two bases (top and bottom) were taken off. Then imagine that we cut along the side *FC* and moved the side *DACF* back. Do the same for side *FCBE*. This is

called spreading it out. Now look at Fig. 7. The part X represents the side $DACF$, the part Z the side $FCBE$, and Y the side $EBAD$. To close the figure up again, fold on edges DA and EB so that line 1 and 2 could be joined. This would form side FC as in Fig. 4.

It is easily seen that the areas of $X+Y+Z$ would be exactly equal to the areas of the three sides in Fig. 4. Also the distance from 1 to 2 in Fig. 7 is exactly the same as the distance from A to C to B and back to A again, Fig. 4. In other words, the length of Fig. 7 is equal to the perimeter of Fig. 4. The length of line FC in Fig. 4 is the same as DA in Fig. 7. So the width of Fig. 7 is the same as the height or altitude (h) of Fig. 4.

The area of Fig. 7 would equal its length (distance from 1 to 2) times its width (distance DA).

Thus we can state a rule for finding lateral area of a prism. This rule applies for all right prisms such as Figs. 1, 3, 4, and 6.

Rule (1).   *The lateral area of a prism equals the perimeter of its base multiplied by its altitude.*

We have seen that the bases of prisms are of different shapes—triangular, rectangular, hexagonal, etc. In Section 12, the methods of finding the areas of such surfaces were discussed. In order to find the area of the bases of a prism, use the method that corresponds to the particular shape of base under consideration.

For example, if a right prism has bases shaped like a triangle (such as Fig. 4) we must find the perimeter of the triangle. If the base happened to be shaped like a hexagon, we would first find the perimeter of the hexagon.

In the case of a cube, the edges are all equal because the faces or sides are all equal. Fig. 6 illustrates this, and you can see that $AB=BC$, $AE=EH$, $HG=GC$, $HD=DA$, etc. Thus we can find the total area of the six faces of a cube as follows:

Rule (2).   *The total area of a cube is equal to six times the square of one edge.*

This can be easily understood if we remember that in a cube all six faces are equal. So, to find the area of one face, we multiply length by width. Then, because all six sides are equal, the total area is six times the area of one face. Now, in a square, the length of each side

is the same. So if we square the length of one side we will have the area of that side.

ILLUSTRATIVE EXAMPLES

1. Find the lateral area of a regular pentagonal prism, its altitude being 7 feet and its base measuring 4 feet on each side.

Solution

Instruction	*Operation*
Step 1	**Step 1**
Find the perimeter of the base. Since a regular pentagon has five equal sides, the perimeter will be five times the length of one side	$4 \times 5 = 20$
Step 2	**Step 2**
Multiply perimeter by altitude to find lateral area	$20 \times 7 = 140$
Lateral area = 140 sq. ft. Ans.	

2. If the lateral area of a regular pentagonal prism is 140 sq. ft. and the base has sides 4 feet long, what is the altitude?

Note

Before going into the step by step solution we should understand certain things that always hold good as far as areas, rules, etc. are concerned.

We know that in Problem 1 we found the area by multiplying the perimeter by the altitude. Now, if we know the area but do not know the altitude, we can find the altitude by dividing the area by the perimeter. Or, if we want to find the perimeter, when the area and altitude are given, we divide area by the altitude.

We can solve Problem 2, as follows:

Solution

Instruction	*Operation*
Step 1	**Step 1**

Since the given area is the product of the altitude and the perimiter of the base, we can, as explained above, find the altitude by dividing the area by the perimeter. From Problem 1 we know the perimeter is 20 ft.

$$140 \div 20 = 7$$

Altitude = 7 ft. Ans.

3. Find the total area of a triangular prism whose base is a right triangle, one of whose sides is 3 inches and the other side 4 inches. The altitude of the prism is 10 inches.

Solution

Instruction	*Operation*
Step 1	**Step 1**

Only two sides of the triangular base are given, so we must first find the third side, or hypotenuse, before we can obtain the perimeter. Apply the formula of the right triangle $H^2 = b^2 + a^2$

$$H^2 = 4^2 + 3^2$$
$$H^2 = 16 + 9 = 25$$
$$H = \sqrt{25} = 5$$

Third side of triangular base is 5 in.

Step 2	**Step 2**

Add the lengths of the three sides to get the perimeter

$$3 + 4 + 5 = 12$$

Perimeter is 12 in.

Step 3	**Step 3**

Multiply the perimeter by the altitude to get lateral area

$$12 \times 10 = 120$$

Lateral area is 120 sq. in.

Step 4

Find area of the bases
Apply the formula for finding the

area of a triangle $A = \dfrac{b \times h}{2}$

Area of each base is 6 sq. in.
Area of both bases is 12 sq. in.

Step 4

$\dfrac{3 \times 4}{2} = 6$

Step 5

Find total area by adding lateral
area and base areas
Total area is 132 sq. in.   Ans.

Step 5

$120 + 12 = 132$

4.   The lateral area of a prism whose base is a regular hexagon is 108 square inches.   If the altitude is 9 inches, what is the length of one side of the base?

Solution

Instruction

Step 1

Since the given area is the product of the altitude and the perimeter of the base and we know the altitude, we can find the perimeter by dividing the lateral area by the altitude
Perimeter of base is 12 in.

Operation

Step 1

$108 \div 9 = 12$

Step 2

A regular hexagon has six equal sides, so to find one side divide the perimeter by 6
Length of one side = 2 in.   Ans.

Step 2

$12 \div 6 = 2$

PRACTICE PROBLEMS

1.   What is the lateral area of a prism whose base is a square having an area of 169 square inches, and whose altitude is 3 feet? Ans. 13 sq. ft.

2. The base of a right prism is in the shape of an equilateral triangle each of whose sides is 6 inches. Find the total area of the prism if its altitude is 15 inches. Ans. 301.17+sq. in.

3. The total area of a prism is 180 square inches. The area of one base is 30 square inches and the altitude is 5 inches. What is the perimeter of the base? Ans. 24 in.

4. What is the total area of a cube one of whose edges measures 6 inches? Ans. 216 sq. in.

Note: Do not go beyond this point in the text until you can solve the above problems and fully understand them.

L e s s o n   2

For Step 1, keep in mind the shapes of various kinds of prisms and their dimensions. For Step 2, learn how to find the volumes of prisms. For Step 3, work the Illustrative Examples. For Step 4, work the Practice Problems.

VOLUMES OF PRISMS

So far in this book, the discussions have been on the surfaces of prisms. Now it is necessary to learn how to find the volumes, or contents, of these solids. The volume of a solid is always given in cubic measure, which you will recall having studied in Section 7.

Volume, or contents, involves the product of three quantities in linear measure or the product of two quantities, one given in square measure and the other in linear measure.

Volume is determined by the number of times the unit of cubic measure (as cubic inch, cubic foot, cubic centimeter, etc.) is contained in the object under consideration. A study of Fig. 8 will illustrate this fact.

In Fig. 8, A represents a cubic inch. It can be seen that there are four cubes similar to A in the bottom layer of B. Also there are three layers in B each containing cubes of the same size as A. Therefore, there are 4×3 or 12 cubes like A in B, or, in other words, there are 12 cubic inches in B.

Each side of the base of B measures 2 inches, so that its area is 2×2 square inches; and since the altitude is 3 inches, the volume is 2×2×3 or 12 cubic inches, as found in the preceding paragraph. The

cubic contents of B, then, can be expressed either as the product of length of base, width of base, and altitude, or as the product of base or end area and altitude.

Rule (3). *The volume of any prism is found by multiplying the area of its base by its altitude.*
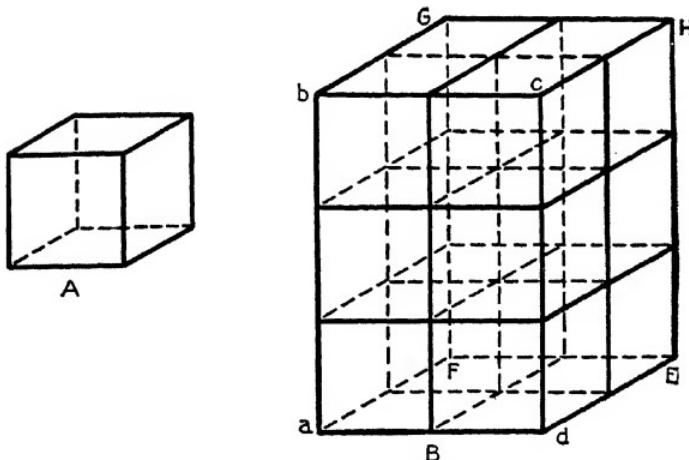
Fig. 8.   A—Cubic Inch; B—Parallelopiped

The above rule, as stated, can be used to find the volume of any shape of prism and is therefore the best rule to remember.

The method of finding the area of the base will depend on what the shape of the base is. The methods of finding areas of various shaped bases can be found in Section 12.

The other method of finding the volume of an object is to multiply length×width×breadth. It is all right to use this second method, but it only applies to objects or prisms that are right parallelepipeds such as Figs. 2, 3, 6, and 8. When finding volumes of prisms shaped like Fig. 1 or Fig. 4, we must use **Rule (3).**

It would be advisable to learn to use **Rule (3)** at all times.

There are variations of **Rule (3)** depending on the problem being considered. Sometimes we know the volume of a given prism and also the altitude. In such a case, we could find the base area by dividing the volume by the altitude. Or, if we knew the volume and base area, we could find the altitude by dividing the volume by base area.

The method of finding the area of the base will depend, of course, on the shape of that base, whether it is triangular, rectangular, pentagonal, etc., as we learned in Lesson 1.

ILLUSTRATIVE EXAMPLES

1. What is the volume of a triangular prism whose height is 20 feet? Each end of the prism is an obtuse-angled triangle, the base of which measures 8 feet and the altitude 5 feet.

Solution

Instruction	*Operation*

Step 1

Find the area of the base or end of the prism by applying the rule for finding the area of a triangle

$$A = \frac{b \times h}{2}$$

Step 1

Substituting in the formula

$$A = \frac{8 \times 5}{2} = 20$$

Area of base of prism is 20 sq. ft.

Step 2

Find the volume of the prism by multiplying the base area by the altitude of the prism

Step 2

$$20 \times 20 = 400$$

Volume of prism is 400 cu. ft. Ans.

2. A certain concrete pillar has a hexagonal cross section. The height of the pillar is 15 feet. Each side of the base measures 2 feet and the line drawn from its center perpendicular to a side is 1.73 feet long. How many cubic yards of concrete are in the pillar?

Solution

Instruction	*Operation*

Step 1

The base of the prism is a hexagon (as Fig. 1), so to find its area we must apply the rule for the area of a hexagon, which we

Step 1

learned in Section 12. $A =$
$3SR$, where S is one side of the
hexagon and R is the apothem.
Substituting in this formula

$$A = 3 \times 2 \times 1.73$$
$$= 10.38$$

Base area of prism is 10.38 sq. ft.

Step 2

Find the volume of the prism by
multiplying the base area by the
altitude

Volume of prism is 155.7 cu. ft.

Step 2

$$10.38 \times 15 = 155.70$$

Step 3

Reduce 155.7 cu. ft. to cubic
yards
5.8 cu. yd.   Ans.

Step 3

$$155.7 \div 27 = 5.8-$$

3.   The volume of a bar of iron is 1680 cubic inches. It is 4
feet long and its cross-section is in the form of a trapezoid. One of
the parallel sides measures 6 inches and the perpendicular distance
between the two parallel sides is 5 inches. What is the length of the
other parallel side of the end of the bar?

Solution

Instruction   *Operation*

Step 1

Read the problem carefully and
draw a diagram to illustrate it,
marking the given values in their
places

Step 1

Step 2

Reduce 4 feet to inches so that
all dimensions will be in the same
denomination
4 feet = 48 inches
This 48 inches is the altitude of
the prism

Step 2

$$4 \times 12 = 48$$

Step 3

Since the volume of the prism equals the base area multiplied by the altitude, the base area will equal the volume divided by the altitude

Base area of prism, that is, the area of the trapezoid end is 35 sq. in.

Step 3

$$1680 \div 48 = 35$$

Step 4

We know now the area of the trapezoid and one of its parallel sides or bases. Recall, from Section 12, a formula for finding the second base of a trapezoid when one base and the area are given. It is formula (9)

$$b_1 = \frac{2A}{h} - b$$

Substitute the values we know in this formula

Step 4

$$b_1 = \frac{2 \times 35}{5} - 6$$
$$= 14 - 6 = 8$$

The other parallel side is 8″. Ans.

PRACTICE PROBLEMS

1. A school room is 40 feet wide and 50 feet long. There are 40 pupils in the room, each requiring 450 cubic feet of air. How high will the room have to be? Ans. 9 ft.

2. Across the bottom, a trough measures 24 inches, across the top 28 inches, and it is 18 inches deep. If the trough when full will hold 243.75 gallons, what is its length? Ans. 10 ft.

3. What is the volume of a prism, the base of which is a right triangle whose two shortest sides are, respectively, 4 feet and 6 feet? The height of the prism is 25 feet. Ans. 300 cu. ft.

4. The perimeter of the base of a regular hexagonal prism is 60 feet. Its height is 20 feet. Find its volume. Find the area of one lateral face. Ans. 5196 cu. ft.; 200 sq. ft.

Lesson 3

For Step 1, recall the method of finding the circumference and area of a circle. For Step 2, acquaint yourself with the form of a cylinder and the method of finding its area and volume. For Step 3, study the Illustrative Examples. For Step 4, work the Practice Problems.

CYLINDERS

A cylinder is illustrated in Fig. 9. Other illustrations are ordinary pipes, such as are used to carry water or gas, a common round lead pencil, and even a broom handle. A cylinder may have any length (h in Fig. 9) and any radius (AO in Fig. 9).

To help illustrate how to find the area of a cylinder, it will be necessary to perform an experiment. On an ordinary piece of paper, draw a rectangle 4 inches long and 2 inches wide. Then cut out this rectangle so you have a piece of paper 2 by 4 inches in size. Letter the corners so that B is in the upper left-hand corner, C in the upper right-hand corner, A in lower left, and D in lower right. Now bend or roll the paper so that CD and BA meet. They should meet so that they form one line, such as BA in Fig. 9. The resulting figure is a cylinder.

A **right cylinder** is one whose side is perpendicular to its base. This means that BA, in Fig. 9, is perpendicular to AO.
A **circular cylinder** is one whose bases are circles.
This means that both the top and bottom, in Fig. 9, are circles.
Only right circular cylinders will be considered in this text.

AREAS OF CYLINDERS

Rule (4). *The lateral area of a cylinder is found by multiplying the circumference of the base by the altitude.*
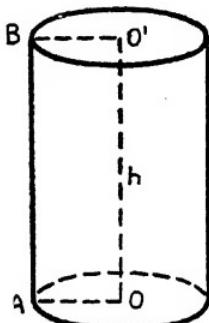
Fig. 9. Cylinder
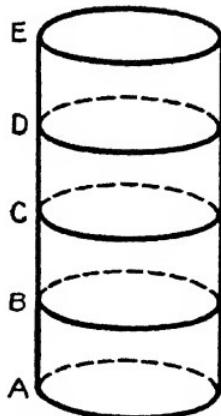
Fig. 10. Cylinder Divided into Equal Sections

Rule (4) can be explained by making use of the experiment you made with the piece of paper 3×2 inches in size. The paper is in the form of a rectangle and you know the area of a rectangle is found by multiplying the length by width. Thus 3×2=6 which is the area of the rectangle. Now, after the paper was rolled so as to form a cylinder, its area was not changed but its appearance was changed. The length after being rolled becomes the circumference of the cylinder. The vertical dimension of the original paper (2″) becomes the height of the cylinder or its altitude. So if we multiply the circumference (3″) by the altitude (2″) we get 6 sq. inches which is the same area as the original piece of paper.

There are variations of **Rule (4)** as there were to **Rule (3)**. If we know the lateral area and the circumference, we can find the altitude by dividing the lateral area by the circumference.

Also, if we know the lateral area and the altitude, we can find the circumference by dividing the lateral area by the altitude.

The total area of a cylinder, as in the case of a prism, is the sum of the lateral area and the areas of the two bases. The area of each base, for all purposes of this text, is the area of a circle and is found by the methods learned in Section 12.

VOLUMES OF CYLINDERS

In Fig. 10 a cylinder is divided into four equal sections, each one unit high. When the area of one circular base is multiplied by one unit of height, the volume of that section is obtained; so when the area of the base is multiplied by the four units of height, the volume of the four sections or the volume of the cylinder is obtained.

Hence we can state the rule for finding the volume of a cylinder:

Rule (5). *The volume of a cylinder is found by multiplying the area of its base by its altitude.*

Section 12 shows the method of finding the area of such a base.

ILLUSTRATIVE EXAMPLES

1. Find the number of square feet of sheet metal necessary to cover the sides and bottom of a cylindrical tank 15 feet long and 7 feet in diameter.

Instruction	**Solution**	*Operation*

Step 1   **Step 1**

Find the area of the bottom, which, of course, is a circle. Section 12 gives the formula for doing this: $A = \pi r^2$. Find the radius (r) by dividing the diameter by 2

$$7 \div 2 = 3.5$$

Substitute the given values in the formula

$$A = \frac{22}{7} \times (3.5)^2$$

$$= \frac{22}{7} \times 12.25 = 38.5$$

Area of the bottom is 38.5 sq. ft.

Step 2   **Step 2**

Find the lateral area. To do this we must first find the circumference of the base

Formula is $C = \pi d$

Substitute the known values in
this formula

$$C = \frac{22}{7} \times 7 = 22$$

Circumference is 22 ft.

Step 3
Find the lateral area by multi-
plying the circumference by the
altitude

Step 3

$$22 \times 15 = 330$$

Step 4
Find the required area of sides
and bottom by adding the two
areas we have found
368.5 sq. ft. Ans.

Step 4

$$330 + 38.5 = 368.5$$

2. The lateral area of a cylinder is 981.75 square feet. If its
altitude is 50 feet, find the radius of its base.

Solution

Instruction

Operation

Step 1
Since the lateral area is the prod-
uct of the altitude and cir-
cumference, we can find the
circumference by dividing the
lateral area py the altitude
Circumference = 19.635 ft.

Step 1

$$981.75 \div 50 = 19.635$$

Step 2
The formula for finding the
circumference is $C = 2 \times \pi \times r$.
Substitute the known values in
this formula

Step 2

$$19.635 = 2 \times \frac{22}{7} \times r$$

Step 3

To clear of fractions, multiply both sides of the equation by the denominator of the fraction

Step 3

$$19.635 \times 7 = 2 \times \frac{22}{7} \times r \times 7$$

$$137.445 = 44 \times r$$

Step 4

If 44 times r is 137.445, r equals 137.445 divided by 44

Required radius is 3.124 ft. Ans.

Step 4

$$137.445 \div 44 = 3.124-$$

3. When blasting in rock, a hole 95 feet long and 10 feet in diameter was made. How many cubic yards of rock were removed?

Solution

Instruction

Operation

Step 1

The hole will be in the form of a cylinder, so to find its cubic contents or volume, we must first find the area of the end. Our formula is $A = \pi r^2$. r is $\frac{1}{2}$ of 10, or 5 feet.

Substitute the values in the formula

Step 1

$$A = \frac{22}{7} \times 5^2$$

$$= \frac{22}{7} \times 25 = 78.57$$

Base area is 78.57 sq. ft.

Step 2

Find the volume by multiplying base area by the length (i.e., the altitude of the cylinder)

Volume is 7464.15 cu. ft.

Step 2

$$78.57 \times 95 = 7464.15$$

Step 3

Reduce this to cubic yards by dividing by 27

Step 3

$$7464.15 \div 27 = 276.45$$

276.45 cu. yd.   Ans.

4. A contractor removed 10,000 cubic yards of clay from the location for a drain. The drain is 6 feet in diameter. How long is it?

Solution

Instruction   *Operation*

Step 1

We know that the volume of the cylindrical-shaped drain is the product of the area of the end and the length, so if we find the area of one end we can find the length. Radius is 3 ft. $A = \pi r^2$ Substitute the given values in the formula

Step 1

$$A = \frac{22}{7} \times 3^2$$

$$\frac{22}{7} \times 9 = 28.28+$$

Area of end is 28.28 sq. ft.

Step 2

Find the length by dividing the volume by this area. Since the volume is given in cubic yards, it must be reduced to cubic feet. Divide this volume by area of end

Step 2

$$10,000 \times 27 = 270,000$$
$$270,000 \div 28.28 = 9547.3+$$

Length of drain is 9547.3+ft.   Ans.

PRACTICE PROBLEMS

1. What must be the diameter of a tank with a circular base if it contains 2000 cubic feet and is 20 feet high?   Ans.   11.28 ft.

2. A cistern is 6 feet in diameter and 8 feet deep. How many gallons of water will it hold? Ans. 1697.1 gals.

3. Find the number of square inches of material necessary to construct a box (including the lid) if the base is circular and the material is used two-ply thick in making the base. The box is 5 inches high and has a diameter of 3 inches. Ans. 68.32+ sq. in.

4. Assume a cylindrical shaped post measuring 72 inches long and having a radius of 4 inches. If a hole 3 inches in diameter was bored through the center of the post, what would be the volume of the part remaining? Give answer in cubic inches.

Ans. = 3110.50 cu. inches

Lesson 4

For Step 1, recall what you learned in Book No. 12 about triangles, because the faces of a pyramid are triangles. For Step 2, study the construction of pyramids and the rules for finding their area and volume. For Step 3, work the Illustrative Examples. For Step 4, solve the Practice Problems.
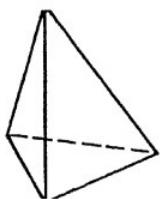
Fig. 11. Triangular Pyramid   Fig. 12. Quadrangular Pyramid   Fig. 13. Pentagonal Pyramid
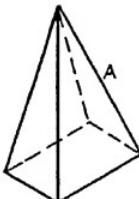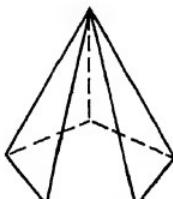
PYRAMIDS

A pyramid is a solid whose base is a polygon and whose sides are triangles. The triangles meet in a common point to form the **vertex** of the pyramid.

This means that any solid figure whose base is a polygon and whose sides are triangles such as *ADB*, in Fig. 14, or *DOC*, in Fig. 15, is a pyramid. The triangles must all meet at one point, called the vertex, such as *D* in Fig. 14.

The **altitude** of the pyramid is the perpendicular distance from the vertex to the base, such as line *OE* in Fig. 15.

Pyramids are named according to the kind of polygon forming the base, namely, triangular, Fig. 11, quadrangular, Fig. 12, pentagonal, Fig. 13, hexagonal, Fig. 14.

A **regular pyramid** is one whose base is a regular polygon and whose vertex lies in a perpendicular erected at the center of the base, Figs. 13, 14, and 15.
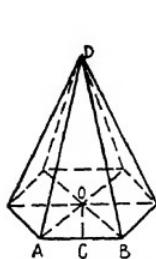
Fig. 14. Hexagonal Pyramid
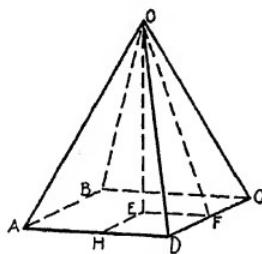
Fig. 15. Pyramid Showing Altitude and Slant Height
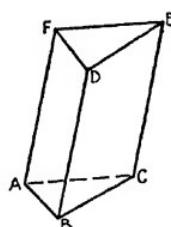
Fig. 16. Triangular Prism

This means that if a line from the center of the base and at right angles to the base is erected, the vertex of the pyramid must be on this perpendicular line in order to call the pyramid *regular*.

The **slant height** of a regular pyramid is a line drawn on a side from the vertex and perpendicular to a side line of the base. (See the line *OF*, Fig. 15.) In other words, it is the altitude of one of the triangles which form the sides.

The **lateral edges** of a pyramid are the intersections of the triangular sides.

The triangles forming the sides of the pyramid are called the **faces.**

AREAS OF PYRAMIDS

The lateral area is the combined area of all the triangles forming the sides. Now, the base of each of these triangles is one side of the base of the pyramid and the altitude of each of these triangles is the slant height of the pyramid, since the slant height is perpendicular to the base. (Distinguish carefully between the altitude of the pyramid

itself and the altitude of one of its faces.) In Fig. 15, the altitude of the pyramid itself is shown by the dotted line *EO*, while the altitude of one of the sides or faces is shown by the dotted line *OF*.

Rule (6).   *The lateral area of a pyramid is equal to the perimeter of the base multiplied by one-half the slant height.*

This rule seems right when we remember that the area of a triangle is the base times ½ the altitude. In a pyramid where there are several triangles, we add all the bases and multiply this sum by ½ the slant height.

If the slant height is not given, we can easily find it by the law of the right triangle. For example, suppose in Fig. 15 we know the length of each side of the base and the length of the altitude or line *OE*. Then, line *EF* is equal to one-half line *AD* because *E* is the center point of the bases. Thus, knowing lengths of *EF* and *EO* is the right triangle of *OEF*, we can easily find the hypotenuse or slant height of *OF*.

Keep in mind that lateral area means only the area of all the sides or faces and does not include the area of the base.

The total area of a pyramid is equal to the sum of the lateral area and the area of the base. The method of finding the area of the base will be determined by the shape of the base.

VOLUMES OF PYRAMIDS

A study of Fig. 16 will show that a triangular prism may be divided into three equal pyramids—*DABC, CEFD,* and *ACDF.*

We have already learned, **Rule (3)**, that the volume of a prism is found by multiplying the area of the base by the altitude. Fig. 14 is a prism so this would be true of it. In Fig. 17, we see that the prism is divided into three separate pyramids. Each of the pyramids in Fig. 17 is one-third of Fig. 16. So, the following rule will be clear.

Rule (7).   *The volume of a pyramid is one-third its base area times its altitude.*

It should be remembered that there are variations of **Rule (7).** For example, if we knew the volume and the altitude, we could find one-third of base area by dividing the volume by the altitude.

The student should study over these rules and variations until he feels sure he thoroughly understands them in order to prevent confusion at a later time in his studies.
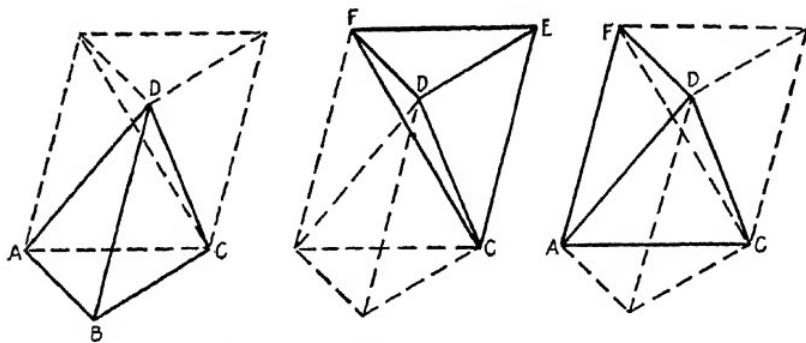
Fig. 17. Showing the three prisms that make up the prism shown in Fig. 16.

ILLUSTRATIVE EXAMPLES

1. Find the lateral area of the pyramid shown in Fig. 15. Its altitude OE is 12 feet and each side of the square base is 8 feet.

Instruction	Solution	Operation

Step 1

Step 1

The rule for finding the lateral area involves the slant height, so our first step is to find the slant height. From the center of the square base to the center of one side is half one side. So the line EF is 4 feet. The line OE is 12 feet and OEF is a right triangle, of which the slant height OF is the hypotenuse.

Apply the law of the right triangle

$$(OF)^2 = 4^2 + 12^2$$
$$(OF)^2 = 16 + 144 = 160$$
$$OF = \sqrt{160} = 12.65-$$

Slant height = 12.65 ft.

Step 2

Find the lateral area by applying **Rule (6)**. Since the base of the pyramid is a square, 8 feet on each side, its perimeter is 32 ft.

Step 2

$$\text{Area} = 32 \times \tfrac{1}{2} \text{ of } 12.65$$
$$= 202.4$$

Lateral area is 202.4 sq. ft. **Ans.**

2.   The volume of a pyramid with a square base is 135 cubic feet.   Its altitude is 15 feet.   Find the perimeter of its base.

Instruction   **Solution**   *Operation*

Step 1

Step 1

Since the volume of this pyramid (135 cu. ft.) is one-third of the product of the altitude and the base area, **Rule (7)**, then the product of the base area and the altitude is three times 135

$$135 \times 3 = 405$$

Step 2

Step 2

Since the base area multiplied by the altitude is 405 cu. ft. and the altitude is 15 feet, the base area will equal 405 divided by 15 Base area is 27 sq. ft.

$$405 \div 15 = 27$$

Step 3

Step 3

The base is a square, so we can find the length of one side by taking the square root of the area One side of the base of the pyramid is 5.196 ft.

$$\sqrt{27} = 5.196$$

Step 4

Step 4

The perimeter of the square base will be four times the length of one side

$$5.196 \times 4 = 20.784$$

Perimeter of base of pyramid is
20.784 ft. Ans.

    3. Find the volume of a regular hexagonal pyramid. One side
of the base measures 6 feet and the altitude of the pyramid is 12 feet.

Solution

Instruction	*Operation*

Step 1   **Step 1**

Refer to Fig. 14. Before we
can find the volume we must
know the area of the hexagon
that forms the base of the pyra-
mid. Recall what you learned
about hexagons in Book No. 12.
The triangle AOB is equilateral,
each side being 6 feet. The
apothem OC bisects the base, so
that CB will measure 3 feet, and
OCB is a right triangle.

Apply law of right triangle to
find apothem OC

$$(OC)^2 = (OB)^2 - (CB)^2$$
$$(OC)^2 = 6^2 - 3^2$$
$$(OC)^2 = 36 - 9 = 27$$
$$OC = \sqrt{27}$$
$$OC = 5.196$$

OC (apothem) $= 5.196$

Step 2   **Step 2**

The area of a hexagon is three
times one side multiplied by the
apothem. Three times one side
is 18 feet

$$18 \times 5.196 = 93.528$$

Area of base of pyramid is 93.528 .

Step 3   **Step 3**

Find the volume of the pyramid
by taking one-third of the

product of the base area and
the altitude $\frac{1}{3} \times 93.528 \times 12 = 374.112$
Volume is 374.112 cu. ft. Ans.

PRACTICE PROBLEMS

1. Find the total area of a pyramid with a square base measuring 3 feet on a side if the slant height is 9 feet. Ans. 63 sq. ft.

2. Find the volume of a pyramid whose base is an equilateral triangle 12 feet on each side and whose altitude is 50 feet. Ans. 1039+cu. ft.

L e s s o n   5

For Step 1, bear in mind what you have learned about circles. For Step 2, note the form of a cone and learn how to find its area and volume. For Step 3, study the Illustrative Examples. For Step 4, solve the Practice Problem.

CONES
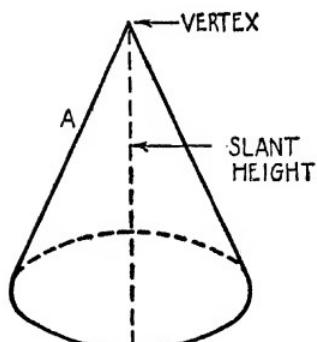
Fig. 18. Circular Cone
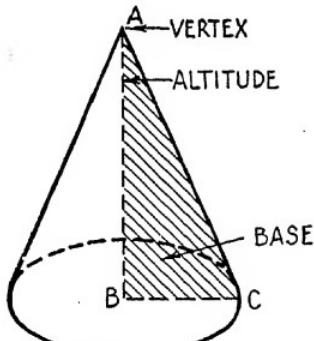
Fig. 19. Cone of Revolution

A cone is a solid whose base is a circle and whose surface tapers from the base to a point called the vertex or top, Figs. 18 and 19. A cone may be considered a pyramid with so many sides or faces that each face would be so small as to make actual counting impossible and so small that the surface or lateral area would appear smooth.

The **altitude** of a cone is the perpendicular distance from the vertex to the base.

The **slant height** is the distance from the vertex to any point on the circumference of the base.

The **lateral area** of a cone is the area of the tapering side.

AREAS AND VOLUMES OF CONES

The rules for finding areas and volumes of cones are the same as the rules for finding areas and volumes of pyramids. The reason for this, as already explained, is because a cone may be considered a pyramid with an unlimited number of sides.

Rule (8). *The lateral area of a cone is found by multiplying the circumference of the base by one-half the slant height.*

Rule (9). *The volume of a cone is one-third of the product of its base area and altitude.*

ILLUSTRATIVE EXAMPLES

1. A conical steeple is 100 feet high and the base is 25 feet in diameter. Find the cost of painting its lateral surface at 40 cents per square yard.

<div align="center">Solution</div>

Instruction	*Operation*

Step 1

The surface to be painted is the lateral area of the cone. So, first, find the circumference of the base by the formula $C = \pi d$

Step 1

$$C = 3.1416 \times 25$$
$$= 78.54$$

Circumference = 78.54 ft.

Step 2

Find the slant height, using the law of the right triangle. (Refer to Fig. 19.) One side of the right triangle is the altitude (100 ft.). The other side of the triangle is half of the diameter, or 12.5 ft. The hypotenuse is the slant height.

Step 2

Apply law of right triangle

$$(\text{Slant height})^2 = 100^2 + 12.5^2$$
$$= 10000 + 156.25$$
$$= 10156.25$$
$$\text{Slant height} = \sqrt{10156.25}$$
$$= 100.77$$

Step 3

Find the lateral area by **Rule (8)**. Multiply the circumference of the base by half the slant height

Step 3

$$78.54 \times \frac{100.77}{2} = 3957.2379$$

Area to be painted $= 3957.24-$ sq. ft.

Step 4

Reduce area in square feet to square yards by dividing by 9
Find cost at 40 cents per sq. yd.
Required cost is $175.87. Ans.

Step 4

$$3957.24 \div 9 = 439.69$$
$$439.69 \times 40 = 17587.60$$

2. Both the circumference of the base and the slant height of a cone are 25 inches. Find its volume.

Solution

Instruction

Operation

Step 1

Finding volume requires both altitude and the base area. Neither are given, so we must find them. (Refer to Fig. 19.) First, we will find the radius so we can find base area. If $\pi \times d =$ circumference, then d will equal circumference divided by π. This is a variation of the formula for finding circumference. Having found d we know $r = \frac{1}{2}$ of d.

Radius $= 7.95 \div 2 = 3.98$

Step 1

$$\pi d = \text{Circumference}$$
$$d = \text{Cir.} \div \pi$$
$$d = 25 \div 3.1416$$
$$= 7.95$$

Step 2
Find area of the base. The base is a circle. From Section 12 we know that area $= \pi r^2$. We found the radius in Step 1.

Step 2

$$\text{Area} = \pi r^2$$
$$= 3.1416 \times (3.98)^2$$
$$= 49.7642$$

Step 3
Now we can find the altitude of the cone by law of right triangle. In Fig. 19, we know the slant height $(AC) = 25$ inches.
This is the hypotenuse of a right triangle ABC. The radius (BC) is 3.98 as we have already found. We wish to find the altitude or line AB. Apply law of right triangle and solve.
Altitude $= 24.6$

Step 3

$$(\text{Altitude})^2 = 25^2 - (3.98)^2$$
$$= 625 - 15.84$$
$$= 609.16$$
$$\text{Altitude} = \sqrt{609.16}$$
$$= 24.6$$

Step 4
Find the volume of the cone by **Rule (9)**. It is one-third of the product of base area and altitude
Volume of cone is 408.06 cu. in. Ans.

Step 4

$$\tfrac{1}{3} \times 49.7642 \times 24.6 = 408.06 +$$

PRACTICE PROBLEM

1. Find the volume and lateral area of a cone 20 feet high, the radius of the base being 5 feet. Ans. Volume is 523.6 cu. ft.; lateral area is 323.58+sq. ft.

Lesson 6

For Step 1, bear in mind what you have learned about pyramids and cones. For Step 2, learn the meaning of "frustum" and the method of finding its lateral area and volume. For Step 3, work the Illustrative Examples. For Step 4, work the Practice Problems.

FRUSTUMS

Refer to Figs. 12 and 18. Suppose that we could, starting at the point A in each figure, saw or cut off the top making sure that the line on which we cut was parallel to the base in each case. The top portion can be moved away. We would then, after cutting Fig. 12, have a figure such as Fig. 20 and after cutting Fig. 18 we would have left a figure such as Fig. 21. These parts (Figs. 20 and 21) are called frustums.
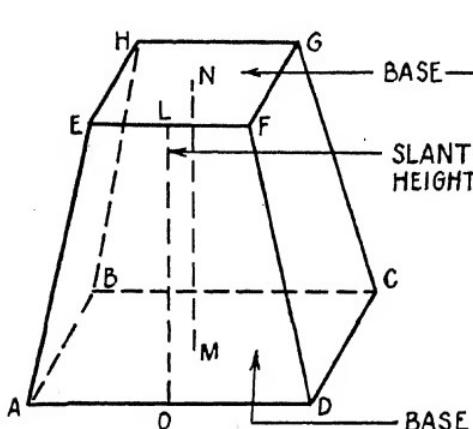
Fig. 20.   Frustum of Pyramid   Fig. 21.   Frustum of Cone
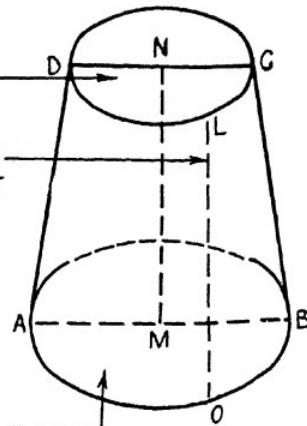
The **altitude** of a frustum is the length of the perpendicular between bases, as lines M N of Fig. 20.

The **slant height** of a frustum is the shortest distance between the perimeters of the bases and is shown by lines OL.

LATERAL AREAS OF FRUSTUMS

In Fig. 20, line OL is the altitude of the trapezoid $AEFD$; therefore, the lateral area of the frustum is equal to the sum of the areas of the four trapezoids composing its faces. Note that the altitude of the trapezoid is the slant height of the frustum. Since the area of one trapezoid is equal to one-half the sum of the bases times the altitude (Section 12), the rule for the lateral area of a frustum is:

Rule (10).   *The lateral area of the frustum of a right pyramid equals one-half the sum of the perimeters of the two bases times the slant height.*

Since a cone may be considered as a pyramid with sides so numerous and so small that the surface appears smooth, a similar rule will be used for finding the lateral area of a frustum of a cone.

Rule (11). *The lateral area of the frustum of a cone is found by multiplying half the sum of the circumferences of the two bases by the slant height.*

The total area of a frustum is the sum of the lateral area and the two bases.

VOLUMES OF FRUSTUMS

The explanation for the rule for finding the volume of a frustum of a pyramid or cone is too difficult to be introduced here. Only the rule and its application in a problem will be given.

Rule (12). *To find the volume of a frustum take the sum of the areas of the two bases; to this add the square root of the product of the two bases; multiply the result by one-third of the altitude.*

ILLUSTRATIVE EXAMPLES

1. Find the lateral area of the frustum of a cone, the slant height of the frustum being 42 feet and the radii of the bases are 12 feet and 4 feet, respectively.

Solution

Instruction	*Operation*
Step 1	**Step 1**
Apply Rule (11). Find the circumference of larger base	$C = 2 \times 3.1416 \times 12$ $= 75.3984$
Step 2	**Step 2**
Find the circumference of smaller base	$C = 2 \times 3.1416 \times 4$ $= 25.1328$

Step 3
Find half the sum of these circumferences

Step 3
$$75.3984 + 25.1328 = 100.5312$$
$$100.5312 \div 2 = 50.2656$$

Step 4
Find lateral area by multiplying by 42
Lateral area of frustum is
2111.15+ sq. ft.   Ans.

Step 4
$$50.2656 \times 42 = 2111.1552$$

2.   What is the volume of the frustum of a square pyramid, the sides of whose bases are 2 feet and 8 feet, the altitude of the frustum being 15 feet.

Solution

Instruction   *Operation*

Step 1
Apply Rule (12).
Find area of the bases, each base being a square

Step 1
$$2 \times 2 = 4$$
$$8 \times 8 = 64$$

Step 2
Find the square root of the product of the two bases

Step 2
$$\sqrt{4 \times 64} = \sqrt{256} = 16$$

Step 3
Find sum of the two bases and this root

Step 3
$$4 + 64 + 16 = 84$$

Step 4
Multiply the sum by one-third of altitude

Step 4
$$84 \times \tfrac{1}{3} \text{ of } 15$$
$$84 \times 5 = 420$$

Volume of frustum = 420 cu. ft.   Ans.

PRACTICE PROBLEMS

1. Find the entire surface area of the frustum of a cone whose slant height is 50 feet and the radii of whose bases are 10 feet and 5 feet. Ans. 2748.9 sq. ft.

2. What is the volume of a frustum of a cone 24 feet in altitude if the radius of its top is 7 feet and the radius of the bottom 14 feet? Use $\pi = \dfrac{22}{7}$ Ans. 8624 cu. ft.

L e s s o n 7

For Step 1, recall what you have learned about circles. For Step 2, note the shape of a sphere and learn the rules for finding its surface area and volume. For Step 3, work the Illustrative Examples. For Step 4, solve all the Practice Problems.

SPHERES

A **sphere** is a solid bounded by a curved surface every point of which is equally distant from a point within called the **center.** In other words, it is a perfectly round ball, Fig. 22.
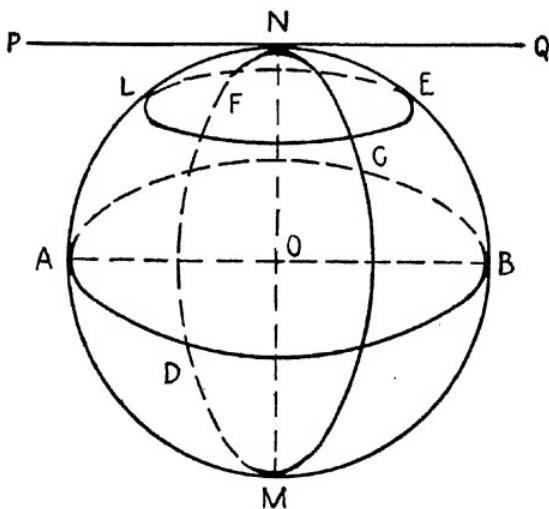
Fig. 22. Sphere

The **diameter** is a straight line drawn through the center and having its extremities in the curved surface, as AB.

The **radius** is a straight line from the center to a point on the surface; it is equal to one-half the diameter. *OM, ON, OA, OB* are all radii.

A plane is **tangent** to a sphere when it touches the sphere at only one point, as plane *PNQ* touching at *N*. The plane *PNQ* can be thought of as a large sheet of paper touching the circle at *N*.

When a plane cuts through a sphere, the section is a circle such as plane *ACBD*.

When the plane cuts through the center of the sphere, the resulting section is called a **great circle**. *A NBM, ACBD,* and *NCMD* are great circles.

When the plane does not pass through the center, the section is called a **small circle**, as *LCEF*.

The **circumference** of a sphere is the same as the circumference of a great circle.

AREAS OF SPHERES

It is proven by Geometry that the following statement is true: The area of the surface of a sphere equals four times the area of one of its great circles.

Now we learned in Section 12 that the area of a circle is found by multiplying the square of the radius by π. We can therefore state that the surface of a sphere is four times the square of the radius multiplied by π.

Put in formula, this would be stated $S = 4\pi r^2$, where S represents the surface of the sphere and r the radius.

Now r can be expressed as $\dfrac{d}{2}$; so if we substitute $\dfrac{d}{2}$ for r in the formula, we get $S = 4\pi \left(\dfrac{d}{2}\right)^2$ or $S = 4\pi \dfrac{d^2}{4}$ which reduces to $S = \pi d^2$. This is the formula most generally used for finding the surface of a sphere and is perhaps the most easily remembered.

Rule (13). *The surface of a sphere is π times the square of the diameter.*

VOLUMES OF SPHERES

The following rule can be proved by Solid Geometry:

Rule (14). *The volume of a sphere equals the area of the surface multiplied by one-third of the radius.*

This can be expressed in formula in this way: $V = \frac{1}{3}Sr$, where V represents the volume, S the surface, and r the radius. But we know that $S = 4\pi r^2$, therefore $V = \frac{1}{3}$ of $4\pi r^2 \times r$ (substituting $4\pi r^2$ for S), which simplifies to $V = \frac{4}{3}\pi r^3$. Again, since $r = \dfrac{d}{2}$, the formula can be put in the form

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\frac{d^3}{8} = \frac{1}{6}\pi d^3$$

Summing up the formulas pertaining to spheres, then, we have

$$S = 4\pi r^2 \quad \text{or } S = \pi d^2$$
$$V = \frac{4}{3}\pi r^3 \quad \text{or } V = \frac{1}{6}\pi d^3$$

These should be memorized so that time will not be lost in looking them up each time one is to be used.

ILLUSTRATIVE EXAMPLES

1. Find the number of square yards of silk necessary to cover a spherical balloon 50 feet in diameter.

Solution

Instruction	*Operation*
Step 1	**Step 1**
Apply the formula for finding surface area of a sphere	$S = \pi d^2$
Substitute the known values in the formula	$S = 3.1416 \times 50^2$
	$= 3.1416 \times 2500 = 7854$
Surface area is 7854 sq. ft.	
Step 2	**Step 2**
Find the number of square yards in 7854 sq. ft.	$7854 \div 9 = 872.7-$
872.7 sq. yd. Ans.	

2. The bottom of a cylindrical water tank is in the shape of a half sphere. The length of the cylindrical part is 22 feet and the diameter is 20 feet. How many gallons will the tank hold?

Solution

Instruction	*Operation*

Step 1

Recall and use the rule for finding the volume of a cylinder. We must first find the area of an end of the cylinder

Substitute the known values

Step 1

$A = \pi r^2$
$A = 3.1416 \times 10^2$
$\quad = 314.16$

Step 2

Find the volume of the cylindrical part

Step 2

Volume $= 314.16 \times 22$
$\qquad = 6911.52$

Step 3

Find the volume of the spherical part. It is a half sphere

Substitute the given value for d and π

Step 3

$V = \frac{1}{2}$ of $\frac{1}{6} \pi d^3$

$V = \frac{1}{2} \times \frac{1}{6} \times 3.1416 \times 8000$
$\quad = 2094.4$

Step 4

Find the total volume of the tank

Volume $= 9005.92$ cu. ft.

Step 4

$6911.52 + 2094.4 = 9005.92$

Step 5

Find the number of gallons (1 cu. ft. $= 7\frac{1}{2}$ gals.)

67,544.4 gals. Ans.

Step 5

$9005.92 \times 7.5 = 67544.400$

PRACTICE PROBLEMS

1. How many square feet of canvas will cover a ball 5 feet in diameter? Ans. 78.54 sq. ft.

2. The outer diameter of a spherical shell is 12 inches. The inner diameter is 8 inches. Find the contents of the shell. Ans. 636.69 cu. in.

3. Find the weight of a cannon ball 15 inches in diameter if a cubic foot weighs 450 pounds. Ans. 460.19+lb.

4. How much material is wasted in cutting the largest possible sphere from a cube 10 inches on each side? Ans. 476.4 cu. in.

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